The importance of lattice QCD for precision physics: Highlights

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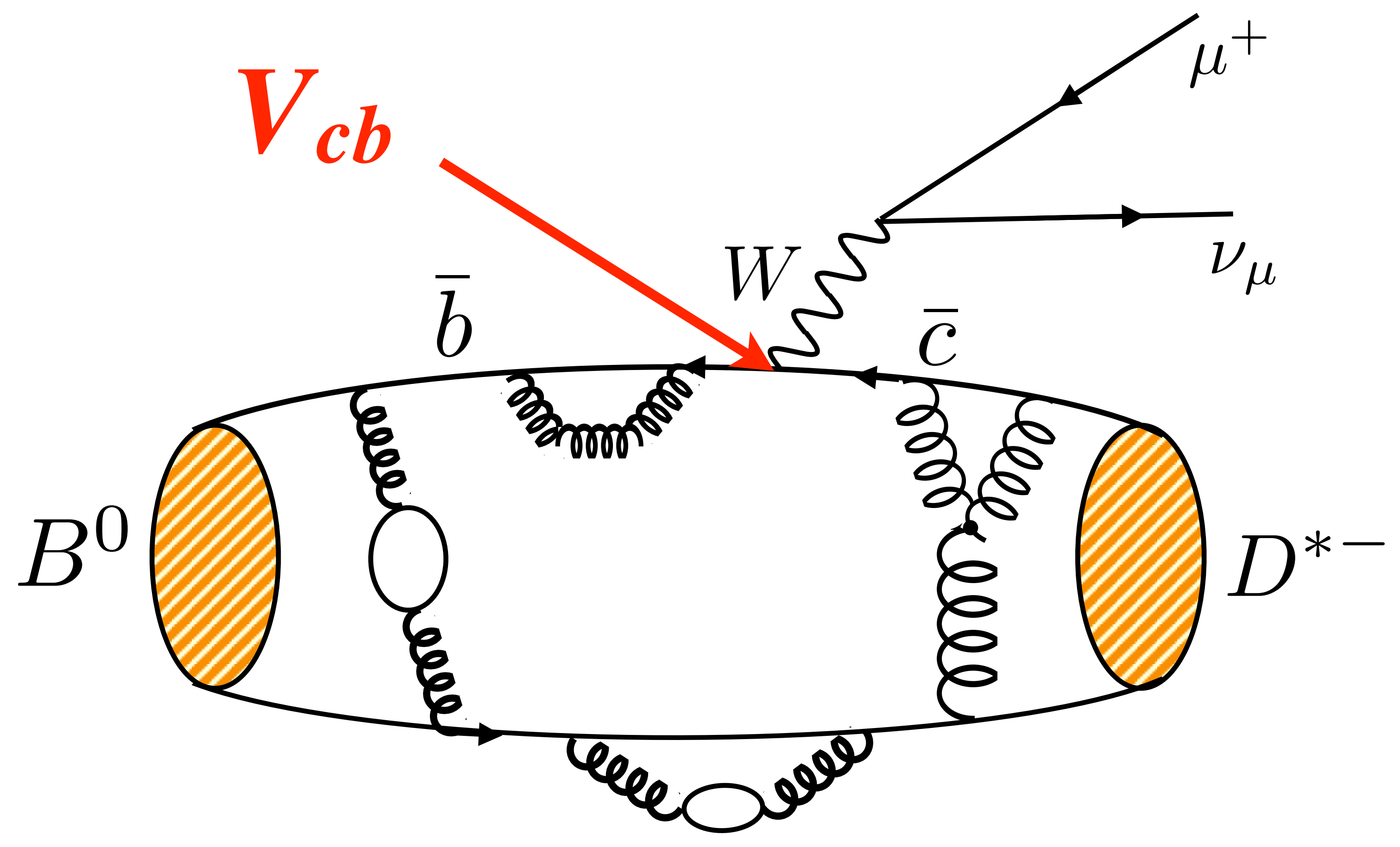
Outline

- The role of (lattice) QCD in precision physics
- Introduction to lattice QCD
- Success stories: two examples
  - $B_{s,d} \to \mu\mu$
  - inputs for Higgs decay rates (m_q, α_s)
- Puzzles: two examples
  - hadronic corrections to muon g-2
  - $|V_{cb}|$
- Summary and Outlook

The role of (lattice) QCD in precision physics

example: $B^0 \to D^{*-}\mu^+\nu_\mu$



Experiment vs. SM theory:

(experiment) = (known) x (**CKM factors**) x (had. matrix element)

$\Gamma\left(K^+ \to \ell^+\nu_\ell(\gamma)\right)$

$d\Gamma(B^0 \to D^{*-}\mu^+\nu_\mu), \ldots$

$B(B_s \to \mu\mu), \ldots$

$\Delta m_{d(s)} \quad \cdots$

The role of (lattice) QCD in precision physics

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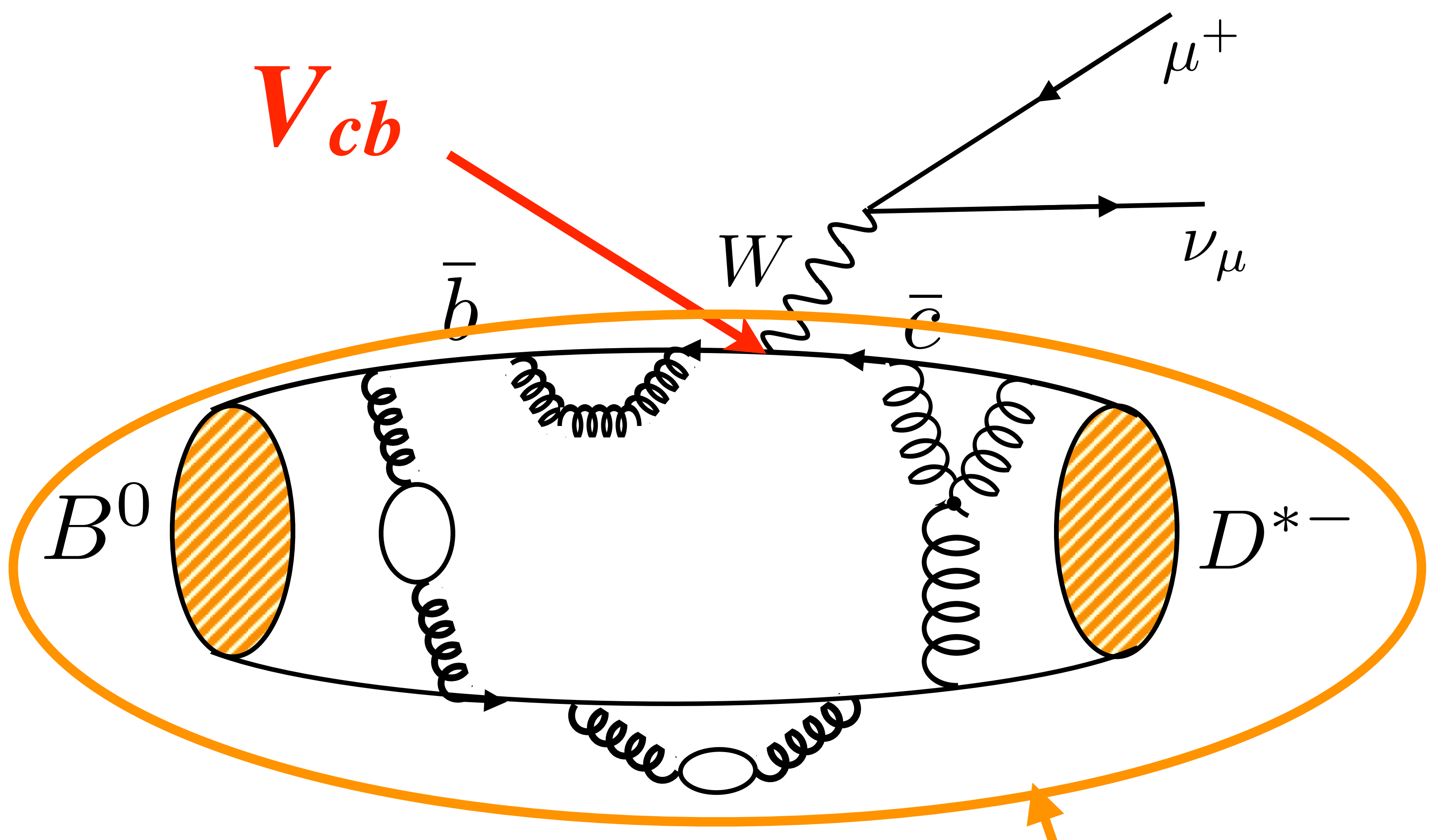

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Lattice QCD

parameterize the MEs in terms of form factors, decay constants, bag parameters, ...

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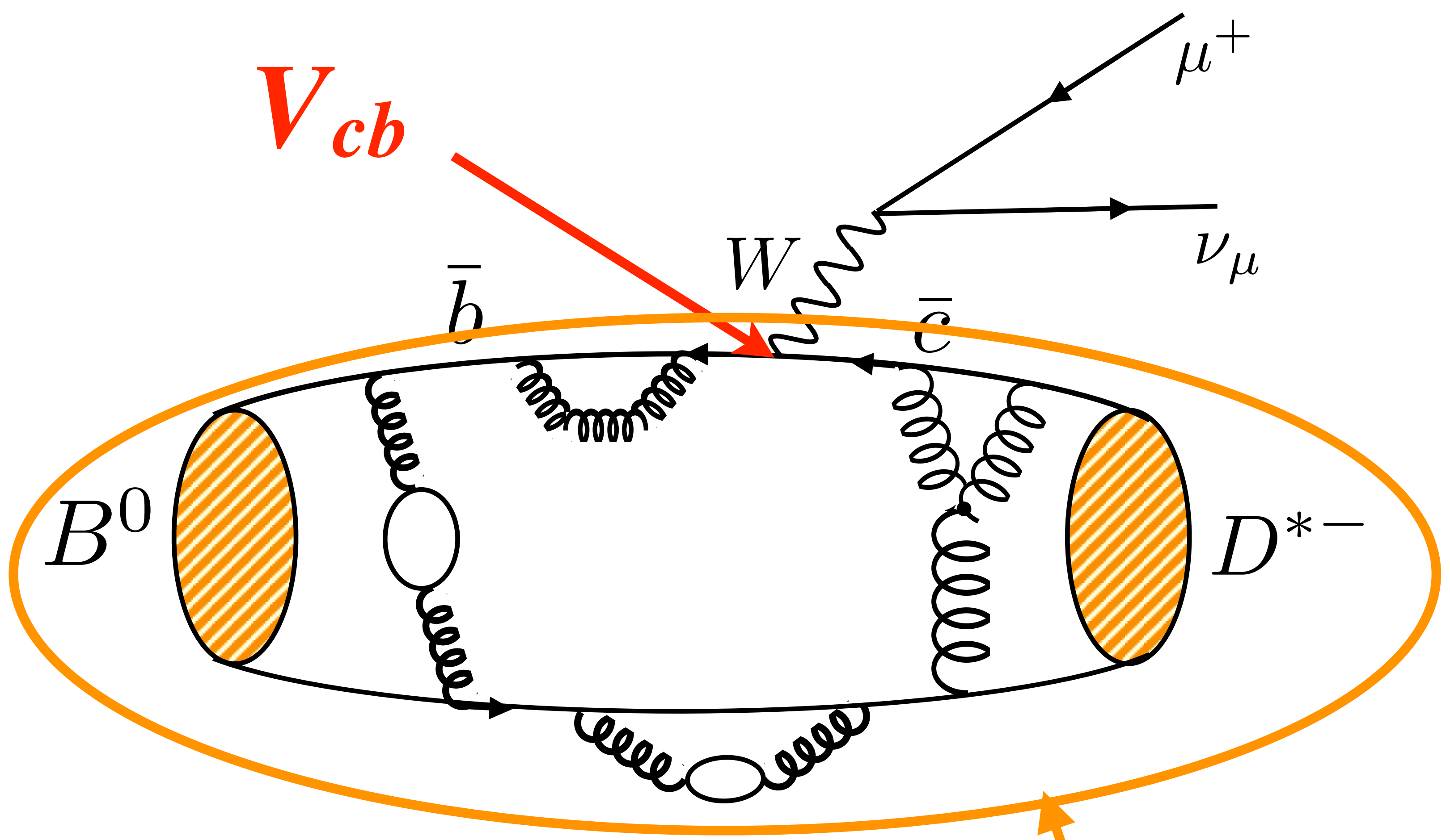

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Two main purposes:

- ✦ combine experimental measurements with LQCD results to determine SM parameters.
- ✦ confront experimental measurements with SM theory using LQCD inputs.

Lattice QCD

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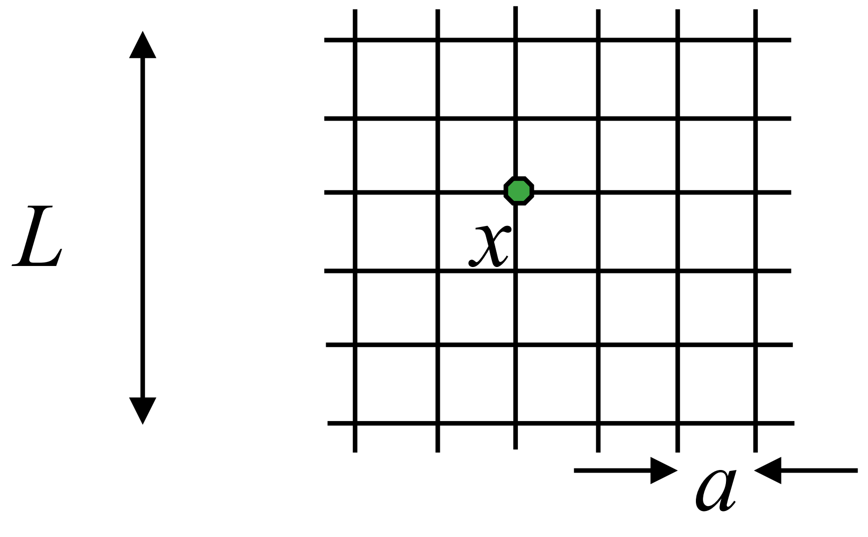
Lattice QCD Introduction

$$\mathcal{L}_{\rm QCD} = \sum_f \bar\psi_f (\not{D} + m_f)\psi_f + \frac{1}{4}{\rm tr} F_{\mu\nu}F^{\mu\nu}$$

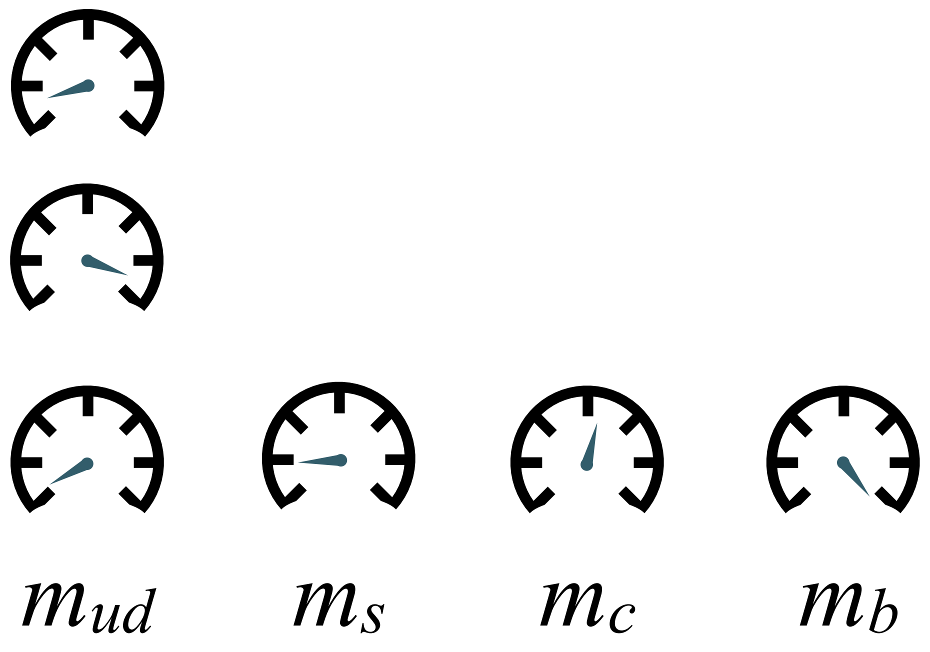


- discrete Euclidean space-time (spacing a)
  derivatives ➙ difference operators, etc…
- finite spatial volume (L)
- finite time extent (T)

Integrals are evaluated numerically using monte carlo methods.

adjustable parameters

- lattice spacing: $a \to 0$
- finite volume, time: $L \to \infty,\ T > L$
- quark masses (m_f): $M_{H,\rm lat} = M_{H,\rm exp}$
  tune using hadron masses
  extrapolations/interpolations
  $m_f \to m_{f,\rm phys}$



Extrapolations/interpolations guided by EFT description of QCD

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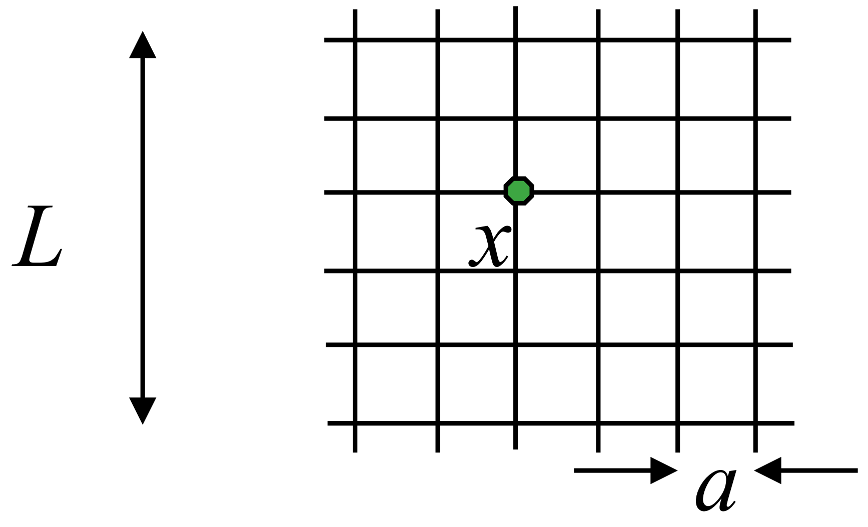

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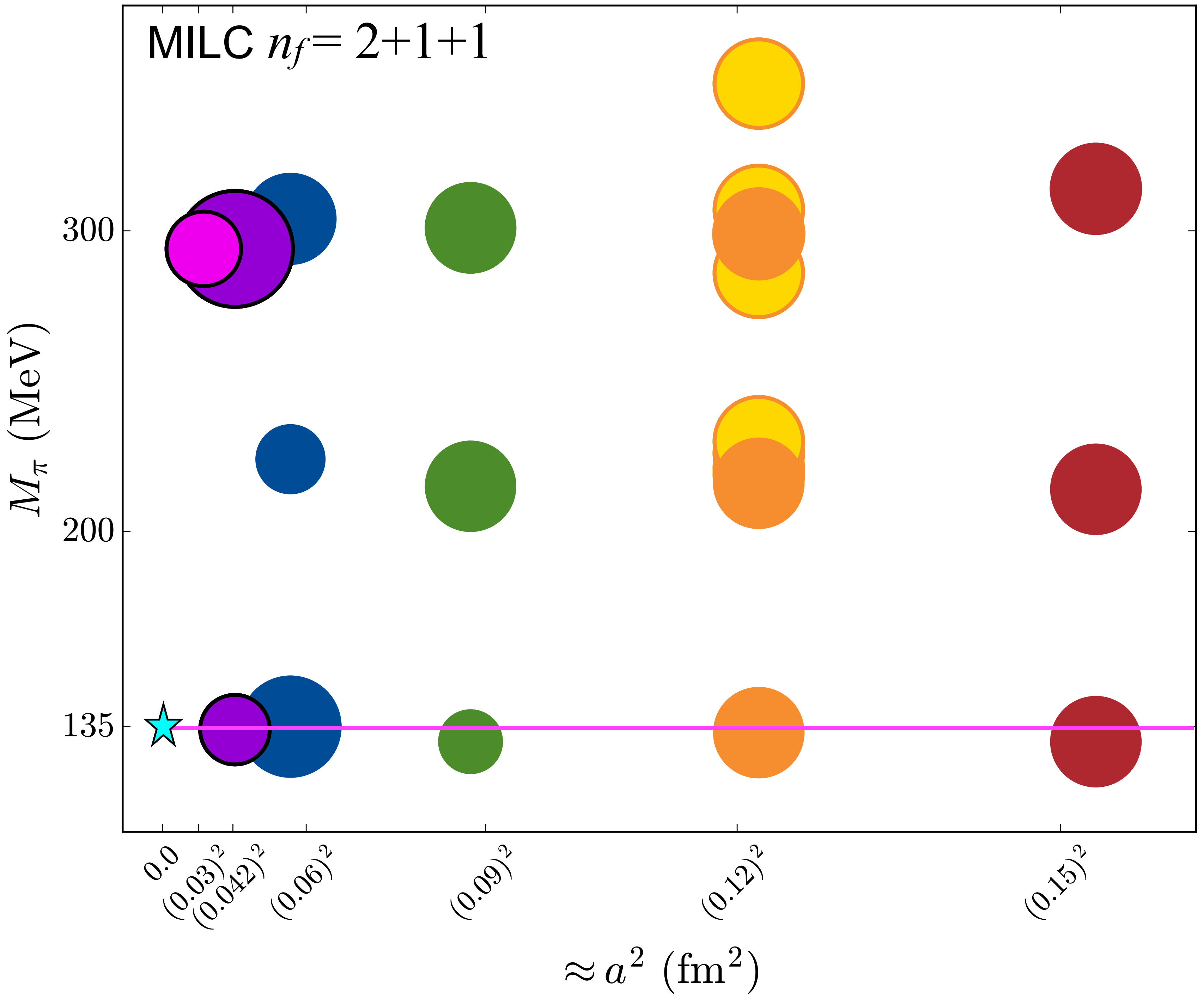

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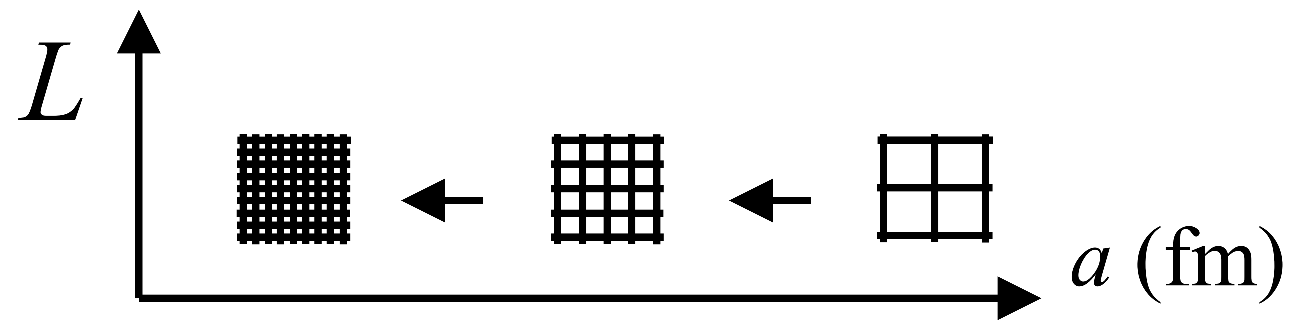

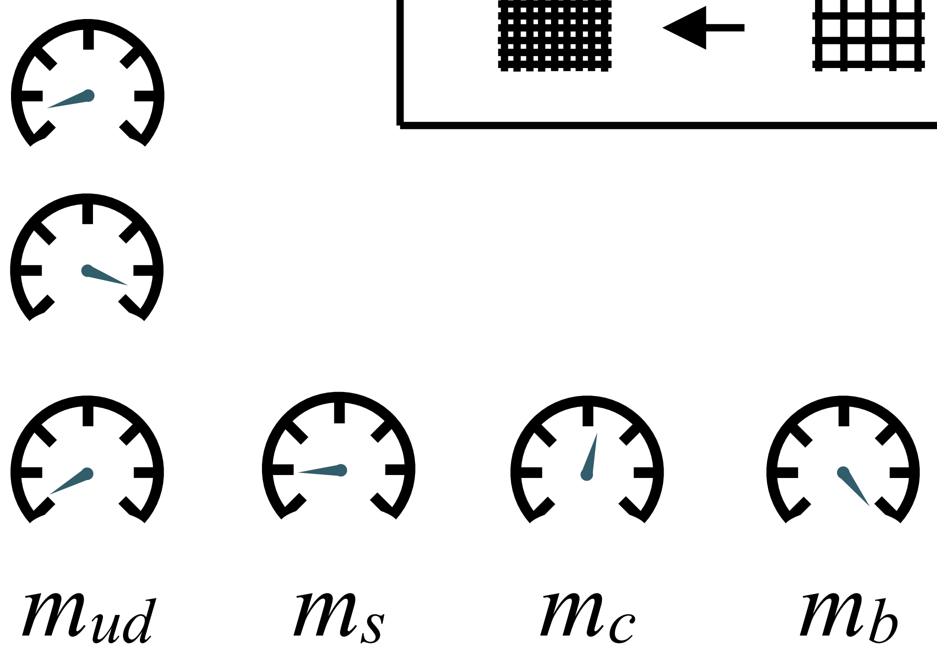

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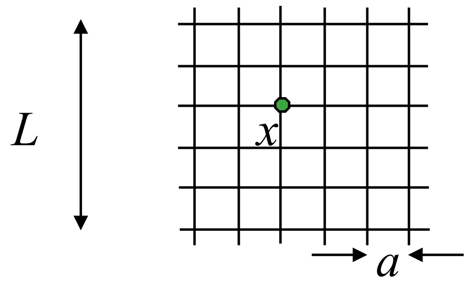
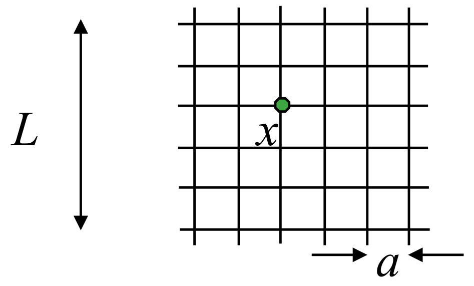


Lattice QCD Introduction

systematic error analysis

...of lattice spacing, chiral, heavy quark, and finite volume effects is based on Effective Field Theory (EFT) descriptions of QCD ➙ ab initio

- finite a: Symanzik EFT
- light quark masses: ChPT
- heavy quark effects: HQET
- finite L: finite volume EFT



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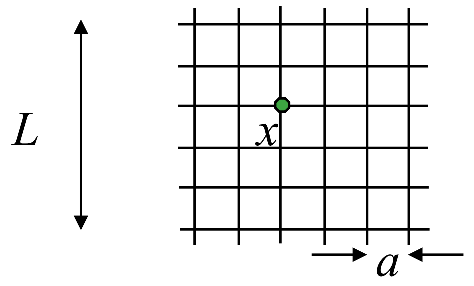
In practice:
stability and control over systematic errors depends on the underlying simulation parameters, available computational resources, analysis choices, …



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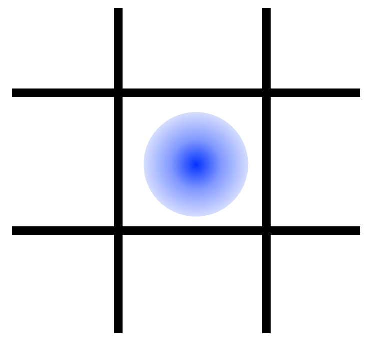
Flavor Lattice Averaging Group: S. Aoki et al [FLAG 2021 review, arXiv:2111.09849, EPJC 2022]

- quality criteria for inclusion
- averages include sys. and stat. correlations
- if using a FLAG average, please cite the underlying lattice results!

- reviews over 60 quantities
- ~ biannual schedule + web update

Finding Beauty

b quark

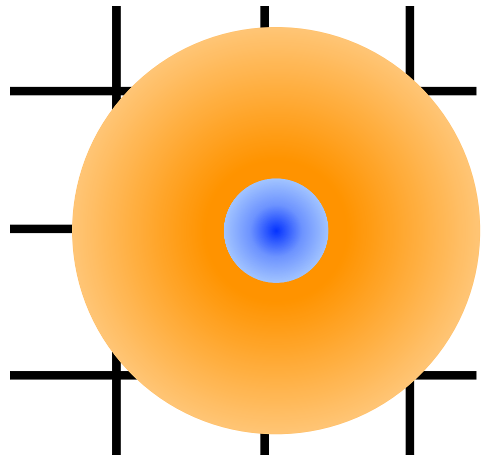


generic disc. errors $\sim (a\Lambda)^n$ (n depends on lattice action)

$m_b \gg \Lambda$ ➠ leading discretization errors $\sim (am_b)^n$ (using same action as for light quarks

If $m_b \gtrsim a^{-1}$ uncontrolled errors

B meson

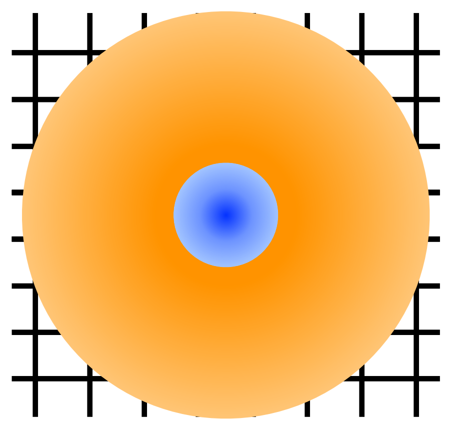


EFTs co-developed lattice / continuum

use EFT (HQET, NRQCD) ➠ Λ/m_b expansion

- lattice HQET, NRQCD: use EFT to construct lattice action
  complicated continuum limit
  nontrivial matching and renormalization ➠ (few-5)% errors
- relativistic heavy quark approach (Fermilab, Columbia)
  matching relativistic lattice action via HQET to continuum
  nontrivial matching and renormalization ➠ (1-3)% errors

Now

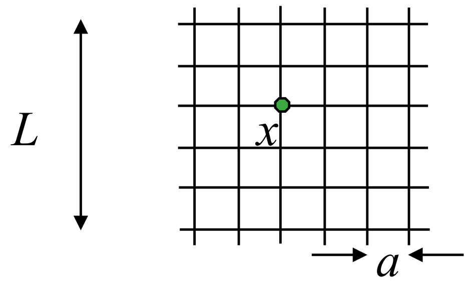


$a^{-1} \gtrsim m_b \gg \Lambda$ + highly improved light quark action

➠ same action for all quarks

➠ simple renormalization (Ward identities)

➠ < 1% errors



Lattice QCD Introduction

The State of the Art

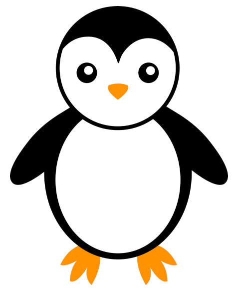
Lattice QCD calculations of simple quantities (with at most one stable meson in initial/final state) that **quantitatively account for all systematic effects** (discretization, finite volume, renormalization,…) in some cases with

- sub percent precision.
- total errors that are commensurate (or smaller) than corresponding experimental uncertainties.

Progress due to a virtuous cycle of theoretical developments, improved algorithms/methods and increases in computational resources (``Moore's law")

Scope of LQCD calculations is increasing due to continual development of new methods:

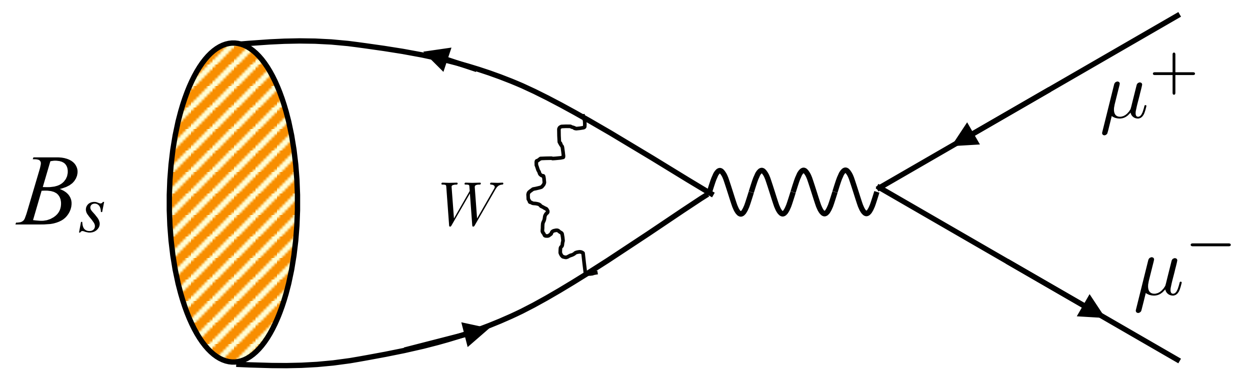
- nucleon matrix elements
- nonleptonic kaon decays ($K \to \pi\pi$, ϵ',…)
- resonances, scattering ($\pi\pi \to \rho$,…)
- long-distance effects (ΔM_K, …)
- QED corrections
- radiative decay rates
- structure: PDFs, GPDs, TMDs, …
- inclusive decay rates ($B \to X_c \ell \nu$,…)
- …



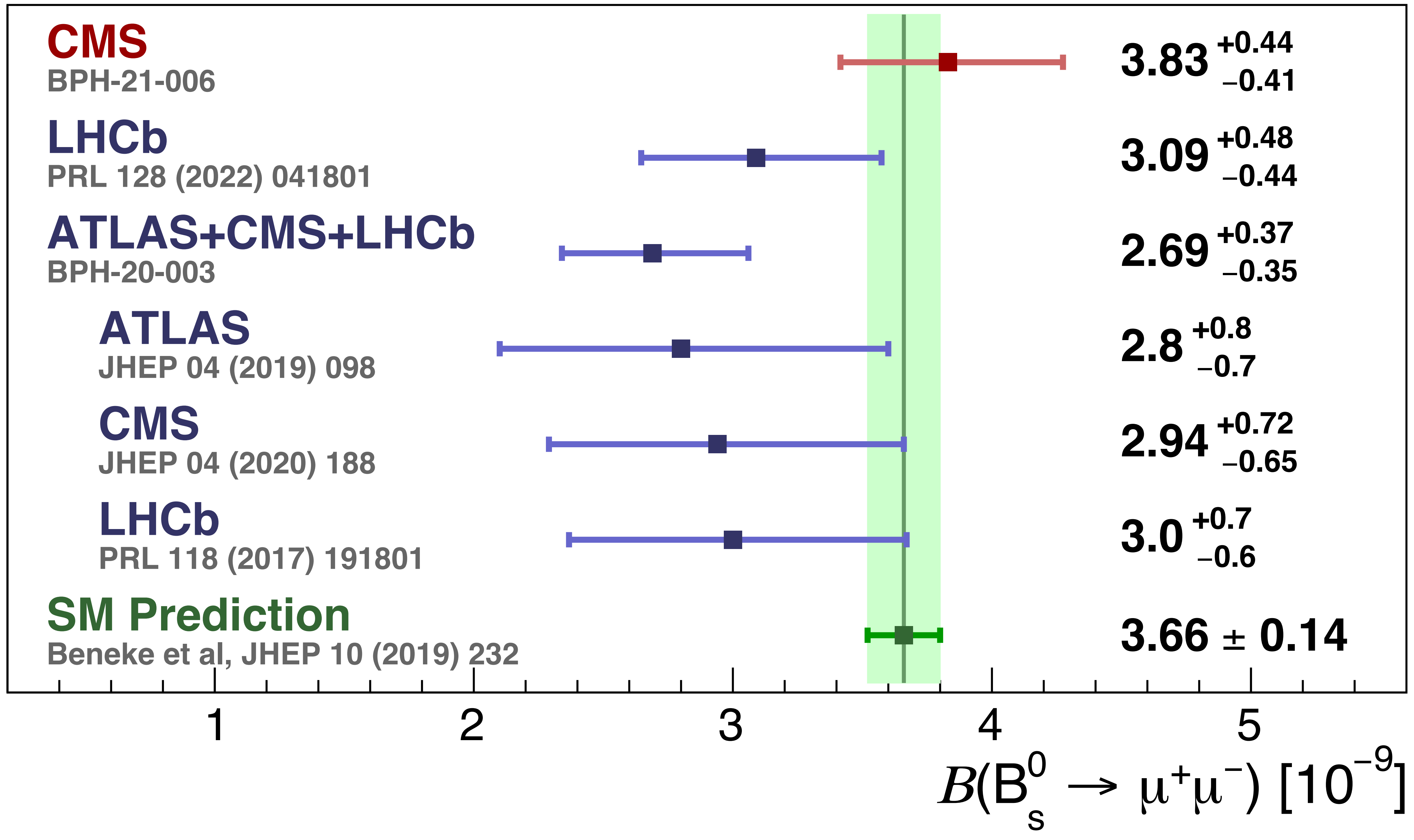
Rare leptonic decay $B_s \to \mu\mu$

SM prediction for rare leptonic decay rate

[Beneke et al, arXiv:1908.07011, JHEP 2019]



Silvano Tosi @ LHCP2023 (Tuesday, Flavor Physics)



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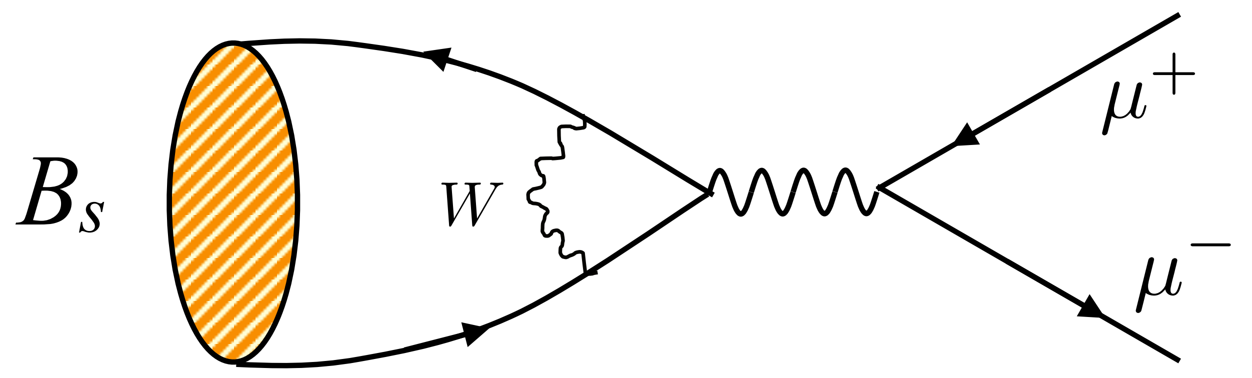

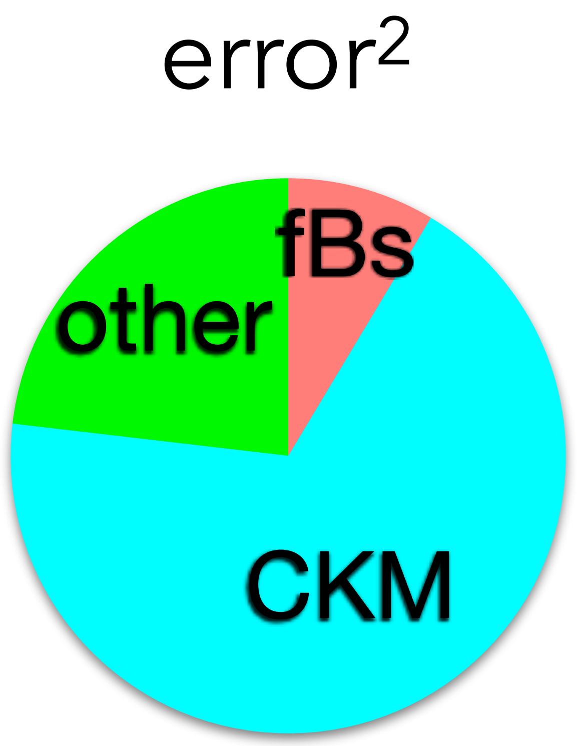

- includes structure-dependent QED corrections
- dominant uncertainty due to $|V_{cb}|$
- LQCD decay constant sub dominant source of uncertainty

S. Aoki et al [FLAG 2021 review, arXiv:2111.09849, EPJC 2022]

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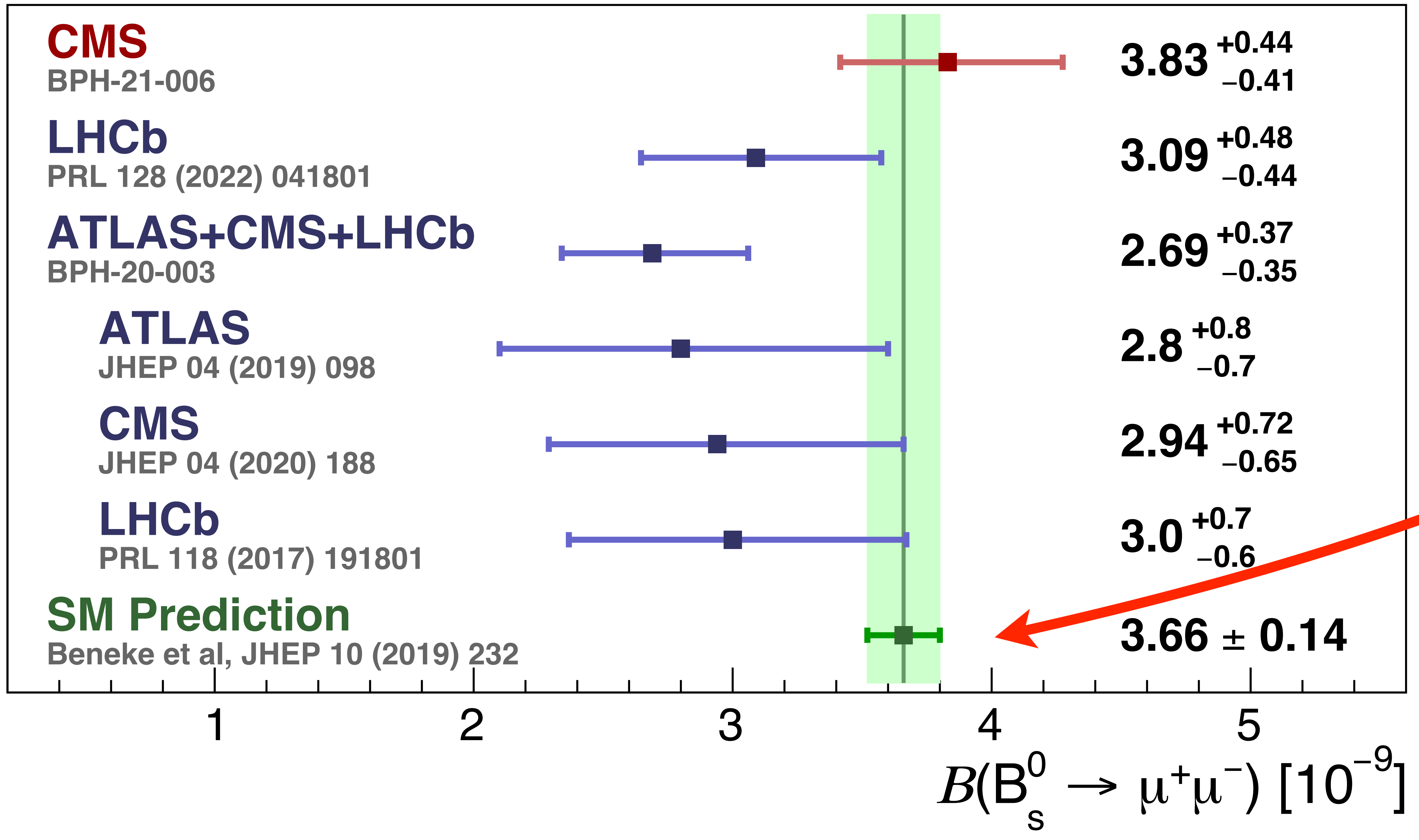

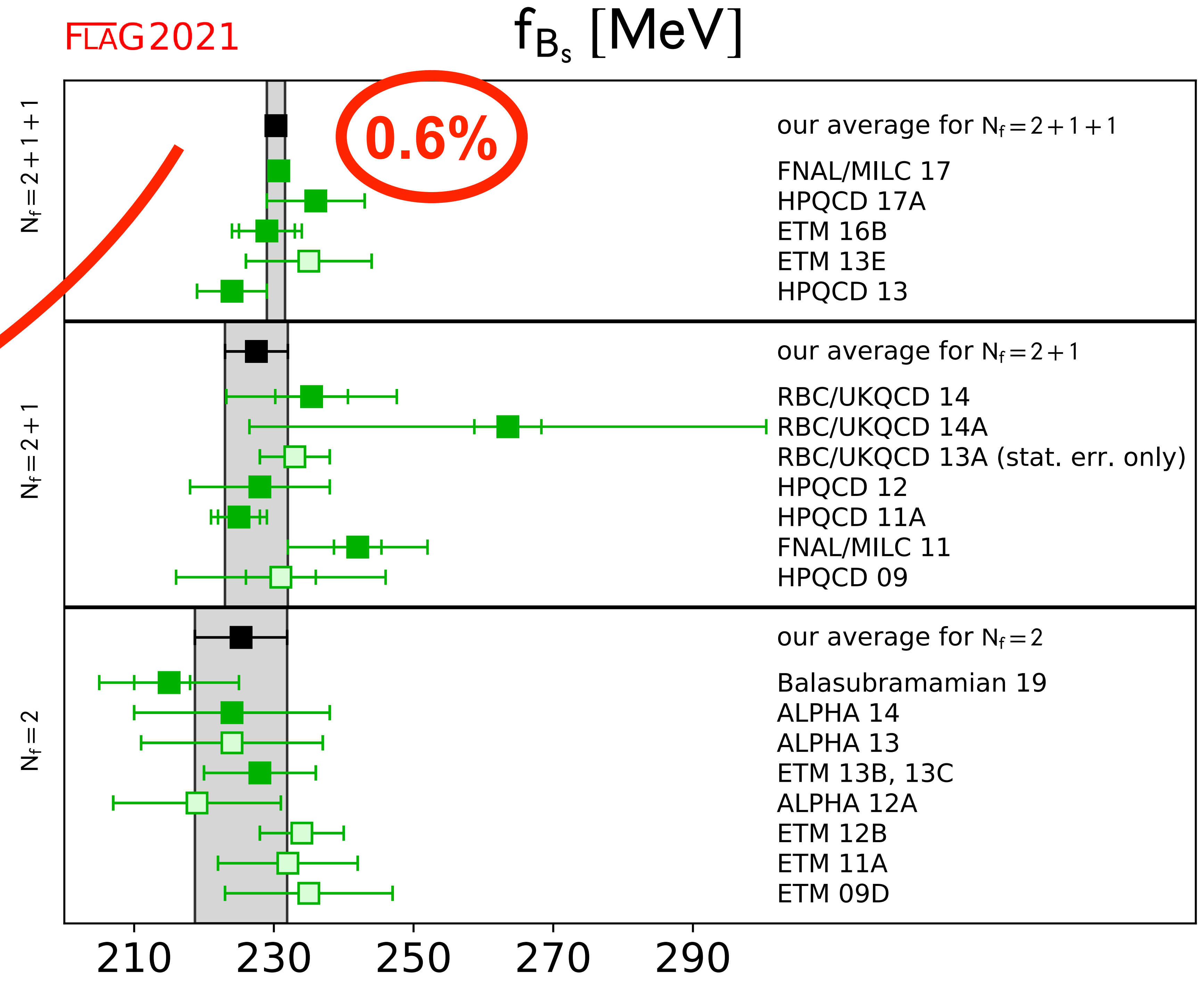

2013 ➠ present

https://www.usqcd.org/documents/13flavor.pdf and [J. Butler et al, arXiv:1311.1076]

Quantity	CKM element	2013 expt. error	2007 forecast lattice error	2013 lattice error	forecast lattice error	2021 FLAG Average
f_K/f_π	$\|V_{us}\|$	0.2%	0.5%	0.4%	0.15%	0.18 %
$f_+^{K\pi}(0)$	$\|V_{us}\|$	0.2%	–	0.4%	0.2%	0.18 %
f_D	$\|V_{cd}\|$	4.3%	5%	2%	$< 1\%$	0.3 %
f_{D_s}	$\|V_{cs}\|$	2.1%	5%	2%	$< 1\%$	0.2 %
$D \to \pi\ell\nu$	$\|V_{cd}\|$	2.6%	–	4.4%	2%	0.7 % [from 2212.12648]
$D \to K\ell\nu$	$\|V_{cs}\|$	1.1%	–	2.5%	1%	0.6 %
$B \to D^*\ell\nu$	$\|V_{cb}\|$	1.3%	–	1.8%	$< 1\%$	~1.5 % [from 2105.14019, 2304.03137]
$B \to \pi\ell\nu$	$\|V_{ub}\|$	4.1%	–	8.7%	2%	~3 %
f_B	$\|V_{ub}\|$	9%	–	2.5%	$< 1\%$	0.7 % (0.6 % for f_{B_s})
ξ	$\|V_{ts}/V_{td}\|$	0.4%	2–4%	4%	$< 1\%$	1.3 %
Δm_s	$\|V_{ts}V_{tb}\|^2$	0.24%	7–12%	11%	5%	4.5 %
B_K	$\text{Im}(V_{td}^2)$	0.5%	3.5–6%	1.3%	$< 1\%$	1.3 %

QED threshold:
QED corrections important/ dominant source of theory error in SM predictions

- Toby Tsang@LHCP (B mixing, Tuesday, Flavor Physics)
- Andreas Jüttner@LHCP (SL B-meson form factors, Friday, Flavor Physics)

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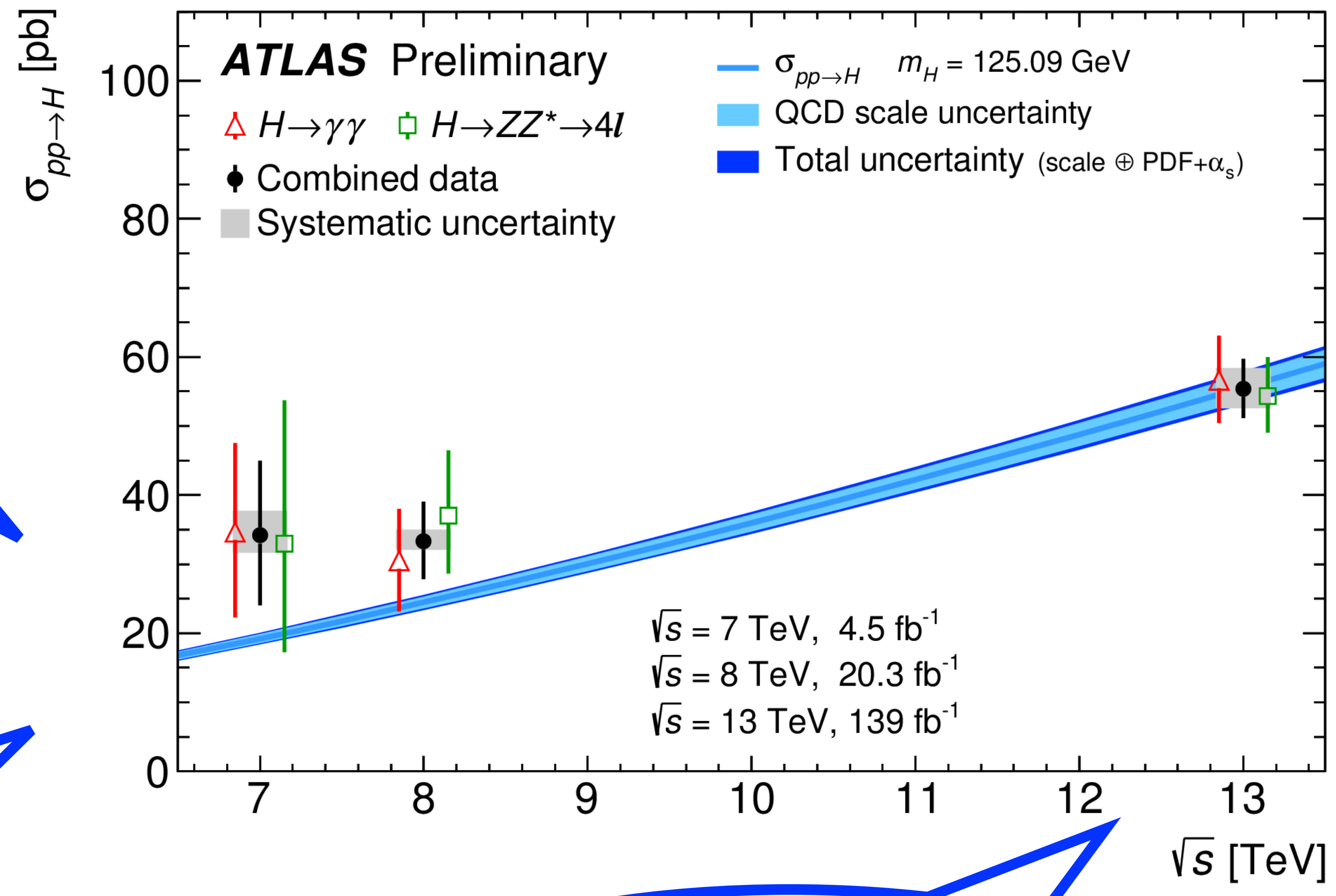
Higgs production and decay

Radja Boughezal @ P5 SLAC town hall

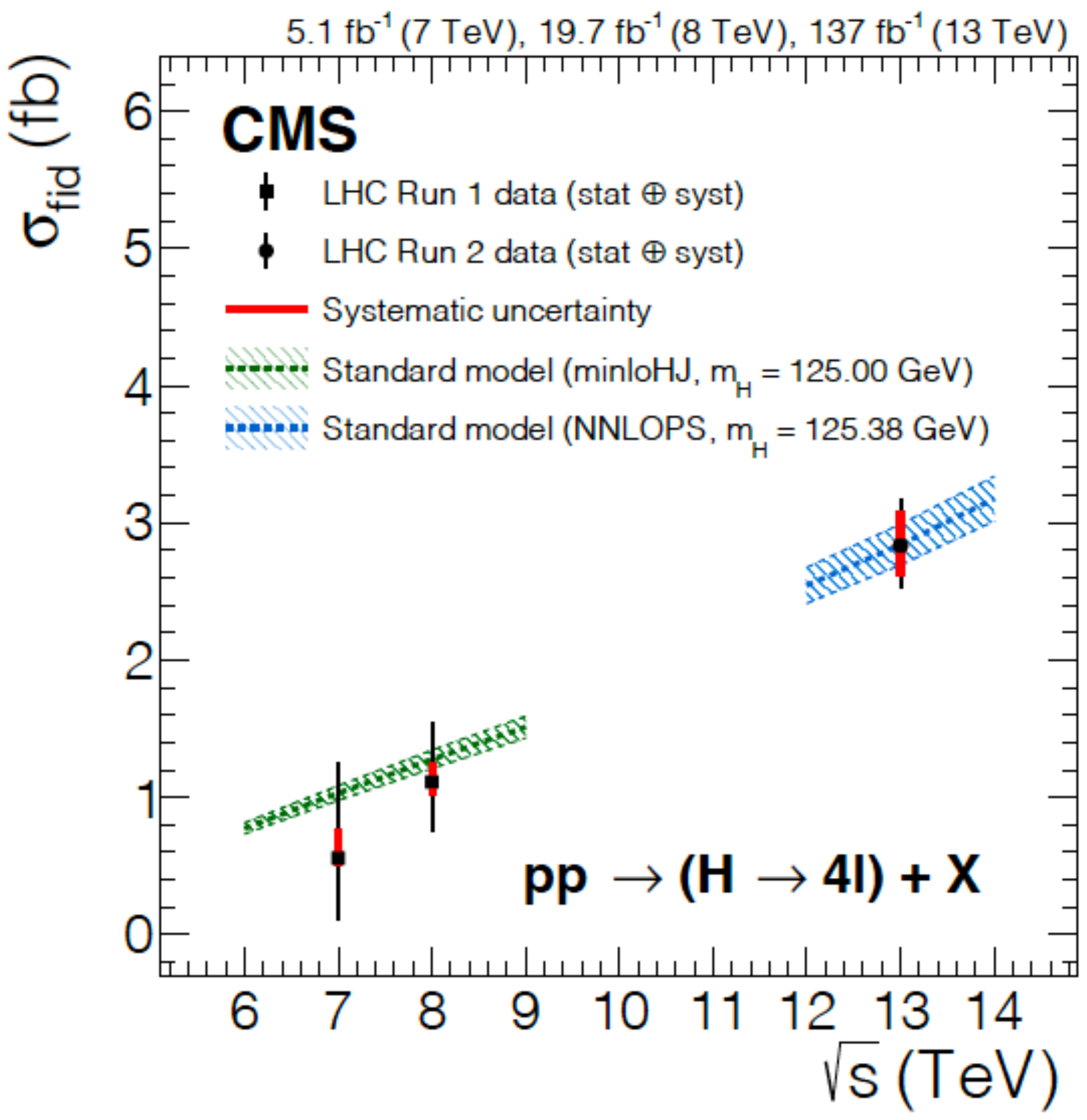
- The computation of the Higgs cross sections and decay modes is an excellent example that highlights all of the theoretical advances needed to maximize the potential of the LHC program.

Electroweak corrections at 2 loops

PDFs@NNLO



Precision determination of quark masses



QCD@N3LO

Precision extraction of α_S

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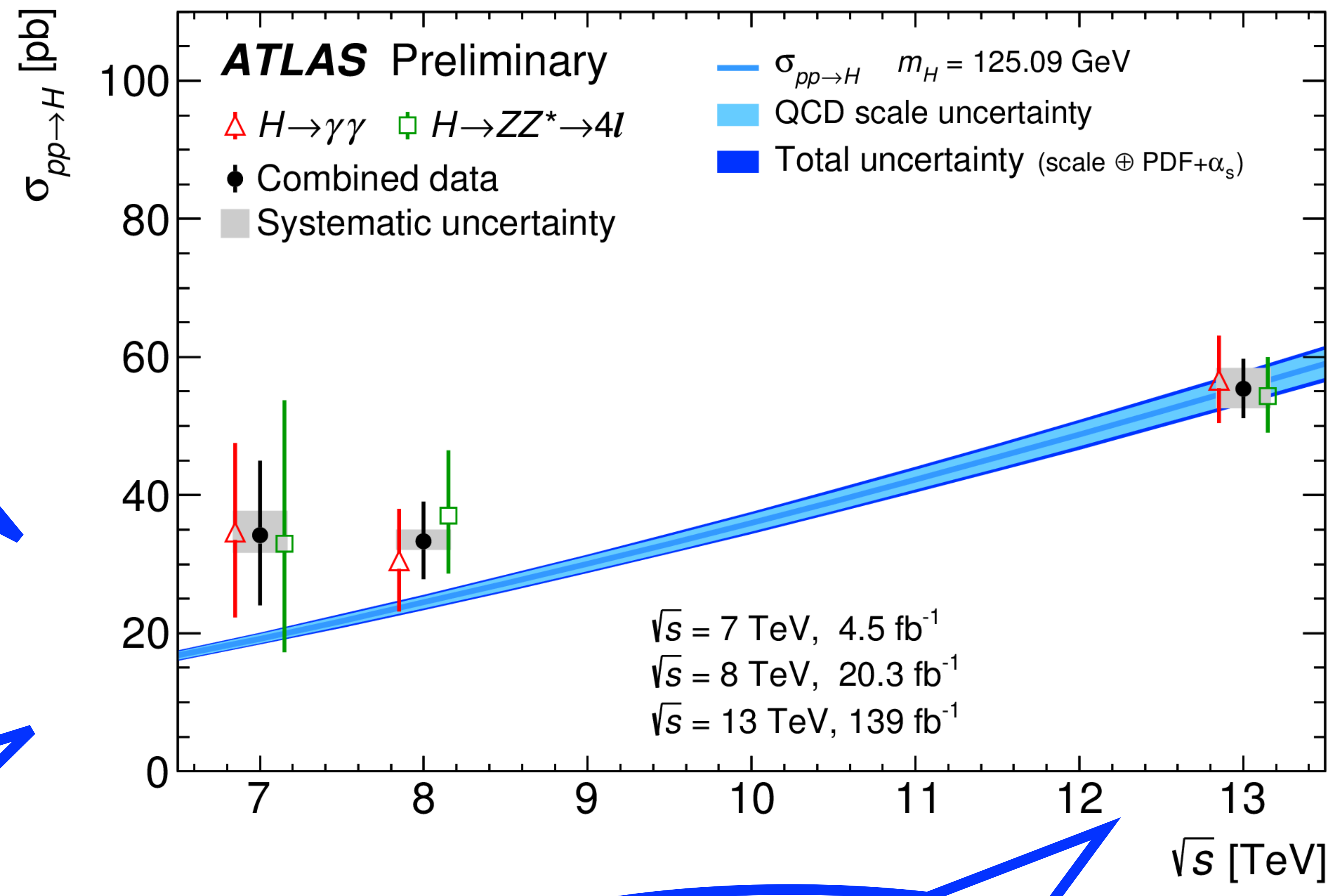

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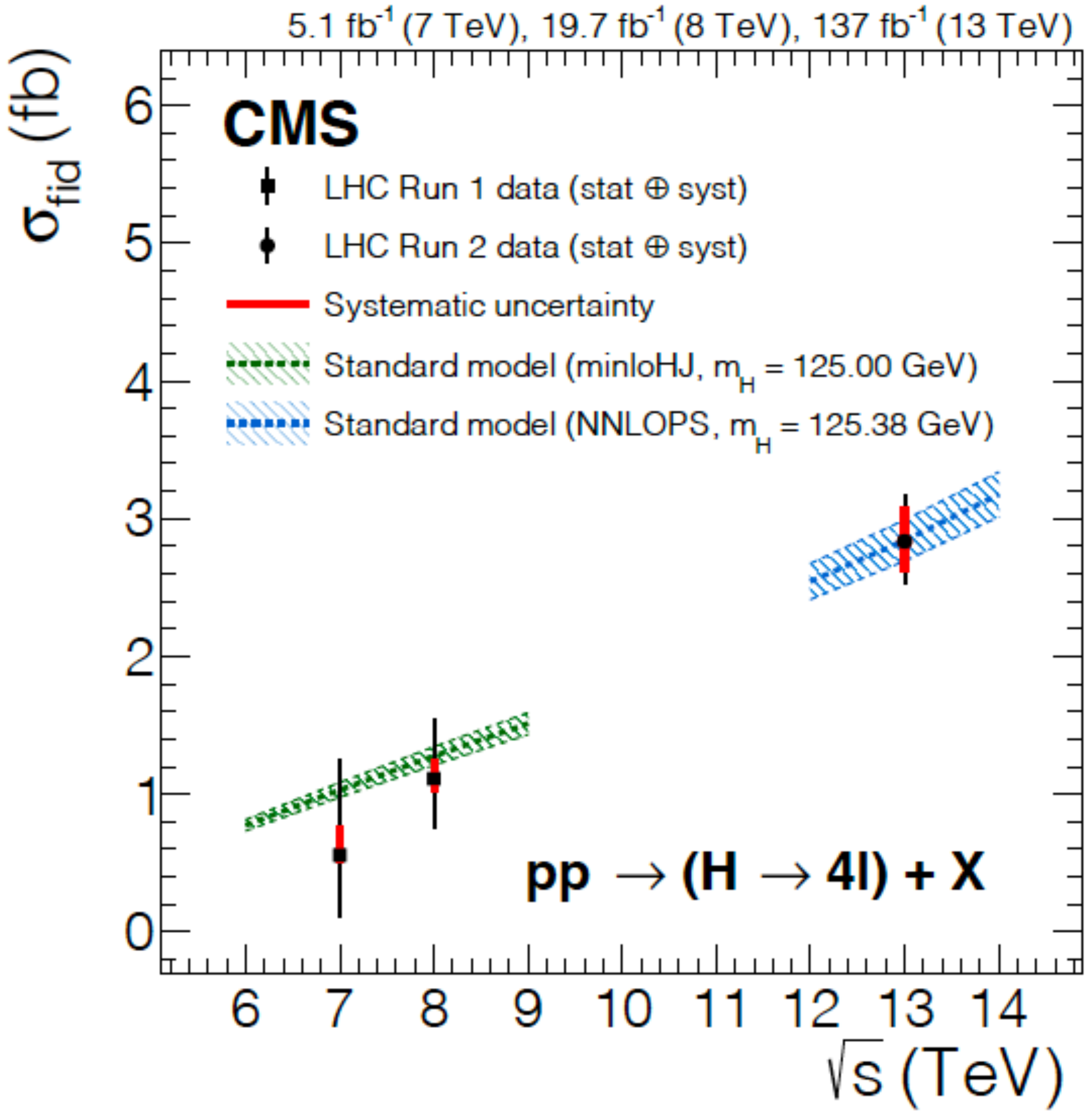

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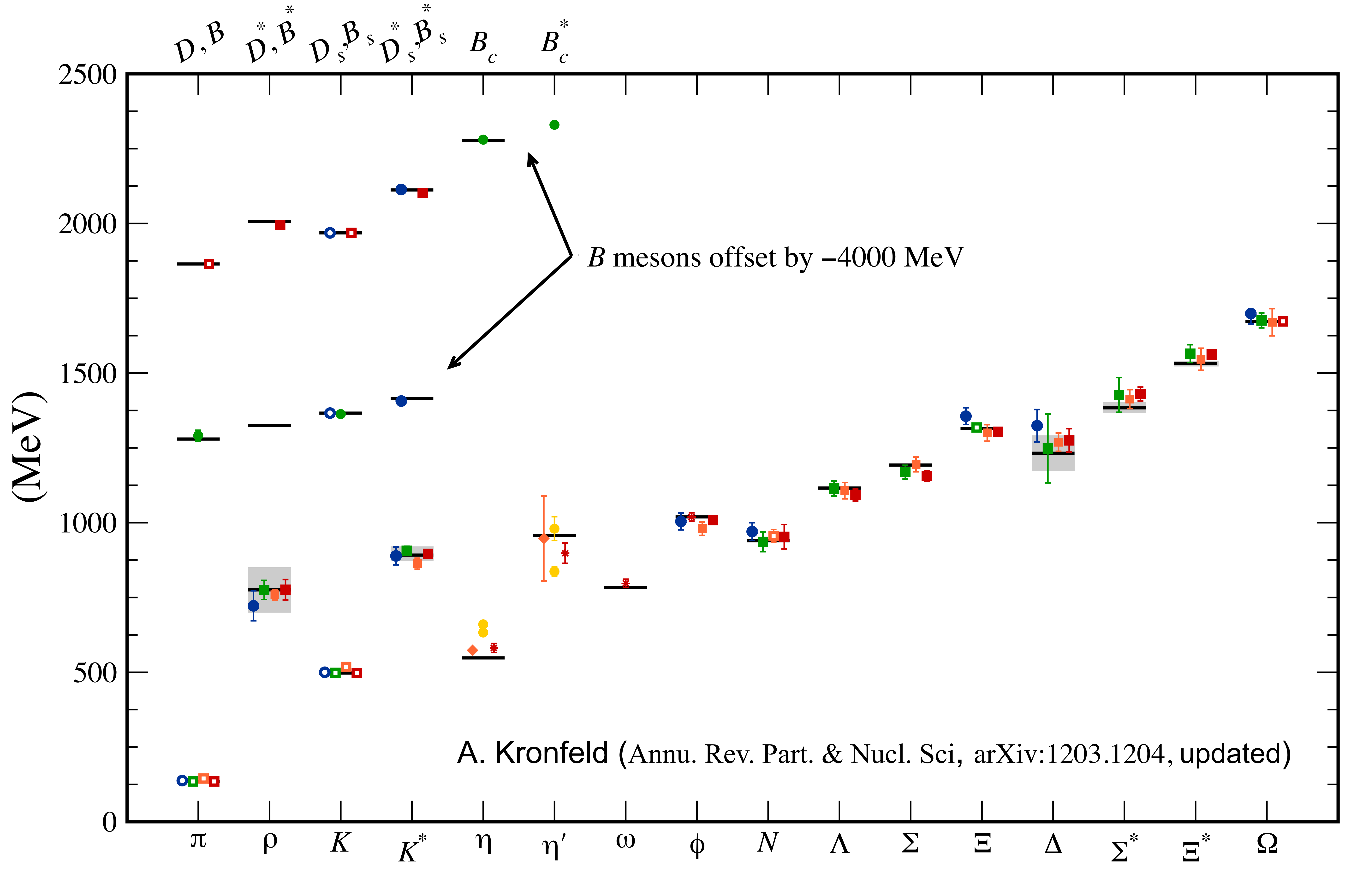
Future lattice QCD inputs

Lattice QCD inputs

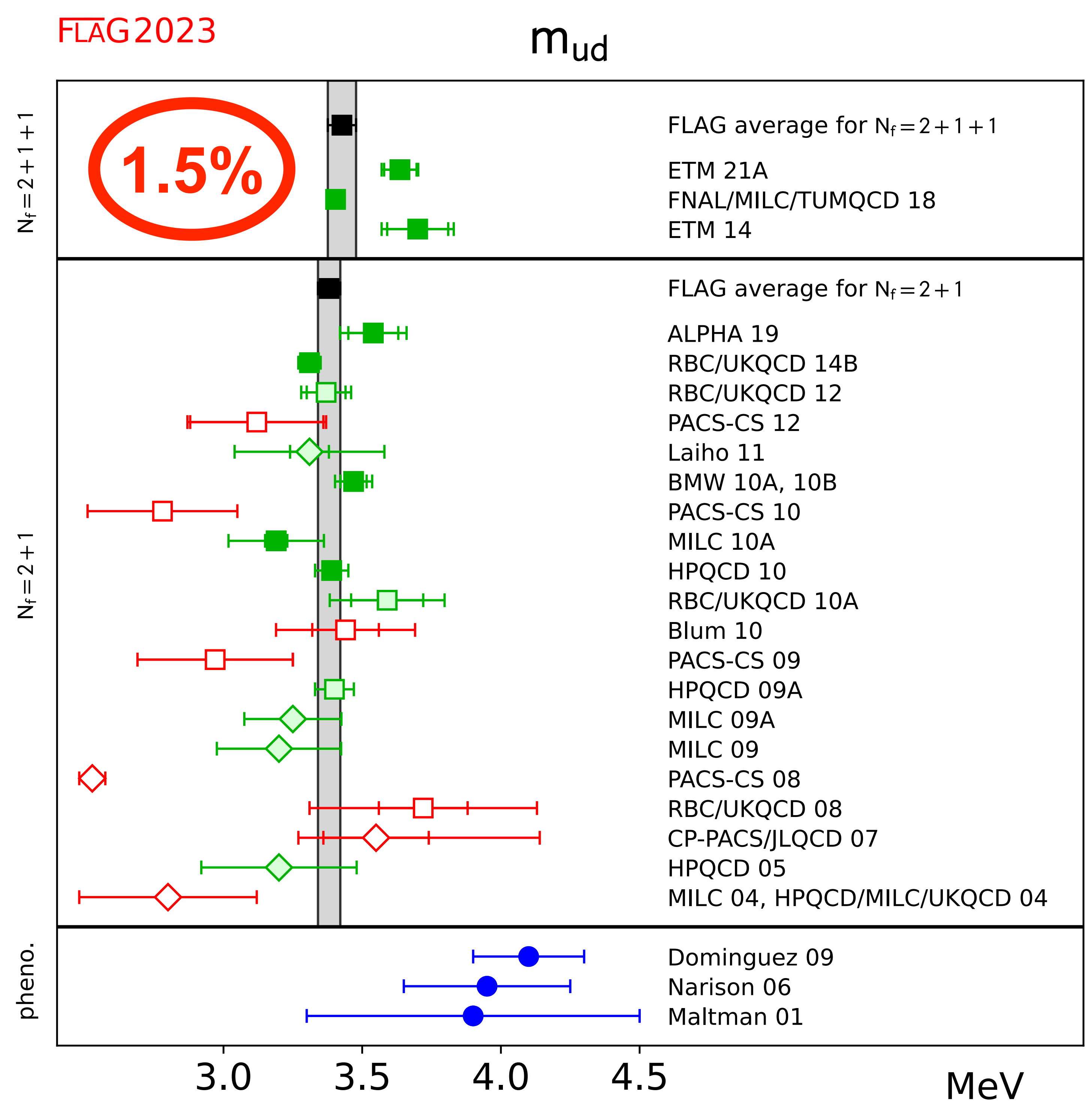
quark masses and α_s

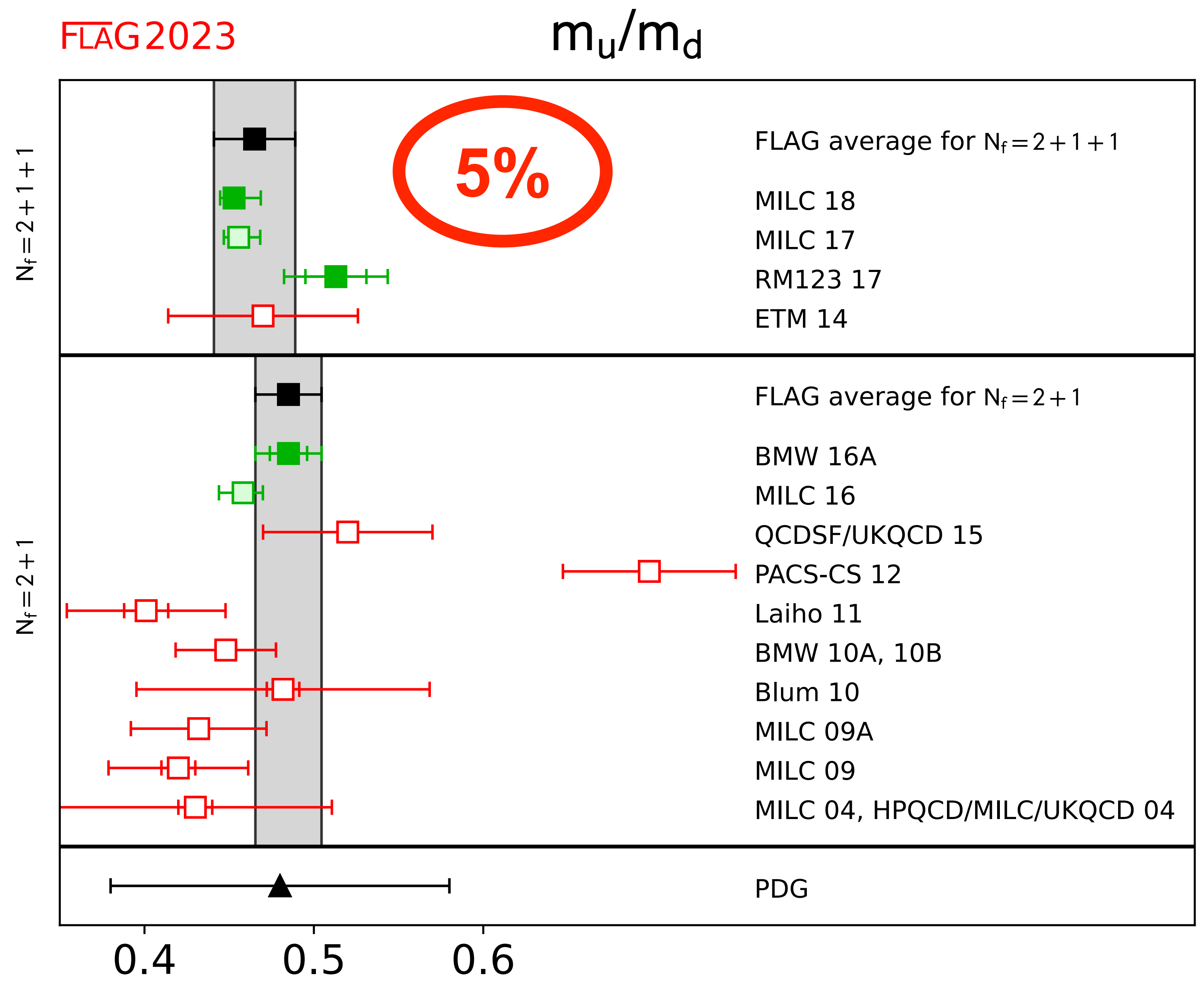
- Inputs to the (lattice) QCD lagrangian
- **bare** quark masses, m_{ud}, m_s, m_c, m_b: fixed with exp. measured hadron masses, e.g., $M_\pi, M_K, M_{D_s}, M_{B_s}$
- lattice spacing in physical units (scale setting): f_π (or M_Ω or …) ➠ α_s

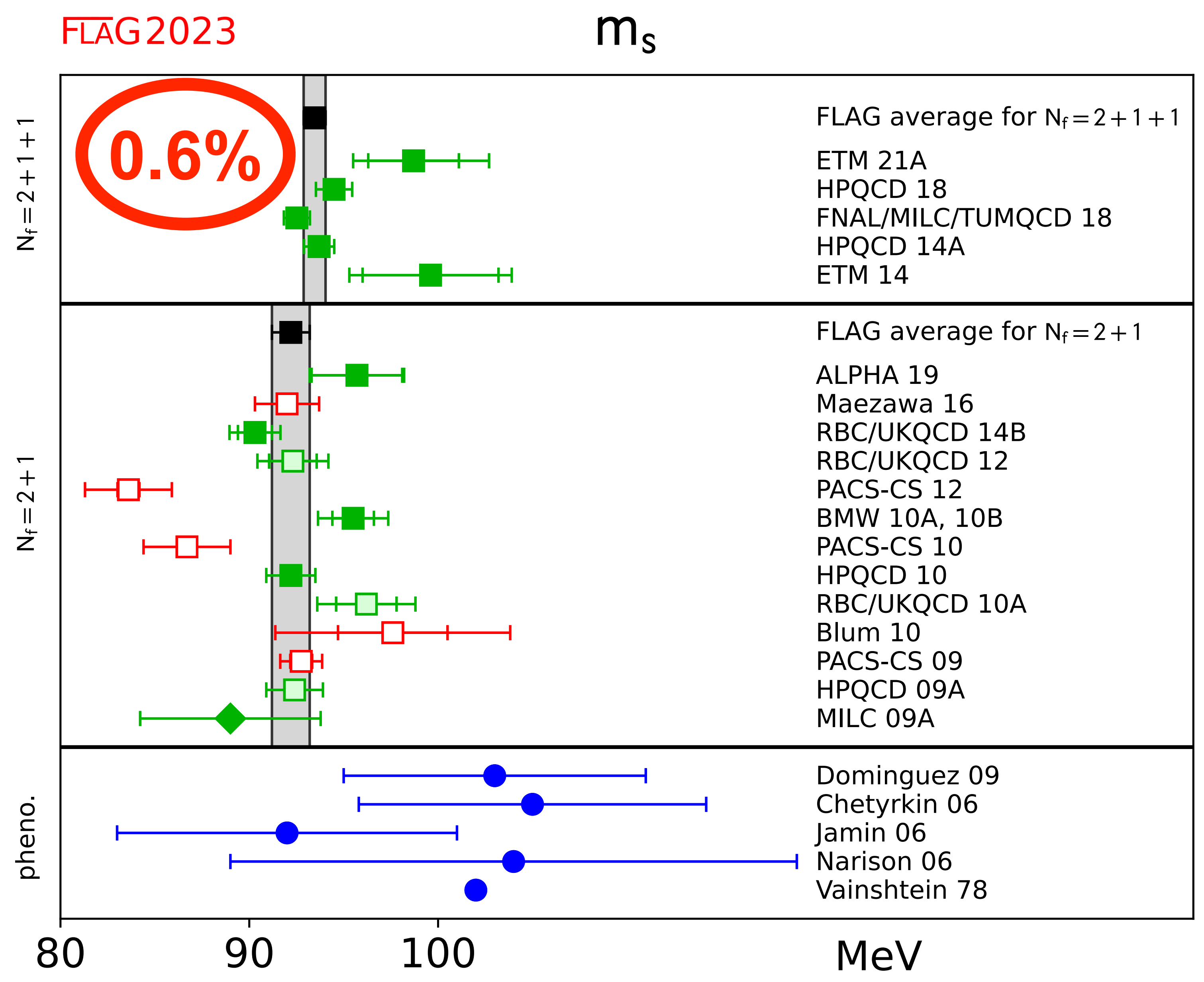


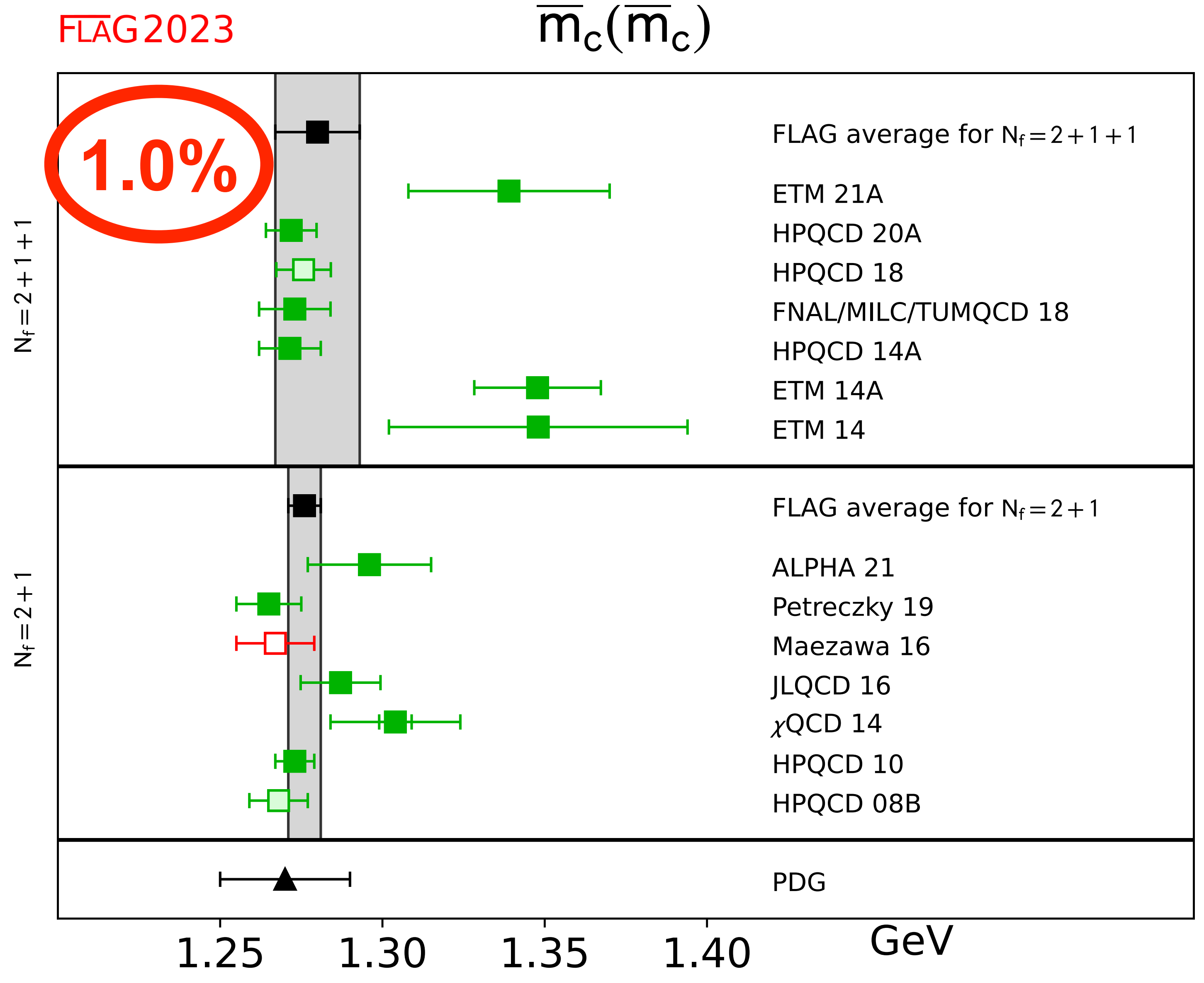
- all other quantities are pre/post dictions that can be compared to experiment.
- determinations of **renormalized** α_s from many different observables/methods: Wilson loops, current correlators, HQ potential, step scaling,…
- m_q:
  different intermediate renormalization schemes (nonperturbative or perturbative) before matching to $\overline{MS}$

quark masses



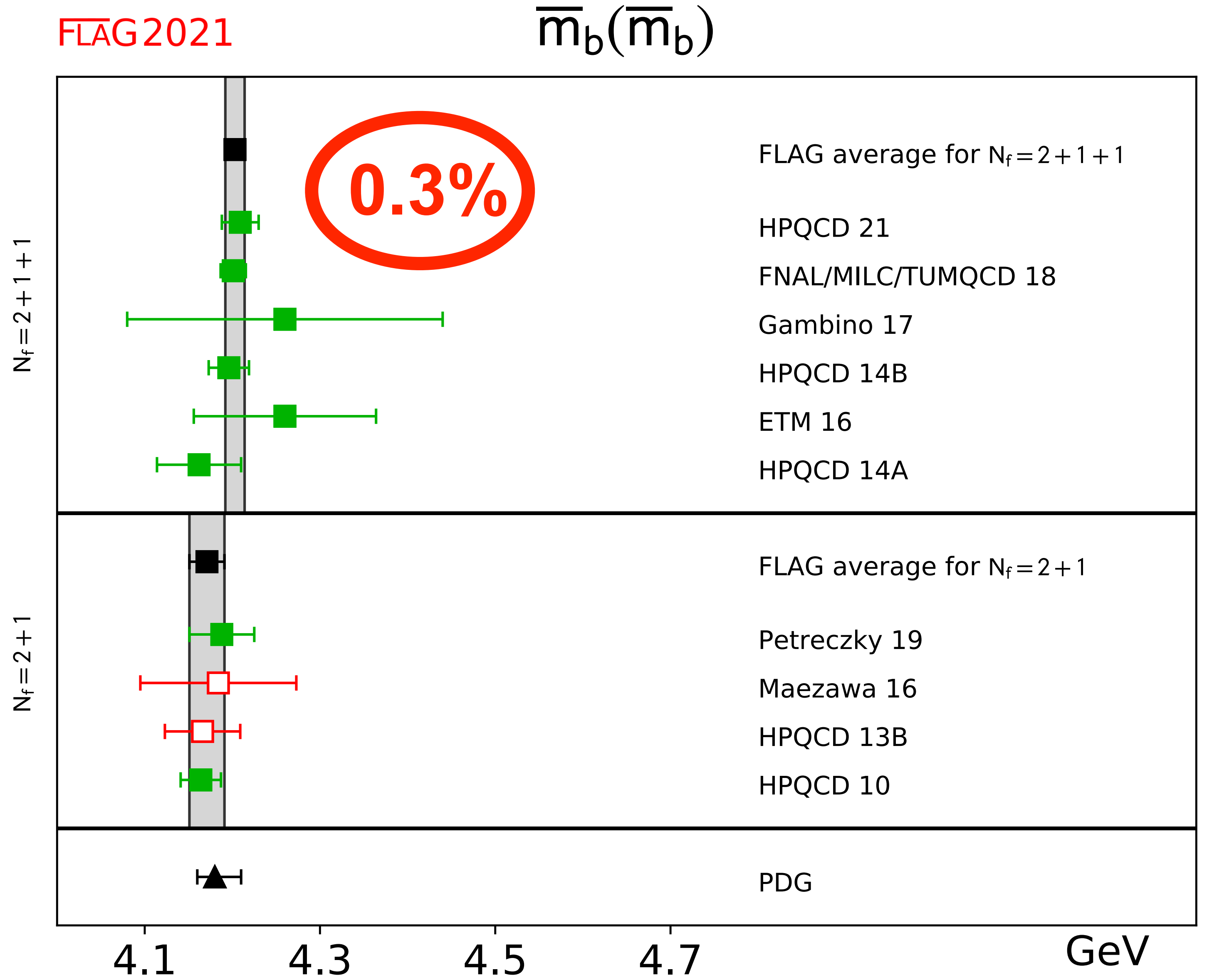






S. Aoki et al [FLAG 2021 review, arXiv:2111.09849, EPJC 2022]

Note: PDG quark mass listings still need to be adjusted.



α_s

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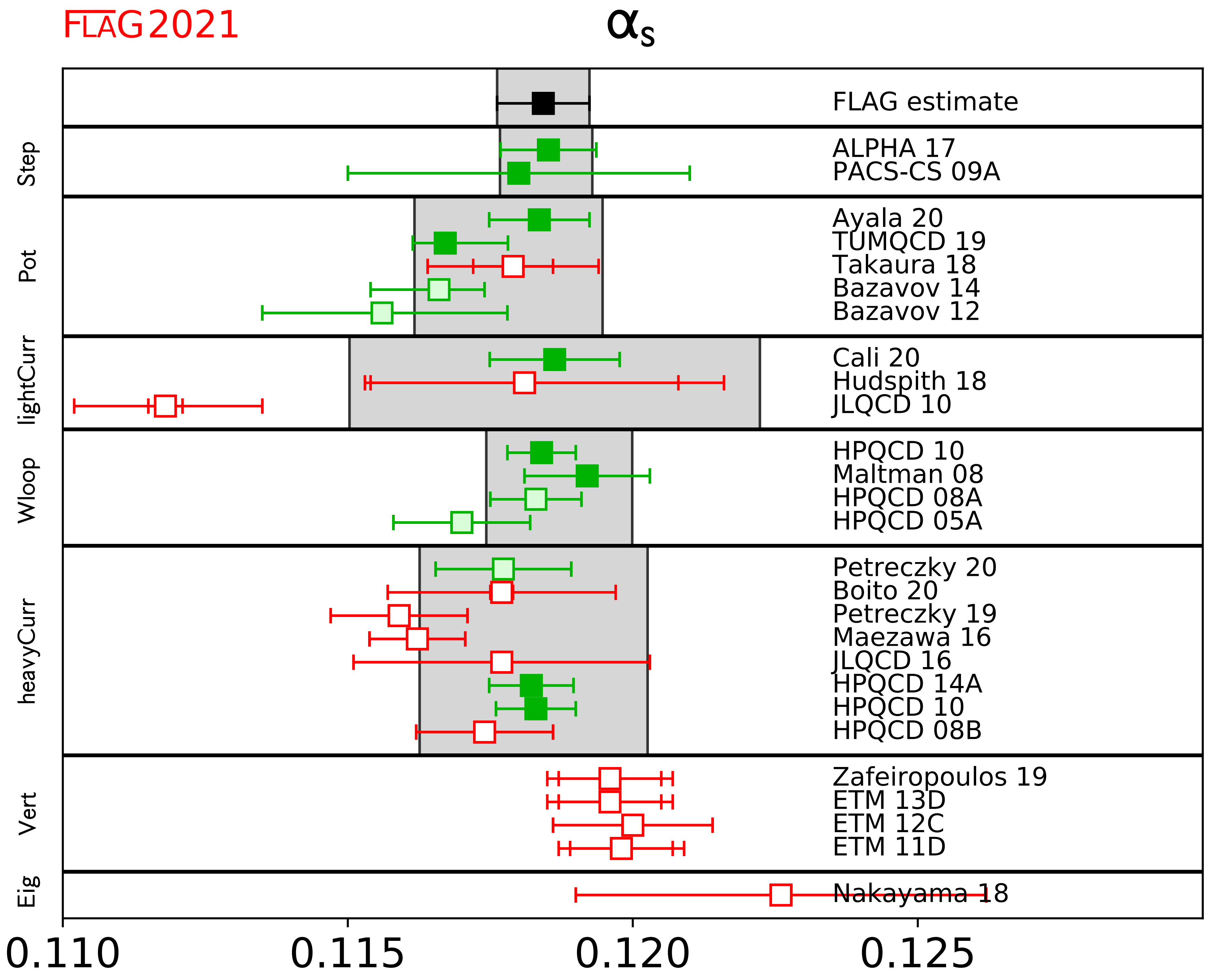
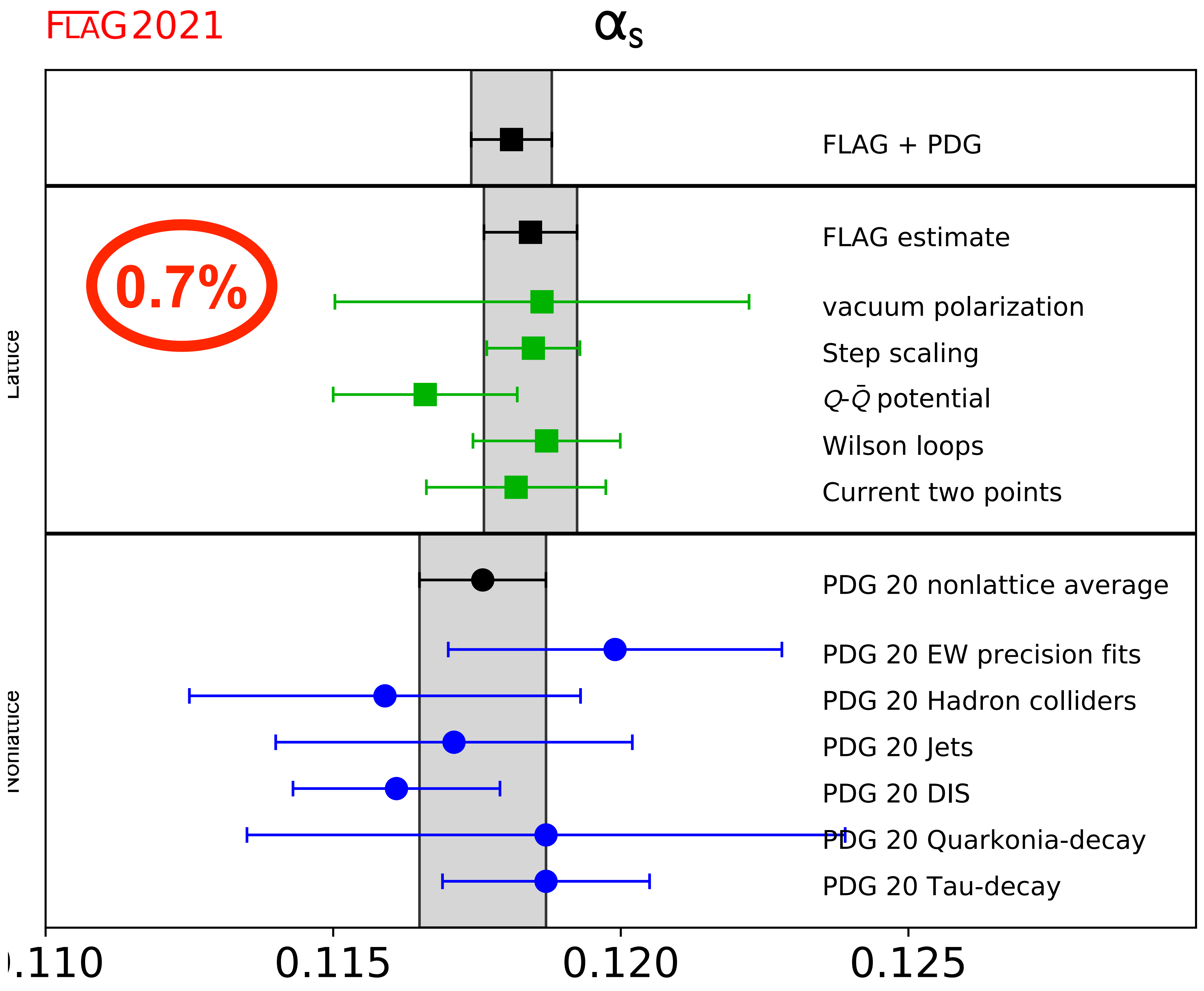
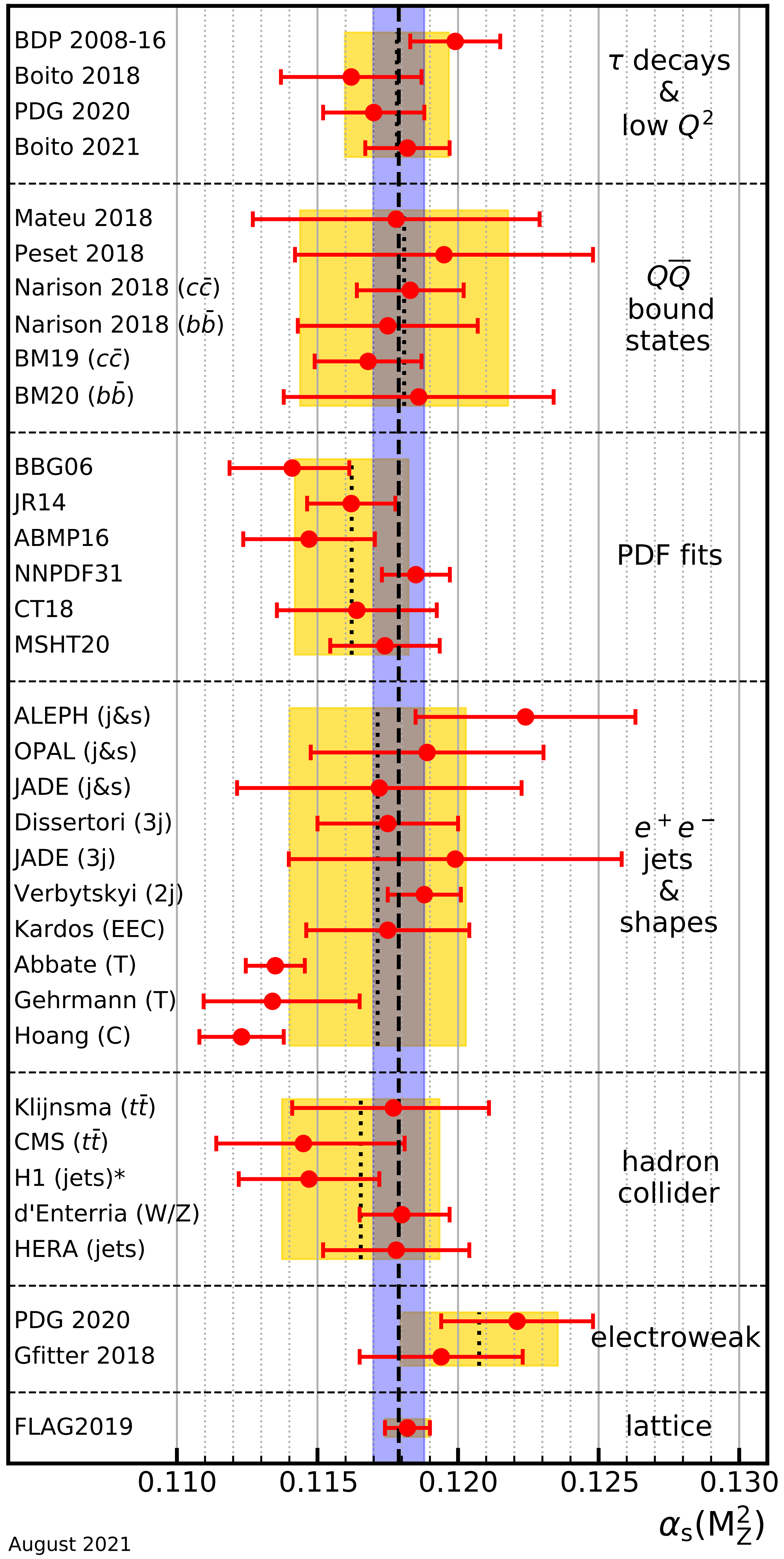




J. Huston, K. Rabbertz, G. Zanderighi
[PDG QCD review]



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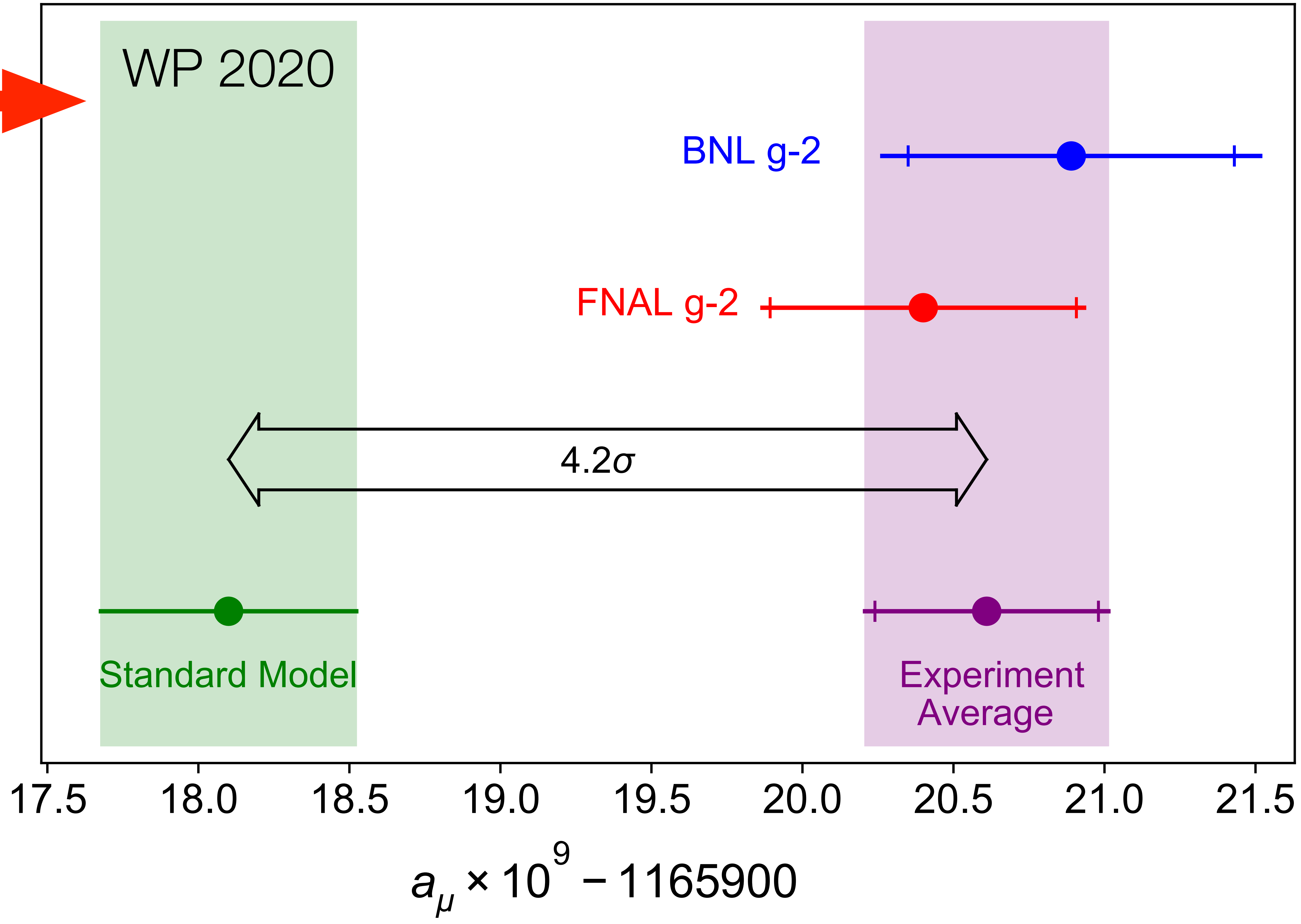
Muon g-2: experiment vs theory

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm Weak} + a_\mu^{\rm HVP} + a_\mu^{\rm HLbL} = 116591810\,(43) \times 10^{-11}$$

[B. Abi et al (Muon g-2 Collaboration), Phys. Rev. Lett. 124, 141801 (2021)]

Muon g-2 Theory Initiative



Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

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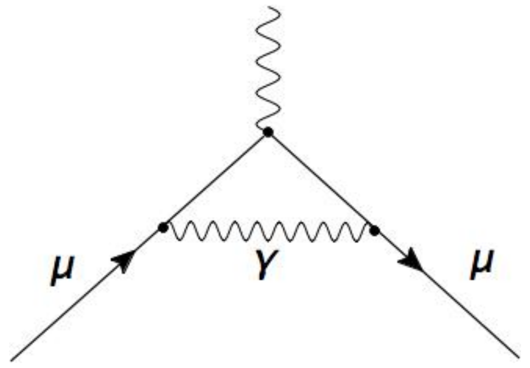
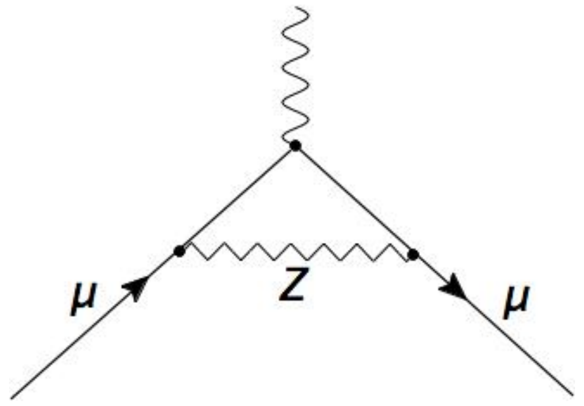
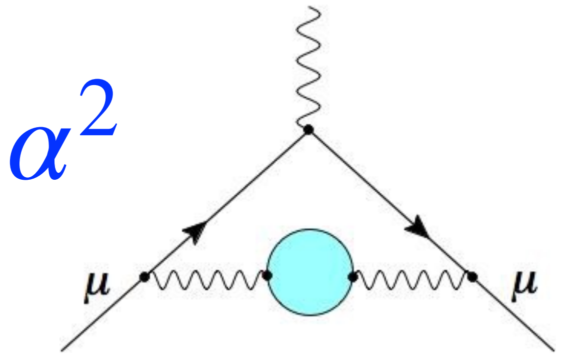
QED 

+… (5 loops) $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

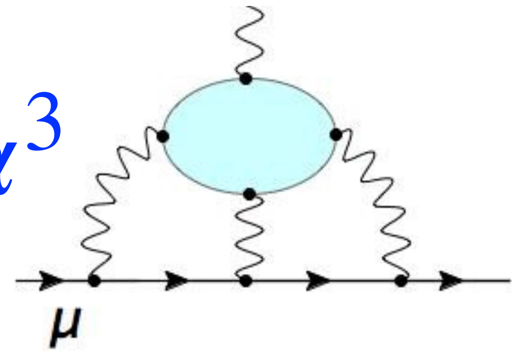
EW 

+… (2 loops) $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic corrections

HVP α^2 

+… (NNLO) $6845\,(40) \times 10^{-11}$ [0.6%] 0.34 ppm

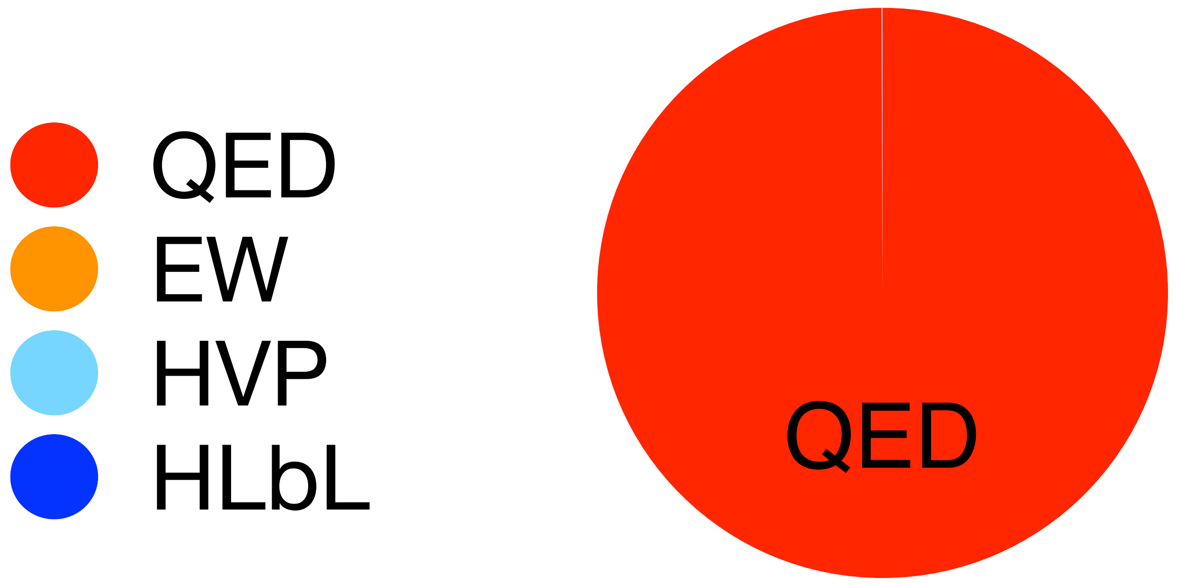
HLbL α^3 

+… (NLO) $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

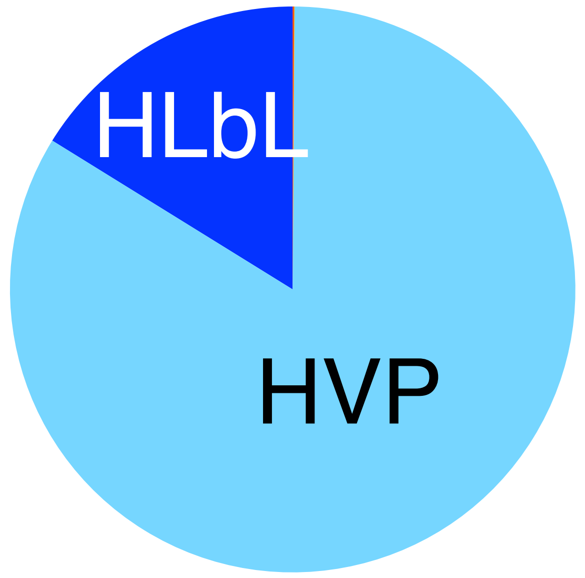
Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

contribution



error²

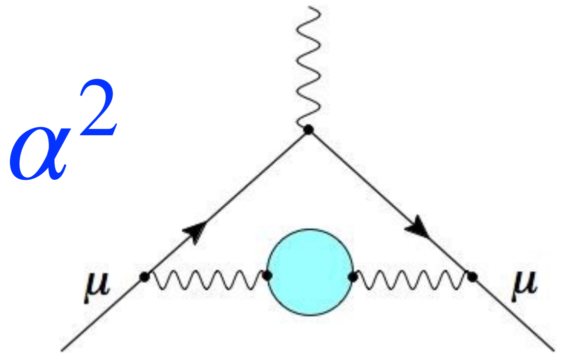


$116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

$153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic corrections

HVP α^2



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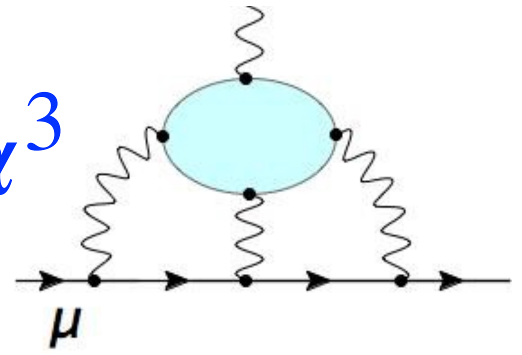
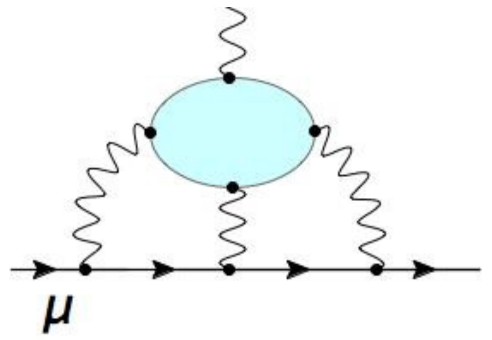
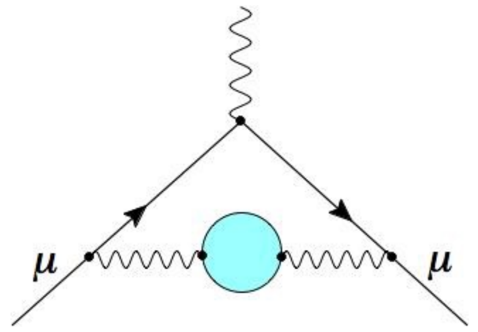
HLbL α^3



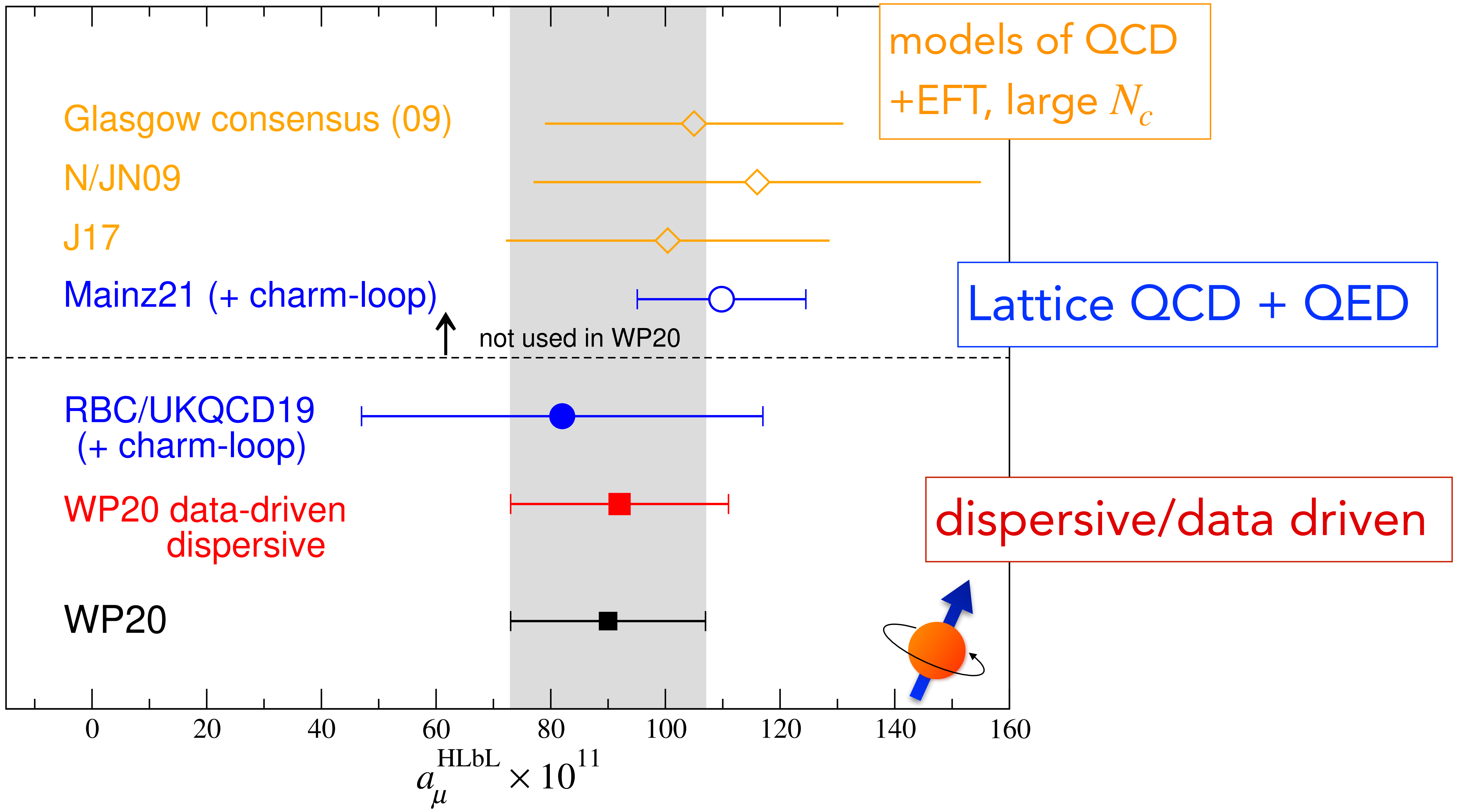
+… (NLO) $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm



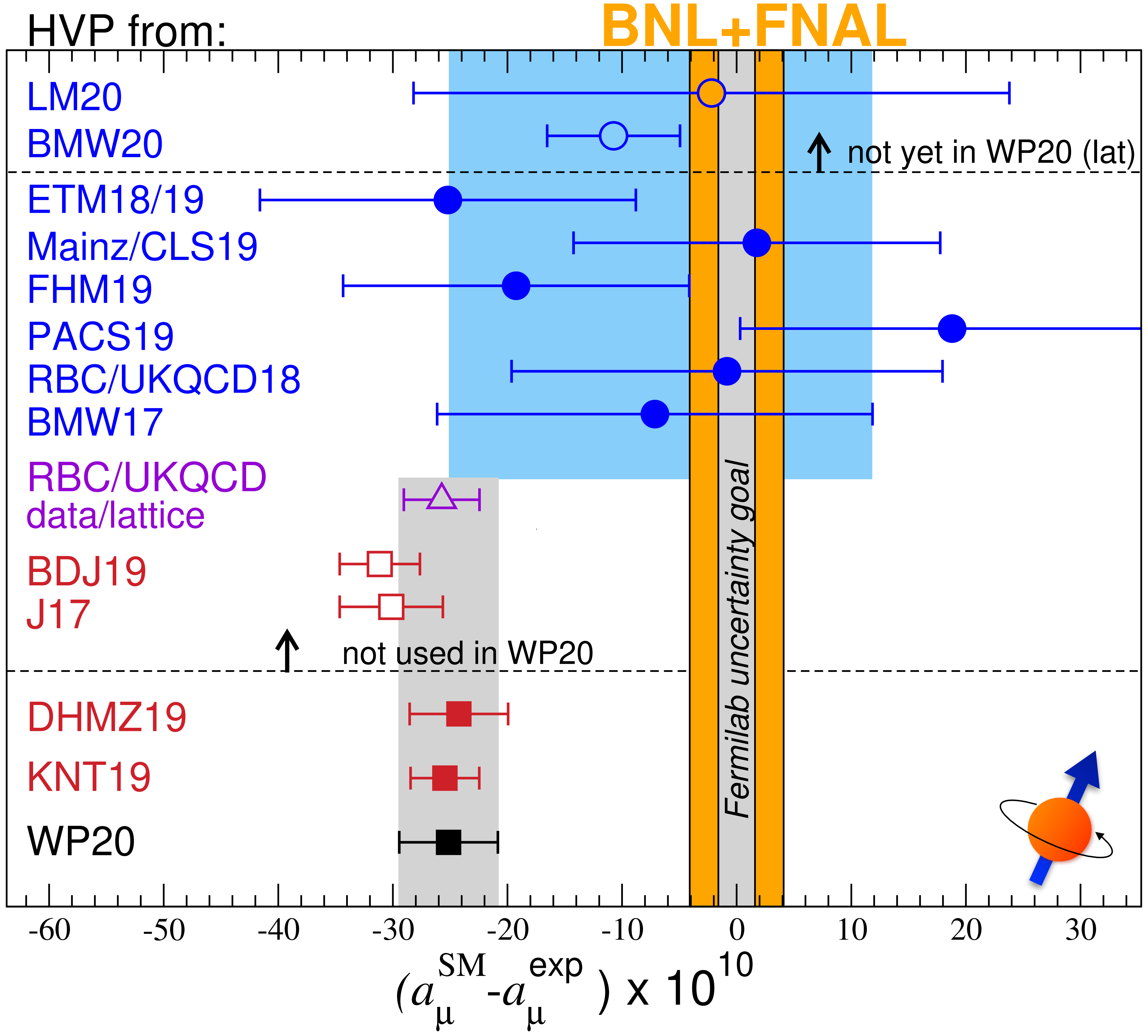
Hadronic Corrections: Comparisons

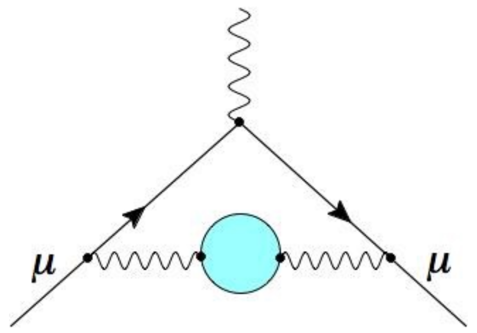


a_μ^{HLbL}



a_μ^{SM} $a_\mu^{\mathrm{HVP}} + [a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{Weak}} + a_\mu^{\mathrm{HLbL}}]$





HVP: data-driven

see appendix

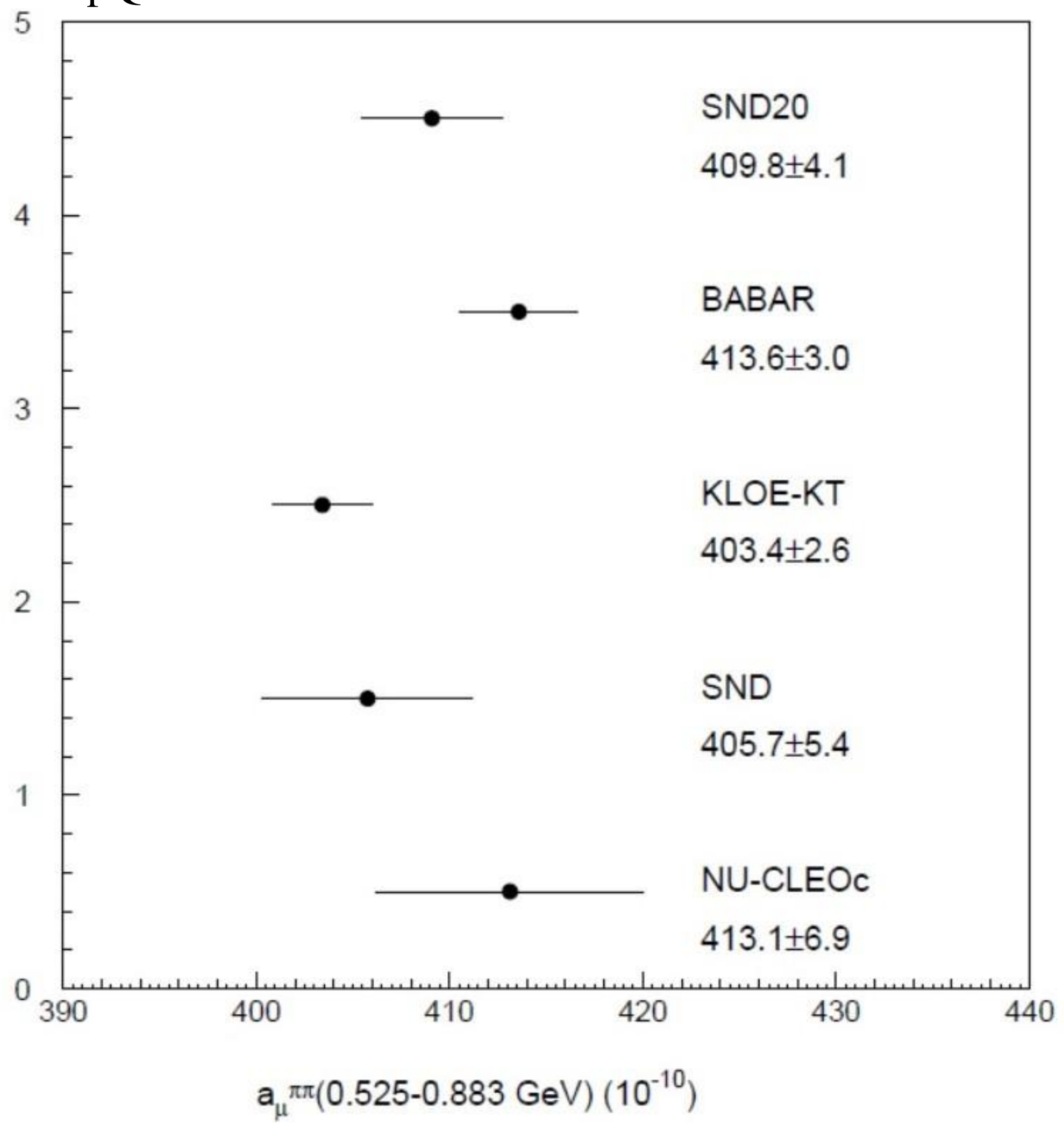
In 2020 WP:

Conservative merging procedure to obtain a realistic assessment of the underlying uncertainties:

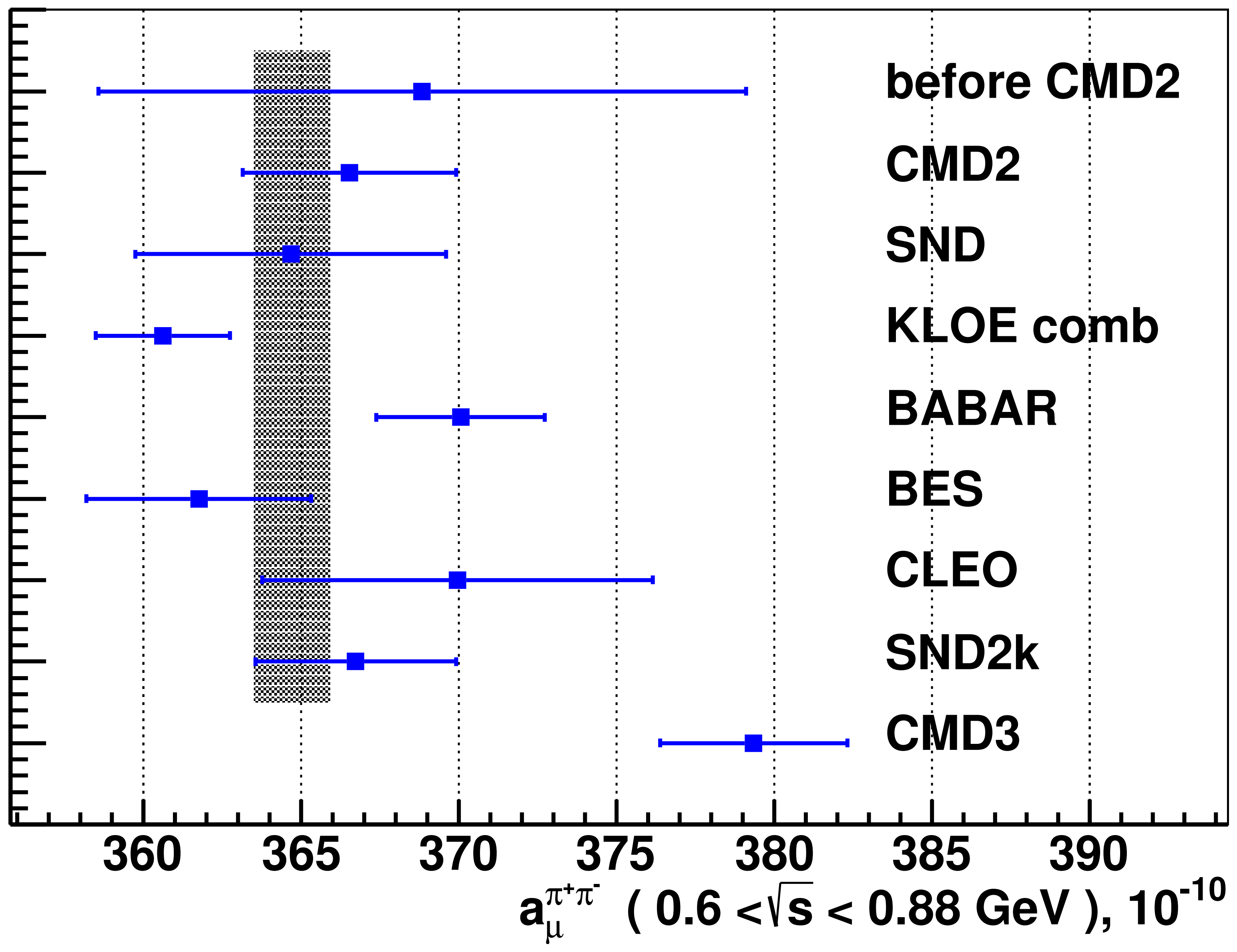
- account for tensions between data sets
- account for differences in methodologies for compilation of experimental inputs
- include correlations between systematic errors
- cross checks from unitarity & analyticity constraints
  [Colangelo et al, 2018; Anantharayan et al, 2018; Davier et al, 2019; Hoferichter et al, 2019]
- Full NLO radiative corrections [Campanario et al, 2019]

$$a_\mu^{\text{HVP,LO}} = 693.1\,(2.8)_{\text{exp}}\,(0.7)_{\text{DV+pQCD}}\,(2.8)_{\text{BaBar-KLOE}} \times 10^{-10}$$
$$= 693.1\,(4.0) \times 10^{-}$$

[M. Davier @ KEK workshop]

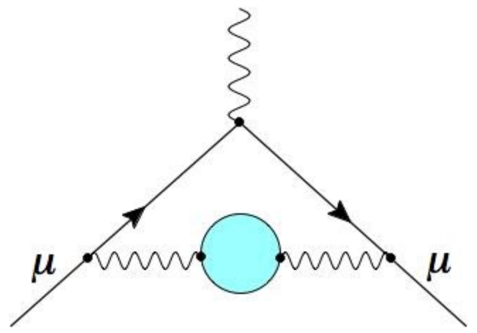


New: from CMD-3 [F. Ignatov et al, arXiv:2302.08834]



A new puzzle!

- discrepancies between experiments now $\gtrsim (3-5)\,\sigma$
  this needs to be understood/resolved
- (virtual) scientific seminar + discussion panel on CMD-3 measurement
  March 27 (8:00 –11:00 am US CDT)
  Discussions are continuing!
- 6th Muon g-2 Theory Initiative workshop (4-8 Sep 2023, Bern)



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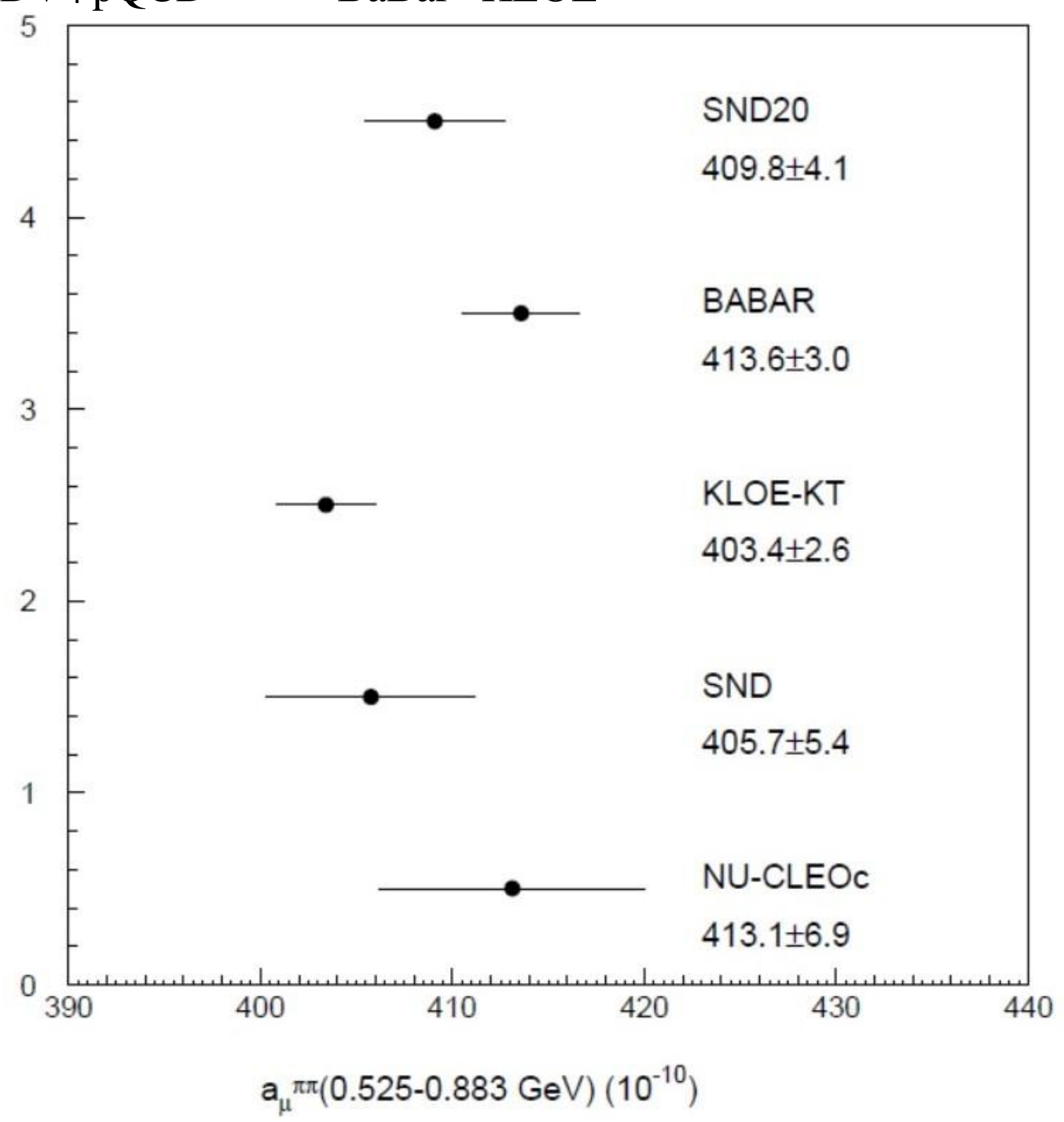
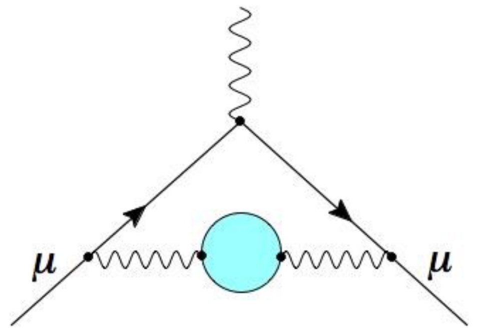


Ongoing work on experimental inputs:

- BaBar: new analysis of large data set in $\pi\pi$ channel, also $\pi\pi\pi$, other channels, other channels
- KLOE: new analysis of large data in $\pi\pi$ channel, other channels
- SND: new results for $\pi\pi$ channel, other channels in progress
- BESIII: new results in 2021 for $\pi\pi$ channel, continued analysis also for $\pi\pi\pi$, other channels
- Belle II: arXiv:2207.06307 (Snowmass WP)
  Better statistics than BaBar or KLOE; similar or better systematics for low-energy cross sections
- Most collaborations proceeding with blind analyses

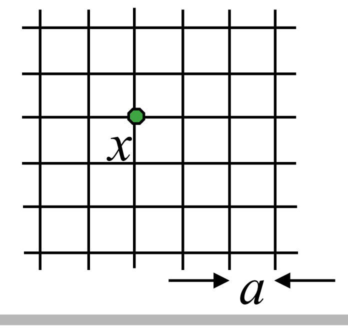
Ongoing work on theoretical aspects:

- Developing NNLO Monte Carlo generators (STRONG 2020 workshop https://agenda.infn.it/event/28089/) [➠ appendix]
- radiative corrections using FsQED (scalar QED + pion form factor)
- charge asymmetry (CMD-3 measurement) vs radiative corrections [Ignatov + Lee, arXiv:2204.12235]
- development of new dispersive treatment of radiative corrections in $\pi\pi$ channel [Colangelo at al, arXiv2207.03495]
- including τ decay data: requires nonperturbative evaluation of IB correction [M. Bruno et al, arXiv:1811.00508]



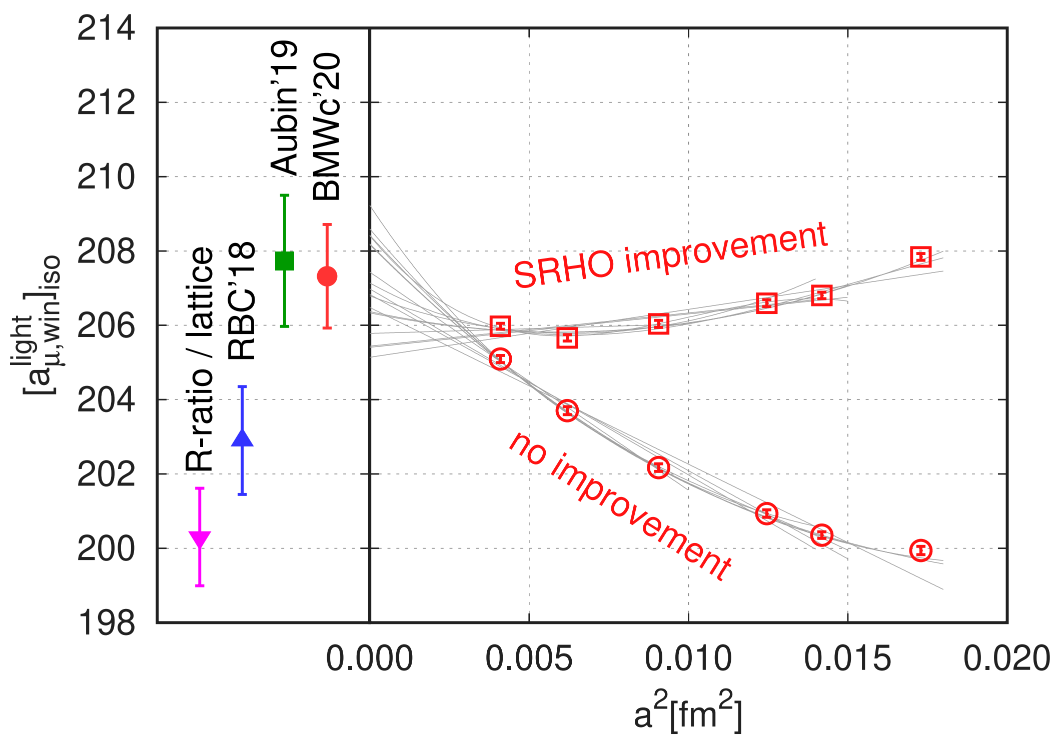
HVP: lattice

see appendix



In 2020 WP:

- Lattice HVP average at $2.6\,\%$ total uncertainty: $a_\mu^{\rm HVP,LO} = 711.6\,(18.4) \times 10^{10}$
- BMW 20 (published in 2021) first LQCD calculation with sub-percent $(0.8\,\%)$ error in tension with data-driven HVP (2.1σ)
- Further tensions for intermediate window



-3.7σ tension with data-driven evaluation

-2.2σ tension with RBC/UKQCD18

$$a_\mu^{\rm HVP,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

- Use windows in Euclidean time to consider the different time regions separately. [T. Blum et al, arXiv:1801.07224, 2018 PRL]

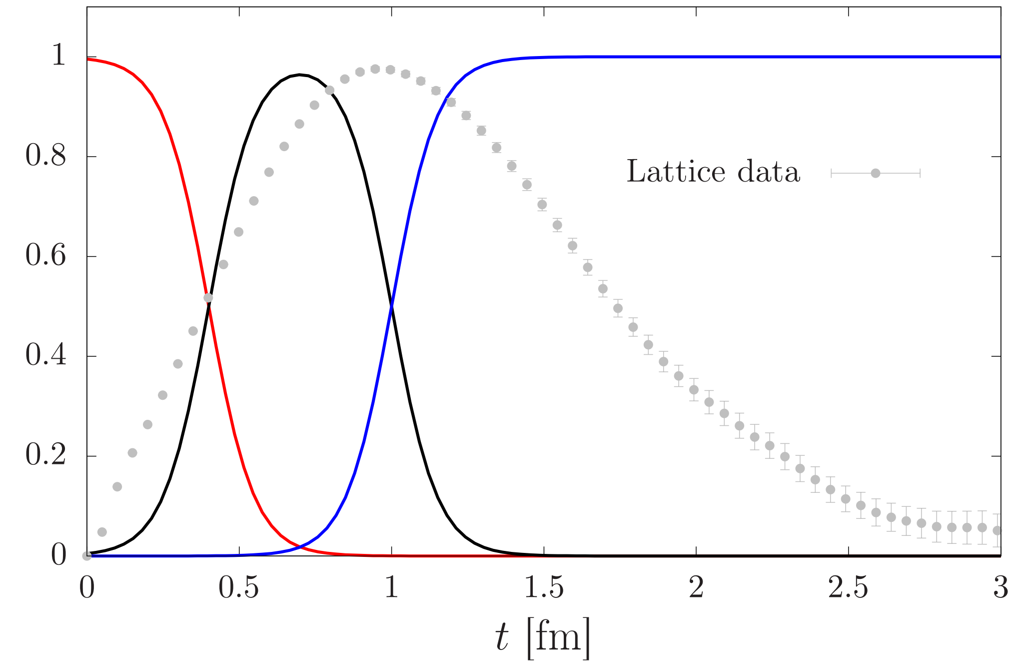
Short Distance (SD) $t: 0 \to t_0$

Intermediate (W) $t: t_0 \to t_1$

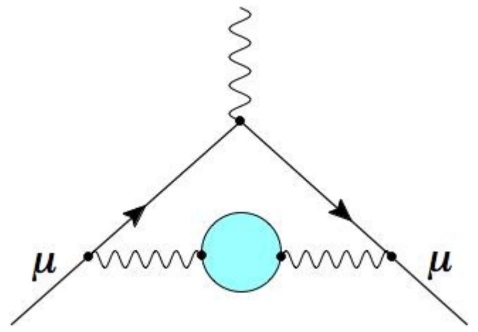
Long Distance (LD) $t: t_1 \to \infty$

$t_0 = 0.4\,{\rm fm}, t_1 = 1.0\,{\rm fm}$

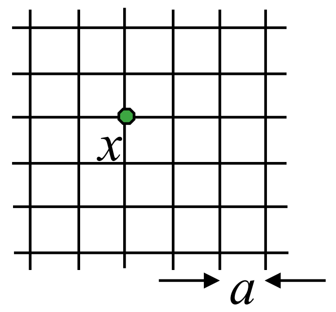
H. Wittig @ Lattice 2021



- disentangle systematics/statistics from long distance/FV and discretization effects
- intermediate window: easy to compute in lattice QCD
- Euclidean windows are also straightforward to evaluate in disperse approach
- Internal cross check: compute each window separately (in continuum, infinite volume limits,…) and combine: $a_\mu = a_\mu^{\rm SD} + a_\mu^{\rm W} + a_\mu^{\rm LD}$



HVP: lattice



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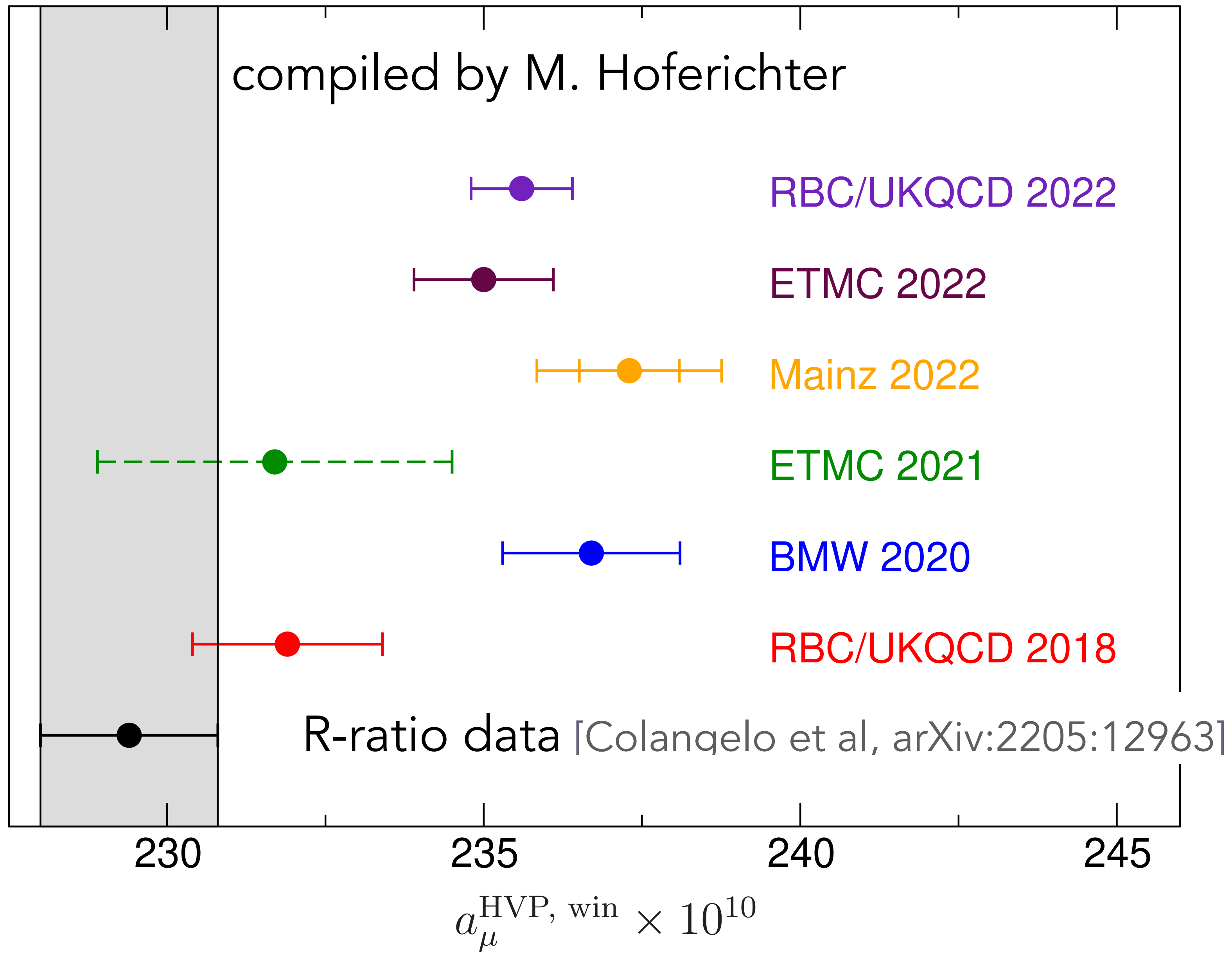

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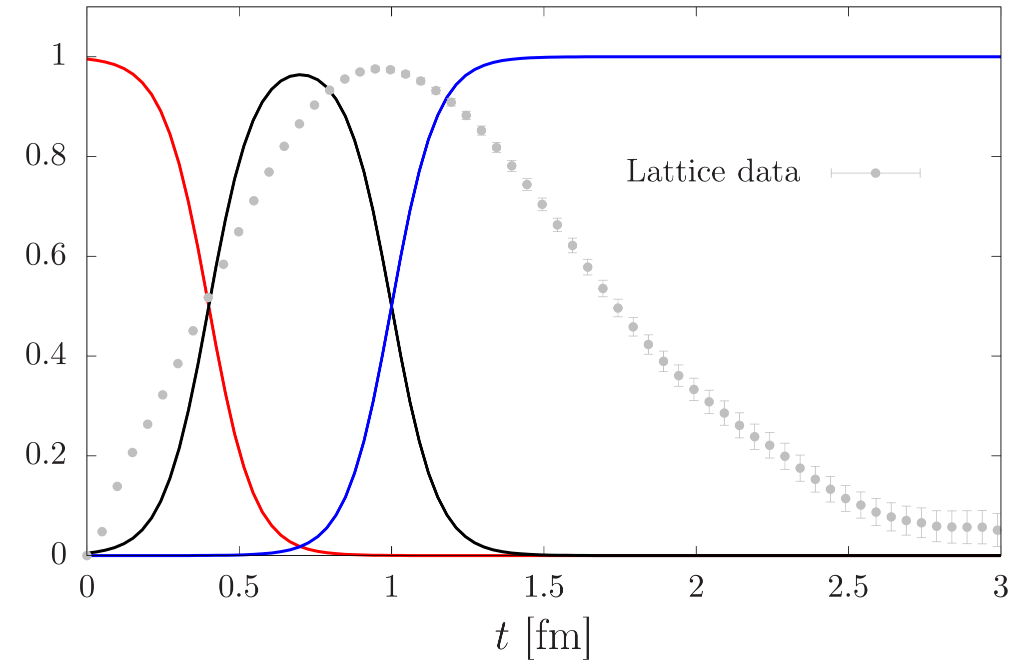

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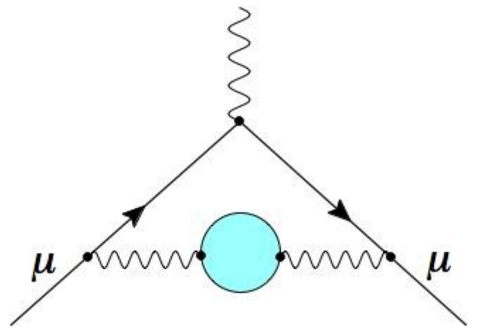

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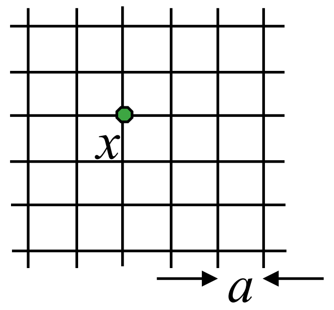

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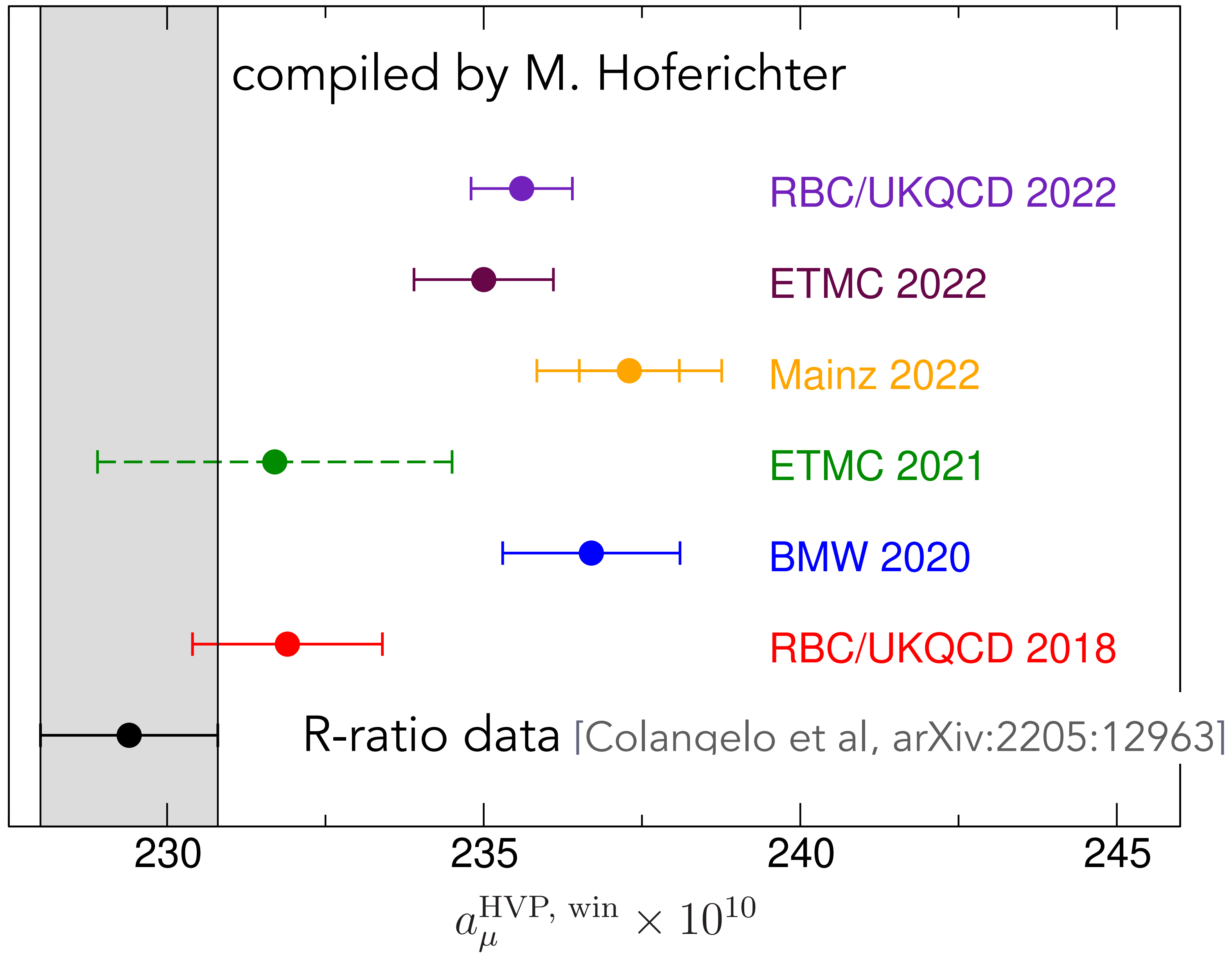
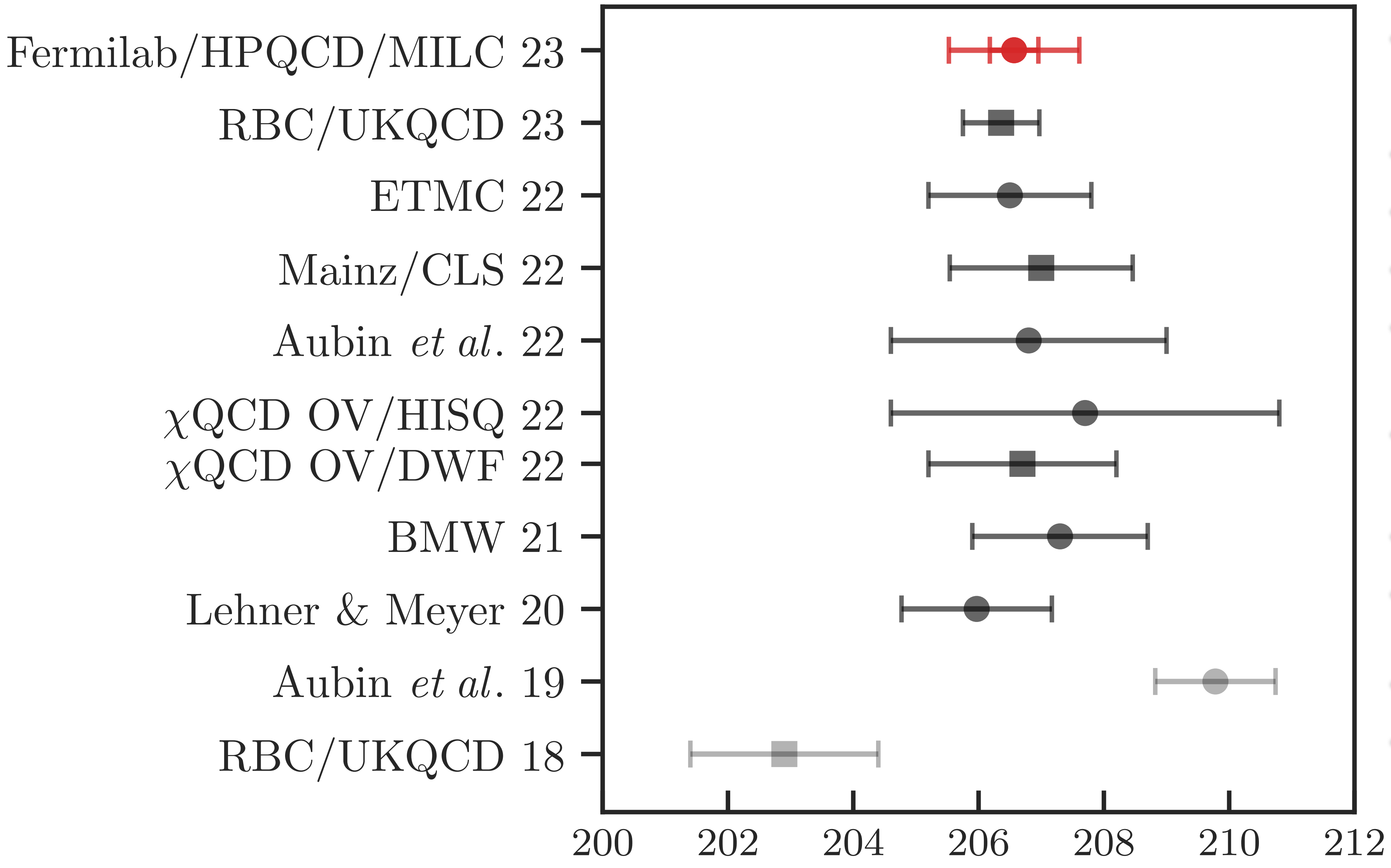
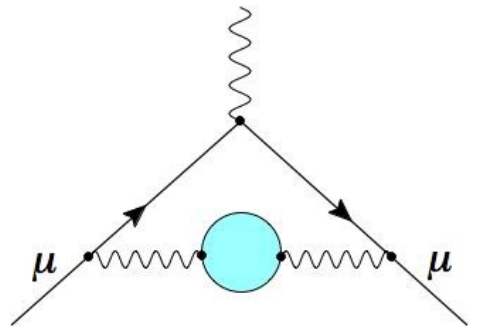


- new results in 2022/23 for intermediate window, a_μ^W from six different lattice groups.
- Most recently announced unblinded results by RBC/UKQCD and Fermilab/HPQCD/MILC
- lattice-only comparison of light-quark connected contribution to intermediate window:



- Fermilab/HPQCD/MILC: arXiv:2301.08274
- RBC/UKQCD 23: arXiv:2301.08696
- ETM: arXiv:2206.15084
- Mainz: arXiv:2206.06582
- Aubin et al: arXiv:2204.12256
- χQCD: arXiv:2204.01280
- BMW 21: arXiv:2002.12347
- L&M: arXiv:2003.04177
- Aubin et al: arXiv:2204.12256
- RBC/UKQCD 18: arXiv:1801.07224



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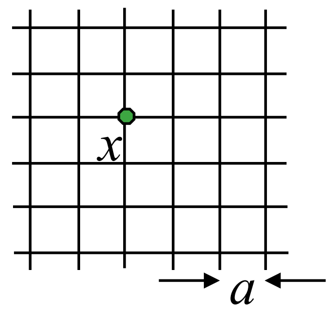

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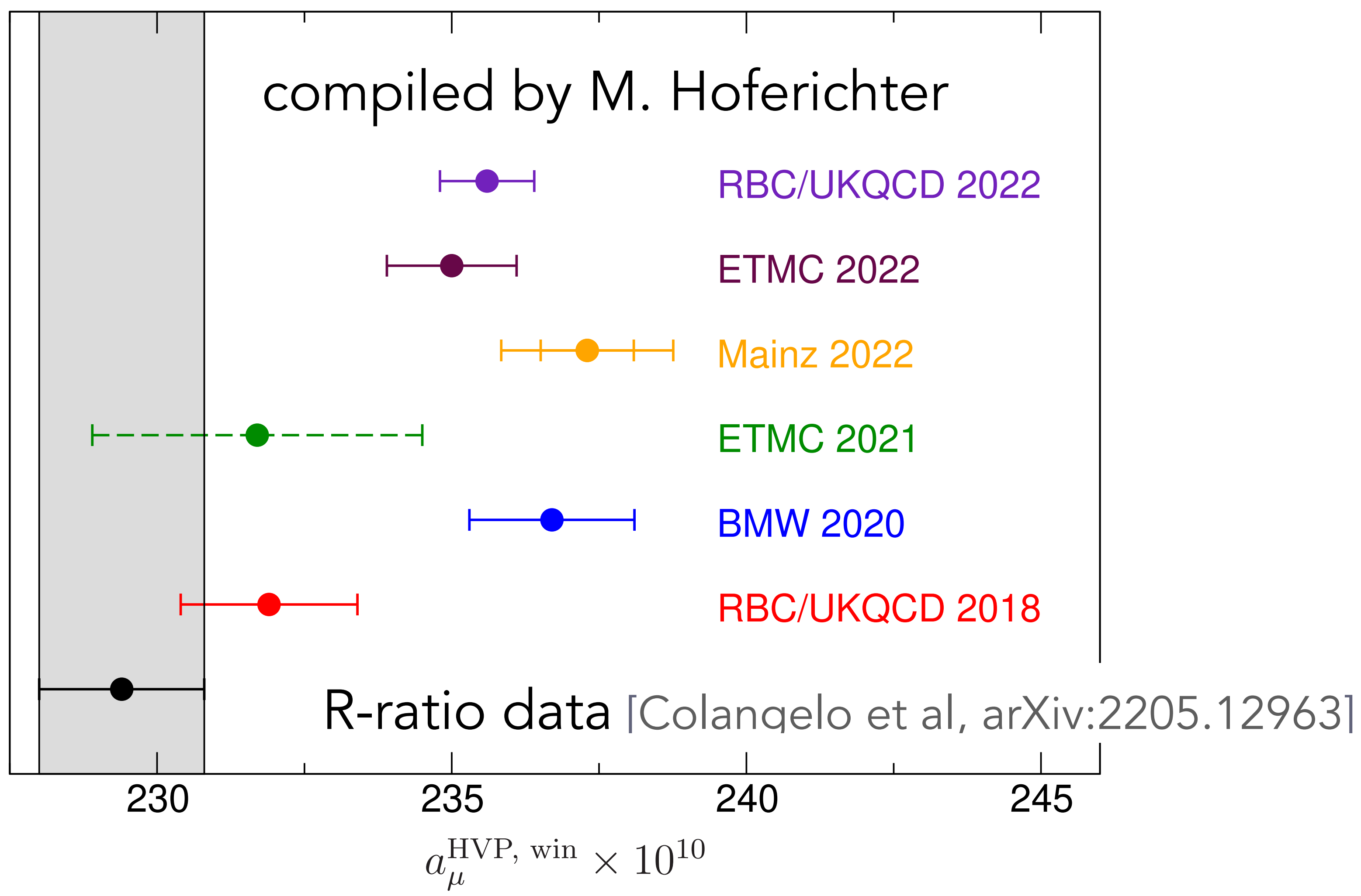


Note: int window ~ 1/3 of $a_\mu^{\mathrm{HVP,LO}}$

Ongoing work:

Evaluations of short-distance windows [ETMC, RBC/UKQCD]

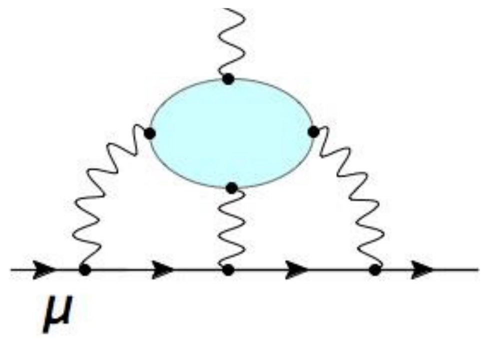
Proposals for computing more windows:

- Use linear combinations of finer windows to locate the tension (if it persists) in $\sqrt{s}$ [Colangelo et al, arXiv:2205.12963]
- Use larger windows, excluding the long-distance region $t \gtrsim 2\,\mathrm{fm}$ to maximize the significance of any tension [Davies at at, arXiv:2207.04765]

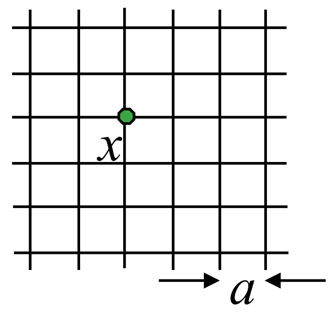
For total HVP:

- Still need independent, precise lattice results for the crucial long-distance contribution (~2/3 of $a_\mu^{\mathrm{HVP,LO}}$) ➠ coming soon!
- Including $\pi\pi$ states for refined long-distance computation [Mainz, RBC/UKQCD, FNAL/MILC]
- all groups plan to include smaller lattice spacings to test continuum extrapolations

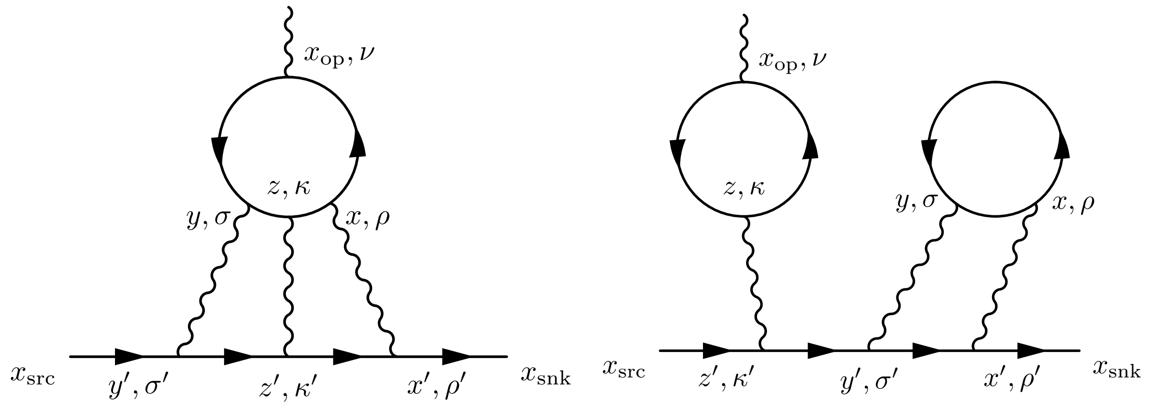
If results are consistent, Lattice HVP (average) with $\sim 0.5\,\%$ errors feasible by ~2025

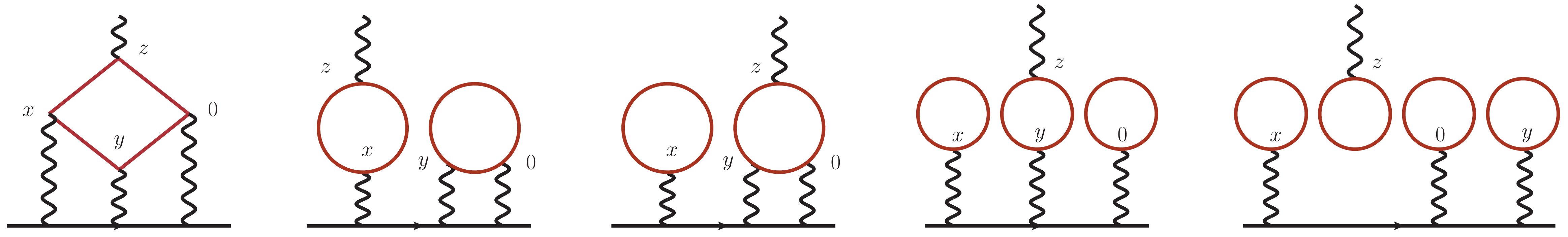


Hadronic Light-by-light



Lattice QCD+QED: Two independent and complete direct calculations of a_μ^{HLbL}





- ✦ RBC/UKQCD
  [T. Blum et al, arXiv:1610.04603, 2016 PRL; arXiv:1911.08123, 2020 PRL]
- ✦ QCD + $\mathrm{QED_L}$ (finite volume)

  DWF ensembles at/near phys mass,
  $a \approx 0.08 - 0.2\,\mathrm{fm},\ L \sim 4.5 - 9.3\,\mathrm{fm}$

- ✦ Mainz group
  [E. Chao et al, arXiv:2104.02632]
- ✦ QCD + QED (infinite volume & continuum)

  CLS (2+1 Wilson-clover) ensembles
  $m_\pi \sim 200 - 430\,\mathrm{MeV},\ a \approx 0.05 - 0.1\,\mathrm{fm},\ m_\pi L > 4$

- ✦ Cross checks between RBC/UKQCD & Mainz approaches in White Paper at unphysical pion mass
- ✦ Both groups are continuing to improve their calculations, adding more statistics, lattice spacings, physical mass ensemble (Mainz)
- ✦ update from RBC/UKQCD [T. Blum et al, arXiv:2304.04423] using QCD+QED (inf).

Lattice HLbL results with $10\,\%$ total uncertainty feasible by ~2025

Form factors for $B \to D^{(*)} \ell \nu_\ell$ and $|V_{cb}|$

$$\frac{d\Gamma(B \to D^* \ell \nu)}{dw} = (\text{known}) \times \eta_{\text{EW}}^2 (1 + \delta_{\text{EM}}) \times |V_{cb}|^2 \times (w^2 - 1)^{1/2} \times \chi(w) |\mathcal{F}(w)|^2$$

$$w = v_B \cdot v_{D^*}$$

$$\frac{d\Gamma(B \to D \ell \nu)}{dw} = (\text{known}) \times \eta_{\text{EW}}^2 (1 + \delta_{\text{EM}}) \times |V_{cb}|^2 \times (w^2 - 1)^{3/2} \times r^3 (1 + r)^2 |\mathcal{G}(w)|^2$$

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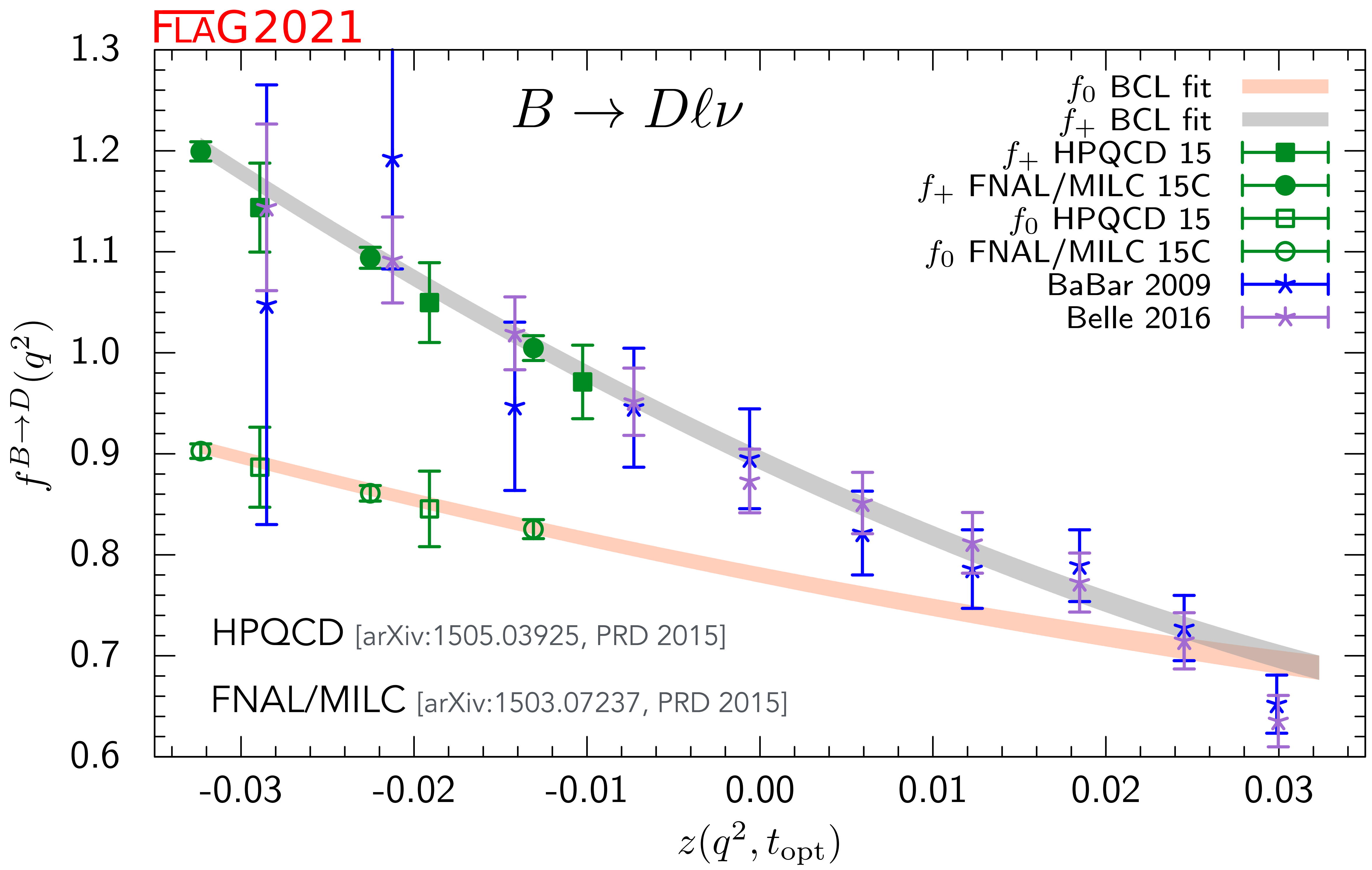

- shape of LQCD form factor agrees with experiment
- fit LQCD form factors together with experimental data to determine $|V_{cb}|$
- The form factors obtained from the combined exp/ lattice fit are well determined over entire recoil range.
- Can be used for an improved SM prediction of *R*(*D*).

Form factors for $B \to D^* \ell \nu_\ell$ and $|V_{cb}|$

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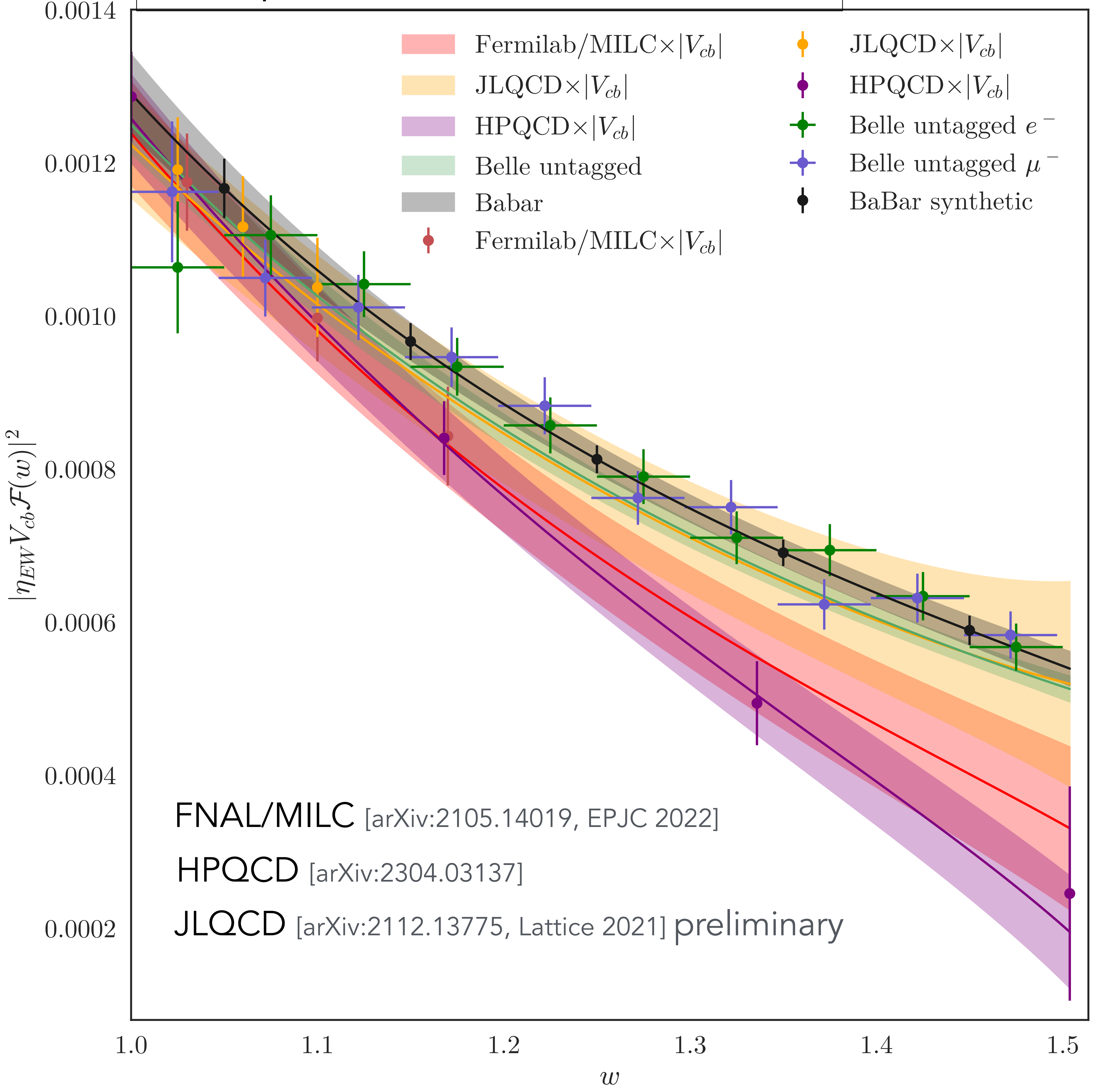
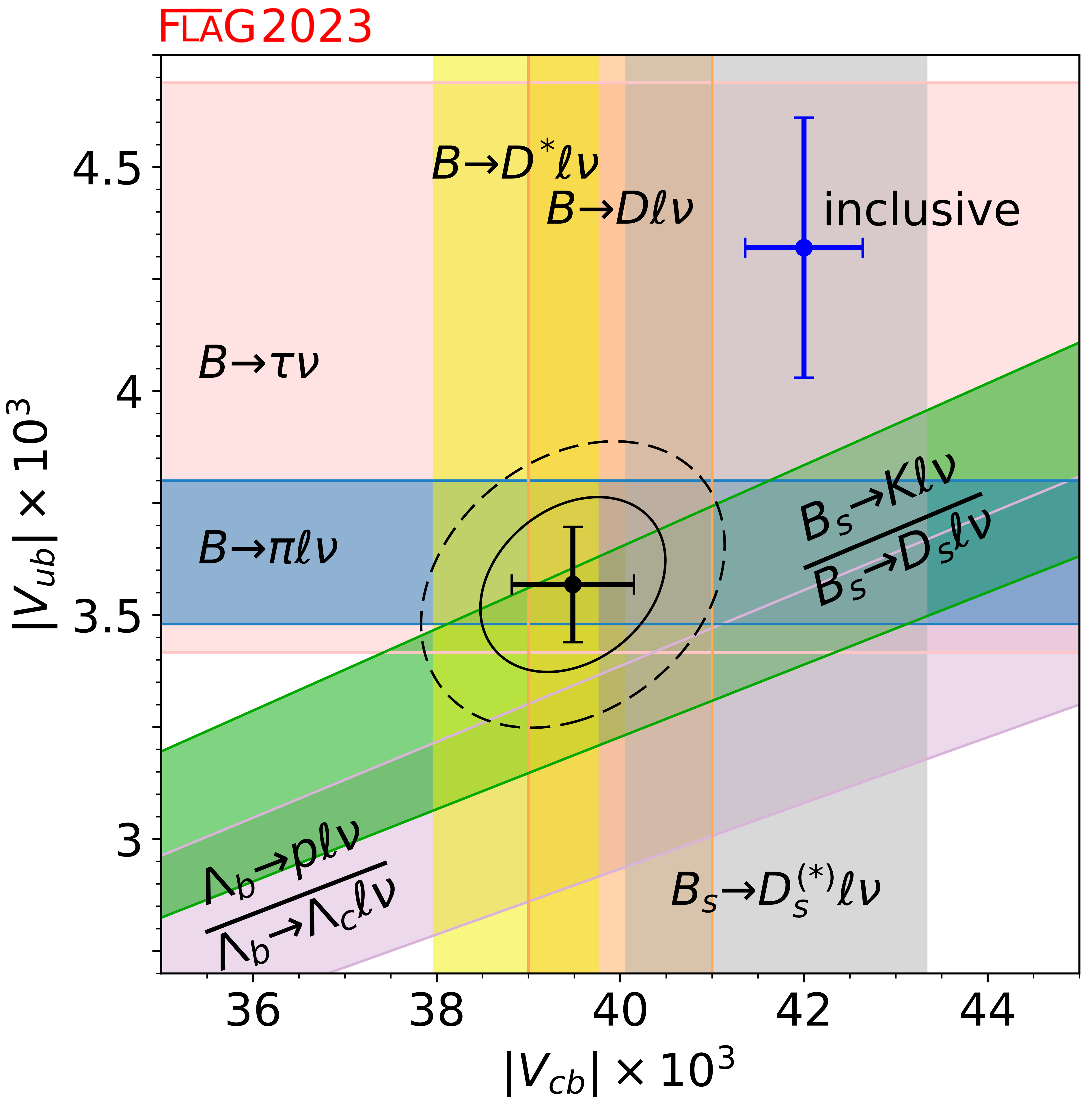
A. Vaquero CERN Flavour@TH 2023



- mild tensions between LQCD form factors from Fermilab/MILC, HPQCD, JLQCD
- shapes of Fermilab/MILC & HPQCD form factors in $\sim 2\sigma$ tension with experimental data
- new determinations of $|V_{cb}|$ and $R(D^*)$ consistent with previous ones.
- new strategies for dealing with truncation effects in z-expansion fits to LQCD form factors [Flynn et al, arXiv:2303.11280]
- see talk by A. Jüttner @ LHCP (Friday, Flavor Physics)

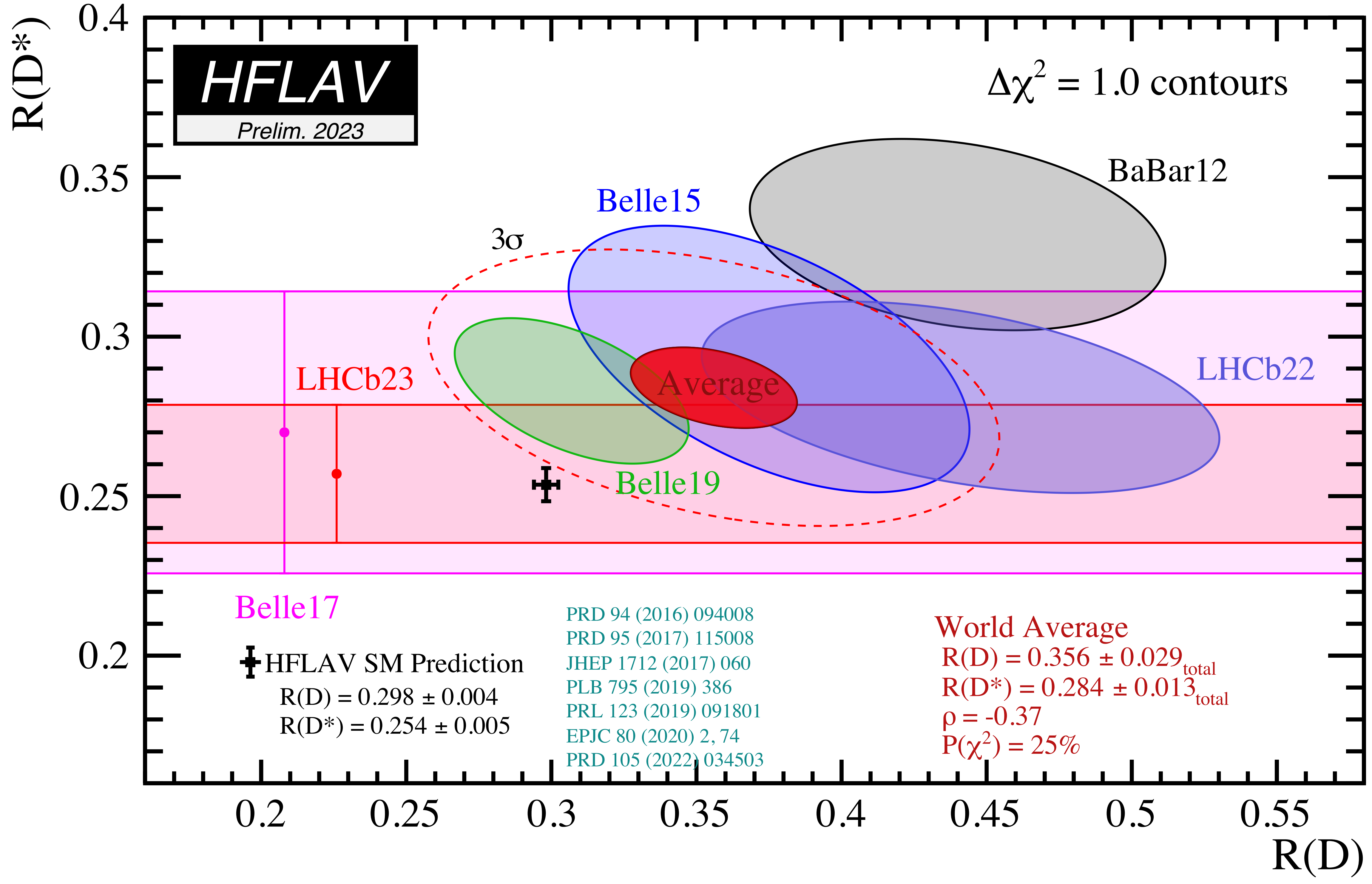
new Belle analysis: arXiv:2301.07529 now in hepdata

Implications: $|V_{cb}|$, $R(D^{(*)})$



Tensions between inclusive and exclusive $|V_{cb}|$ (and $|V_{ub}|$) determinations persist.

$$R(D^{(*)}) = \frac{\mathcal{B}(B\to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B\to D^{(*)}\ell\nu)}$$



$\sim 3\sigma$ tension between SM and Exp.

New LQCD results not yet included

Summary & Outlook

$m_{ud},\, m_u/m_d,\, m_s$ ⬅ $a_\mu^{\text{HVP LO}}$ a_μ^{HLbL} d_n MEs for light nuclei

$m_c,\, m_b$ $\langle \bar{B}_q^0 | \mathcal{O}_i^{\Delta B=2} | B_q^0 \rangle$ PDFs, GPDs, TMDs,.. $\langle NN | O_i | NN \rangle$

$\alpha_{\overline{MS}}(m_z)$ $\langle \bar{D}^0 | \mathcal{O}_i^{\Delta C=2} | D^0 \rangle$ $\Lambda_b \to (p, \Lambda_c, \Lambda)\, \ell\nu$ $D_s \to \ell\nu\gamma$ $B \to \pi\pi\, \ell\nu\, \ldots$

$\hat{B}_K, \ldots$ nucleon form factors, .. $K^+ \to \ell^+\nu\,(\gamma)$ $B \to X_c \ell\nu$, other inclusive decay rates, …

$f_{B_{(s)}}, \ldots$ $g_A,\, g_T,\, g_S$ $\pi\pi \to \rho$ $\pi \to \ell^+\ell^-,\, K \to \gamma\gamma$

$f^{K\to\pi}(q^2)$ $f_{+,0,T}^{B\to D}(q^2), \ldots$ $\langle \pi\pi_{(I=0)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$ $K^+ \to \pi^+\ell^+\ell^-$ …

$f_{K^\pm}$ $f_{+,0,T}^{B\to\pi}(q^2), \ldots$ $\langle \pi\pi_{(I=2)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$ $\Delta M_K,\, \epsilon_K$ $K^+ \to \pi^+\nu\bar{\nu}$

Complexity →

✓ LQCD flagship results

↑ First complete LQCD results, large(ish) errors

↑ First results, physical params, incomplete systematics

↑ new methods, pilot projects, unphysical kinematics

↑ new ideas, first studies

Summary & Outlook

$m_{ud},\, m_u/m_d,\, m_s$ ⇐ $a_\mu^{\text{HVP LO}}$ a_μ^{HLbL} d_n MEs for light nuclei

$m_c,\, m_b$

$\langle \bar{B}^0_q | \mathcal{O}_i^{\Delta B=2} | B^0_q \rangle$

PDFs, GPDs, TMDs,..

$\langle NN | O_i | NN \rangle$

$\alpha_{\overline{MS}}(m_z)$

$\langle \bar{D}^0 | \mathcal{O}_i^{\Delta C=2} | D^0 \rangle$ $\Lambda_b \to (p, \Lambda_c, \Lambda)\, \ell\nu$ $D_s \to \ell\nu\gamma$ $B \to \pi\pi\, \ell\nu$ ...

$\hat{B}_K, \ldots$ nucleon form factors, .. $K^+ \to \ell^+\nu\,(\gamma)$ $B \to X_c \ell\nu$, other inclusive decay rates, ...

$g_A,\, g_T,\, g_S$

$f_{B_{(s)}}, \ldots$ $\pi\pi \to \rho$ $\pi \to \ell^+\ell^-,\, K \to \gamma\gamma$

$f^{K\to\pi}(q^2)$ $f^{B\to D}_{+,0,T}(q^2), \ldots$ $\langle \pi\pi_{(I=0)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$ $K^+ \to \pi^+\ell^+\ell^-$ ...

$f_{K^\pm}$ $f^{B\to\pi}_{+,0,T}(q^2), \ldots$ $\langle \pi\pi_{(I=2)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$ $\Delta M_K,\, \epsilon_K$ $K^+ \to \pi^+\nu\bar{\nu}$

Complexity →

✓ LQCD flagship results

First complete LQCD results, large(ish) errors

First results, physical params, incomplete systematics

new methods, pilot projects, unphysical kinematics

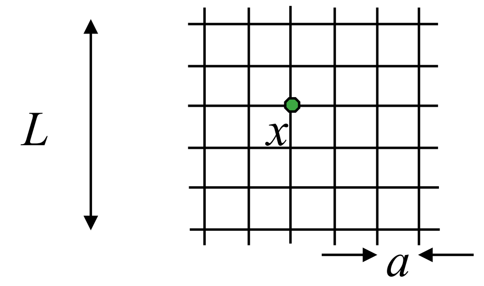
new ideas, first studies

Topics not covered (incomplete list)

- PDFs: huge progress and much new theoretical work since 2013
  [X. Ji arXiv:1305.1535, PRL 2013]
- hot QCD
- hadron spectroscopy, exotics, scattering phase shifts
- inclusive decay rates (appendix)
- Semileptonic B-meson decay form factors (Jüttner) + baryons ffs
- B mixing (Tsang)
- First and second row CKM unitarity
- QED corrections and radiative decay rates
- kaon mixing, $\Delta M_K, \epsilon'$
- nucleon matrix elements and charges
- two and few nucleon systems
- …

Farah Willenbrock

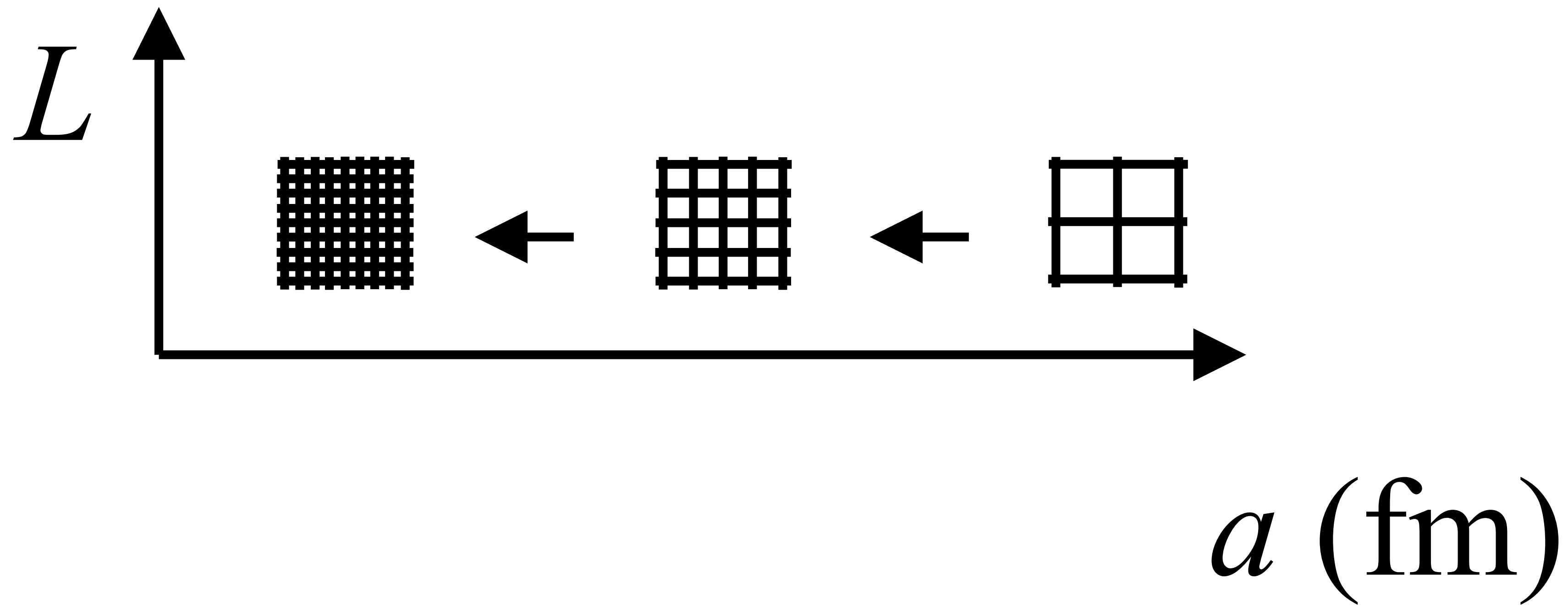
Appendix

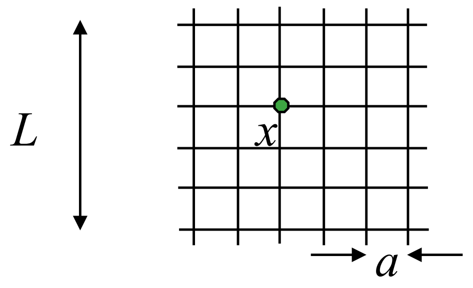


Lattice QCD Introduction

discretization effects — continuum extrapolation

- typical momentum scale of quarks gluons inside hadrons: $\sim\Lambda_{\rm QCD}$
- make a small to separate the scales: $\Lambda_{\rm QCD} \ll 1/a$
- Symanzik EFT: $\langle \mathcal{O} \rangle^{\rm lat} = \langle \mathcal{O} \rangle^{\rm cont} + O(a\Lambda)^n$, $n \geq 2$
  - provides functional form for extrapolation (depends on the details of the lattice action)
  - can be used to build improved lattice actions
  - can be used to anticipate the size of discretization effects





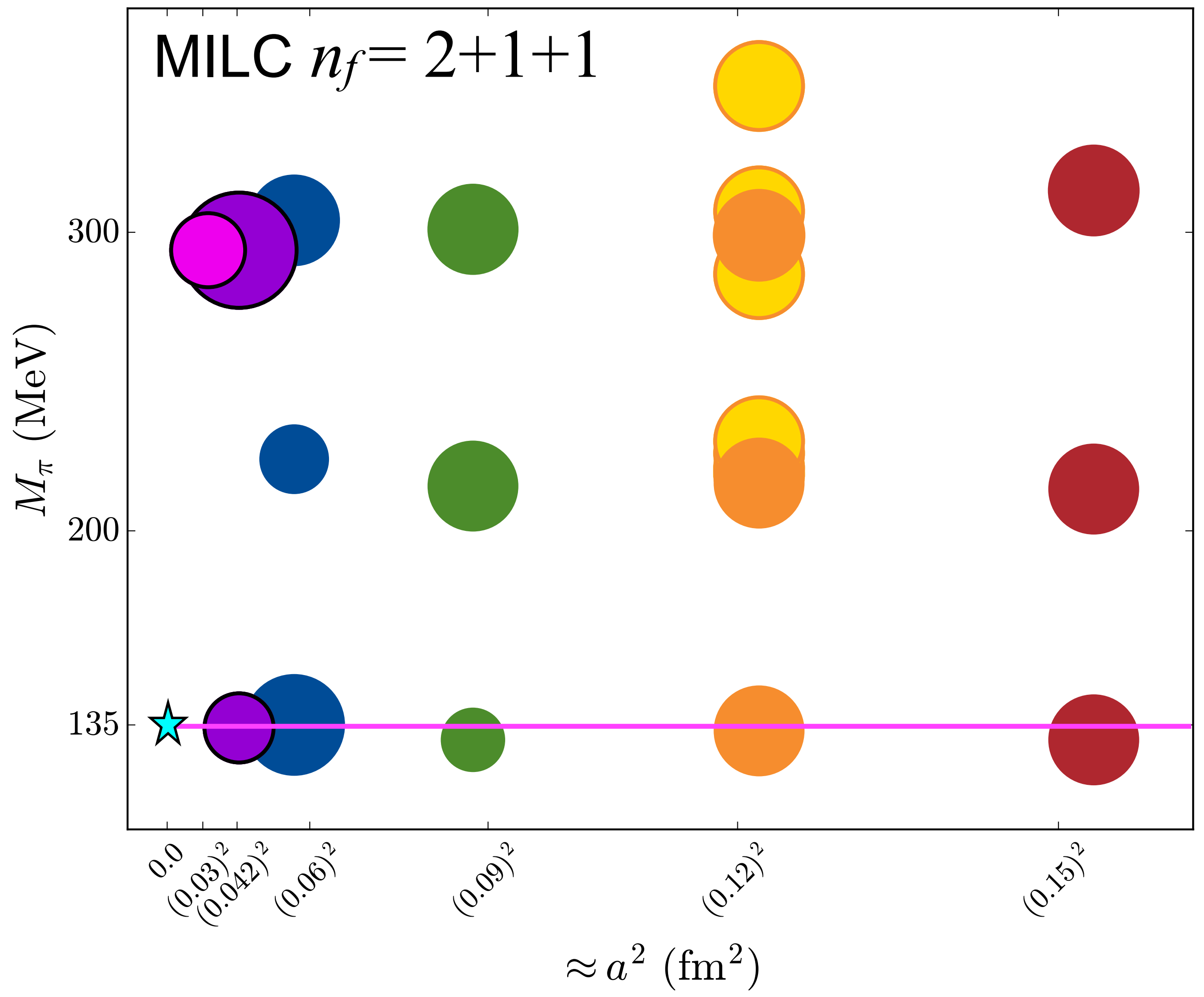
Lattice QCD Introduction: quark discretizations

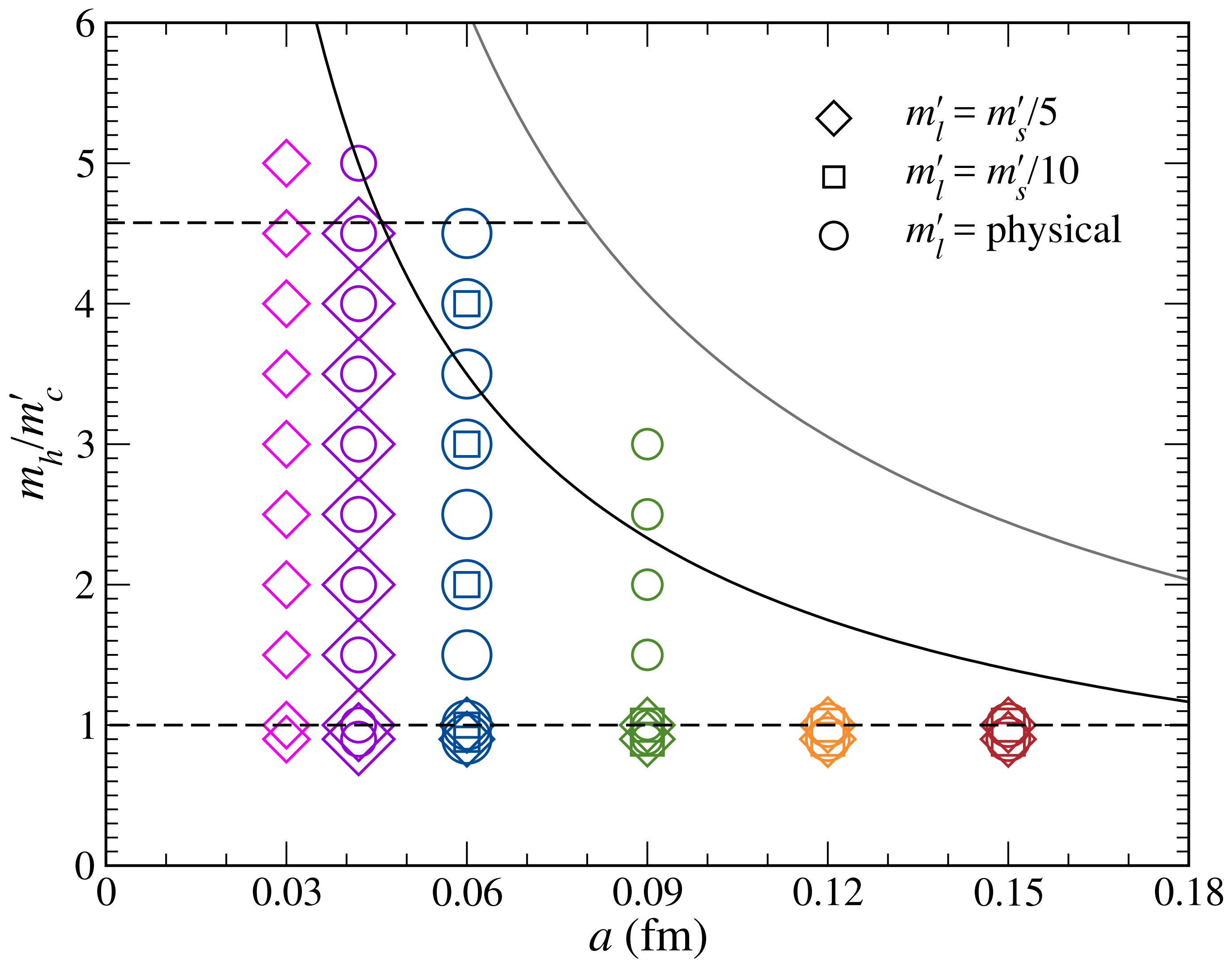
Fermion doubling problem ⇔ chiral symmetry

- Staggered quarks (a.k.a Kogut-Susskind)
  reduce the number of doublers (staggering) but keep some (a.k.a tastes)
  dominant discretization effects due to taste-breaking effects (can be corrected analytically) $\sim O(a^2)$
  various improved versions to reduce taste-breaking effects (HISQ,..)
  computationally inexpensive
- (improved) Wilson quarks
  no doublers, but chiral symmetry broken explicitly
  requires improvement to remove $O(a)$ effects (NP improved, twisted mass, …)
  moderate computational cost
- Domain wall quarks (live in 5 dimensions)
  no doublers, chiral symmetry exponentials suppressed
  small $O(a^2)$ discretization effects
  high computational cost
- …

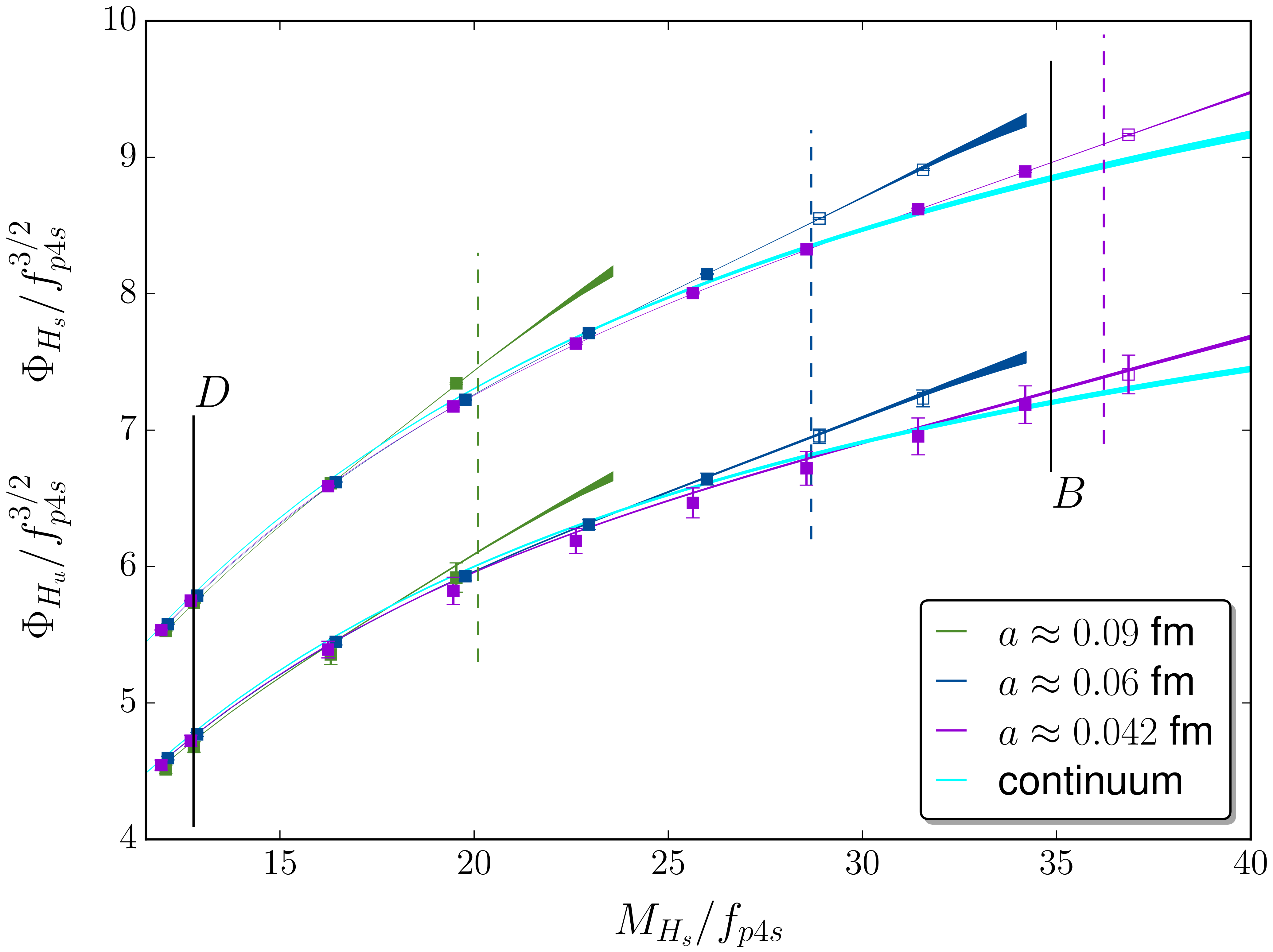
- new ideas:
  workshop on novel fermion actions
  https://indico.mitp.uni-mainz.de/event/314/

B meson decay constants





A. Bazavov et al [FNAL/MILC, arXiv:1712.09262, 2018 PRD]



systematic error budget

Error (%)	f_{B^0}	f_{B_s}	f_{B_s}/f_{B^0}
Statistics and EFT fit	0.39	0.36	0.24
Two-point correlator fits	0.39	0.22	0.17
Fit model	0.34	0.39	0.08
Scale-setting quantities and tuned quark masses	0.10	0.06	0.05
Finite-volume corrections	0.03	0.01	0.02
Electromagnetic corrections	0.02	0.02	0.01
Topological charge distribution	0.07	0.00	0.07
$f_{\pi,\text{PDG}}$	0.14	0.11	0.04

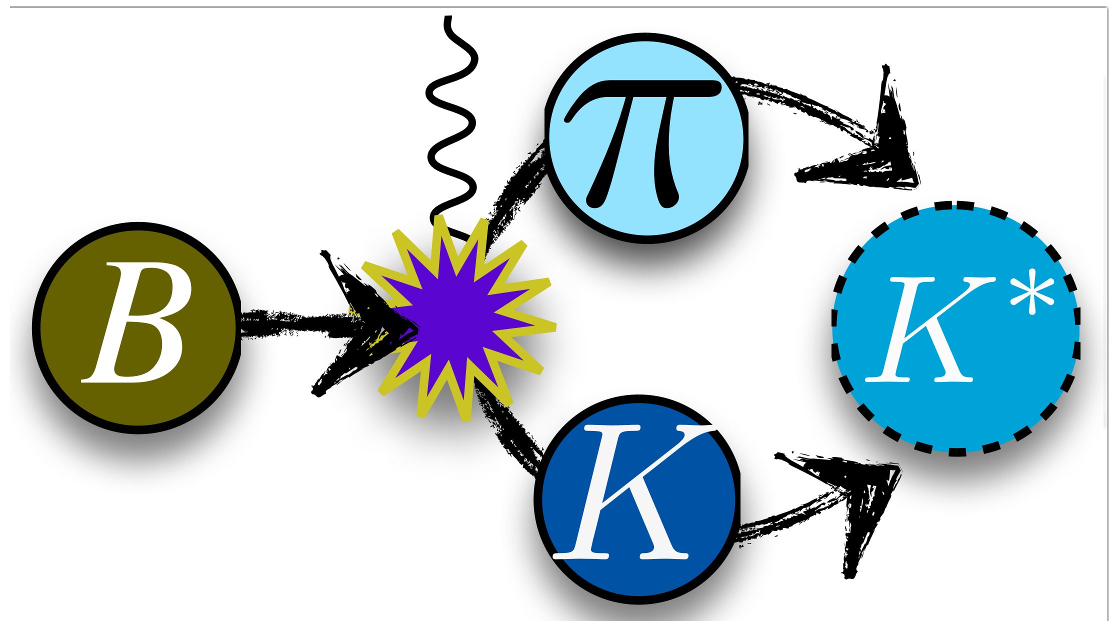
small errors due to **physical light quark masses**
improved quark action with small discretization errors even for heavy quarks
no renormalization (Ward identity)

Semileptonic B decays to vector mesons:

existing LQCD results for $B \to K^*, B_s \to \phi$ form factors assume stable K^*, ϕ (narrow width approximation)
[R. Horgan et al, arXiv:1310.3887, 1310.3722, 1501.00367]

Formalism for multi-channel 1→ 2 transition amplitudes:
[Briceno, Hansen, Walker-Loud, arXiv:1406.5965, PRD 2015;1502.04314, PRD 2015,…]

weak current



[Figure by R. Briceno]

pilot study [Agadjanov et al, arXiv:1605.03386, NPB 2016]

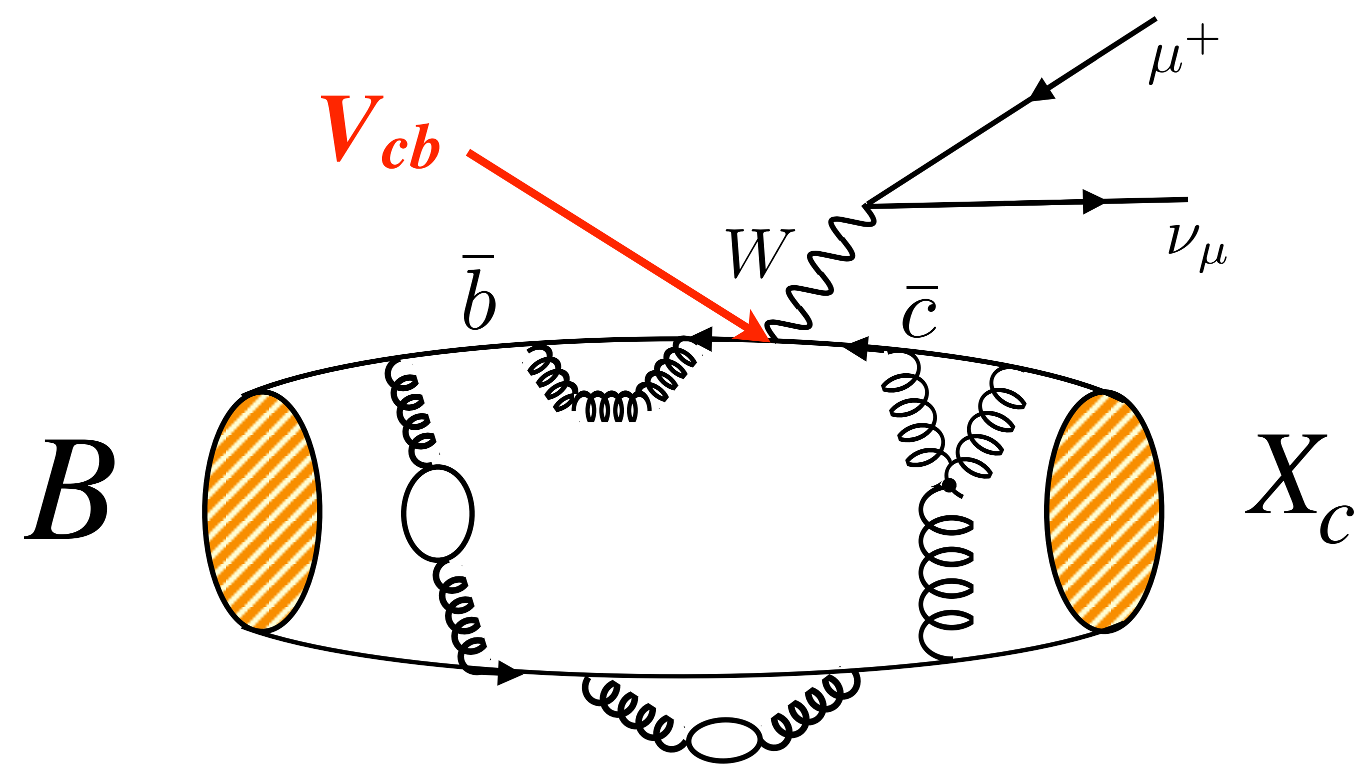
Limitations:
- q^2 reach: small recoil
- invariant mass of two-hadron system: $< 3\ m_H$
- recent work to extend formalism to 3 hadrons
  [M. Hansen et al, arXiv:2101.10246]

studies of $K\pi$ scattering
[G. Rendon et al, arXiv:1811.10750;
D. Wilson et al, arXiv:1904.03188]

preliminary results for $B \to \pi\pi\ell\nu$ form factor with $m_\pi \simeq 320\,\mathrm{MeV}$
[L. Leskovec et al, arXiv:2212.08833]

Inclusive decay rates with lattice QCD

For example: $B \to X_c \ell \nu_\ell$



Sum over final states:

$$X_c = D, D^*, D\pi, D\pi\pi, D^{**}, \ldots$$

Use OPE + pert. QCD to write $d\Gamma$ as a double expansion:

$$d\Gamma \sim \sum_n c_n \frac{\langle O_n \rangle}{m_b^n}$$

- c_n are calculated in perturbation theory
- $\langle O_n \rangle$ are matrix elements of local operators

Target: $d\Gamma \sim |V_{cb}|^2 L^{\mu\nu} W_{\mu\nu}$

$$W_{\mu\nu} = \frac{1}{2M_B} \int d^4x e^{-iqx} \langle B | J_\mu^\dagger(x) J_\nu(0) | B \rangle$$

Start with Euclidean four-point function:

$$C_4(q,\tau) = \sum_x e^{iqx} \frac{1}{2M_B} \langle B | J_\mu^\dagger(x) J_\nu(0) | B \rangle$$

- new methods to perform inverse Laplace transform
[Liu & Dong (PRL 1994);Liu (PRD 200);Jian et al (1710.11145); Hansen, Meyer, Robaina (1703.01881, PRD 2017); M. Hansen et al, arXiv:1903.06476; P. Gambino & S. Hashimoto, arXiv:2005.13730; J. Bulava et al, arXiv:2111.12774]

- first applications to $B \to X_c \ell \nu$: good agreement with OPE
[P. Gambino et al, arXiv:2203.11762; A. Barone et al, arXiv:2305.14092]

Muon g-2 Theory Initiative

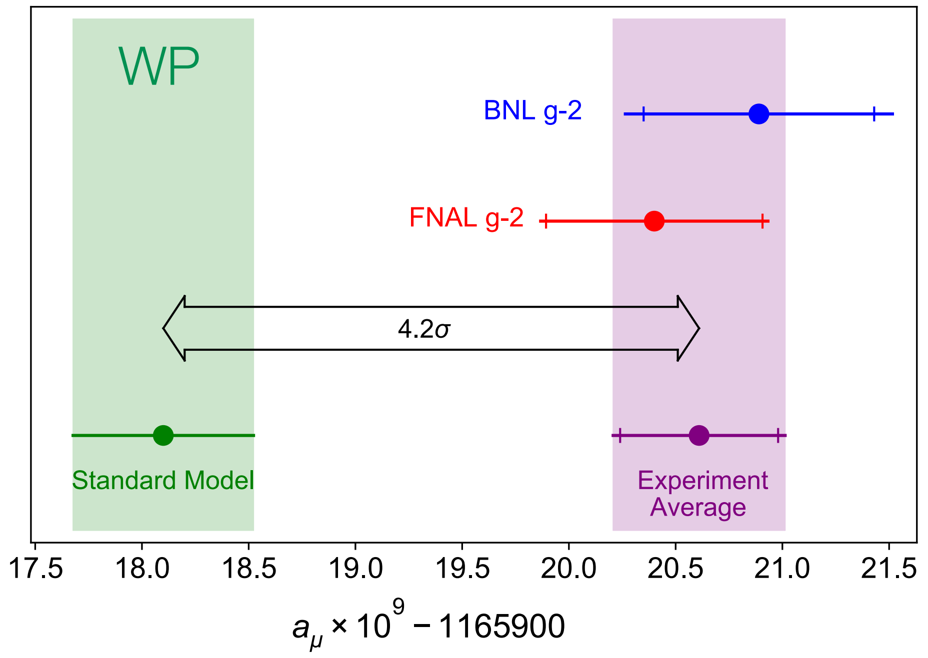
Steering Committee

- Gilberto Colangelo (Bern)
- Michel Davier (Orsay) co-chair
- Aida El-Khadra (UIUC & Fermilab) chair
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University) co-chair
- Laurent Lellouch (Marseille)
- Tsutomu Mibe (KEK)
  J-PARC Muon g-2/EDM experiment
- Lee Roberts (Boston)
  Fermilab Muon g-2 experiment
- Thomas Teubner (Liverpool)
- Hartmut Wittig (Mainz)

https://muon-gm2-theory.illinois.edu

- Maximize the impact of the Fermilab and J-PARC experiments
  ➠ quantify and reduce the theoretical uncertainties on the hadronic corrections
- summarize the theory status and assess reliability of uncertainty estimates
- organize workshops to bring the different communities together:
  First plenary workshop @ Fermilab: 3-6 June 2017
  HVP workshop @ KEK: 12-14 February 2018
  HLbL workshop @ U Connecticut: 12-14 March 2018
  Second plenary workshop @ HIM (Mainz): 18-22 June 2018
  Third plenary workshop @ INT (Seattle): 9-13 September 2019
  Lattice HVP at high precision workshop (virtual): 16-20 November 2020
  Fourth plenary workshop @ KEK (virtual): 28 June - 02 July 2021
  Fifth plenary workshop @ Higgs Centre (Edinburgh): 5-9 September 2022
  Sixth plenary workshop @ University of Bern: 4-8 September 2023
  Seventh plenary workshop @ KEK (Japan): June 2024

Near-term Timeline



Theory Initiative:

- ☆ ongoing activities: develop method average for Lattice HVP
- ☆ CMD-3 seminar (virtual): **27 March 2023** at 8:00am US CDT
- ☆ WP update with all available results ~ late 2023

★ TI workshops:
Jun 2021 @ KEK (virtual)
Sep 2022 @ Higgscentre
Sep 2023 @ Bern
Summer 2024 @ KEK

Updated WP Summary Table

Contribution	Value $\times 10^{11}$	References
Experimental average (E989+E821)	116592061(41)	Phys.Rev.Lett. 124, 141801
HVP LO (e^+e^-)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	2(1)	Ref. [31]
HLbL (lattice, uds)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	90(17)	Refs. [18–30, 32]
QED	116 584 718.931(104)	Refs. [33, 34]
Electroweak	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	92(18)	Refs. [18–32]
Total SM Value	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)	

website: https://muon-gm2-theory.illinois.edu

Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

QED

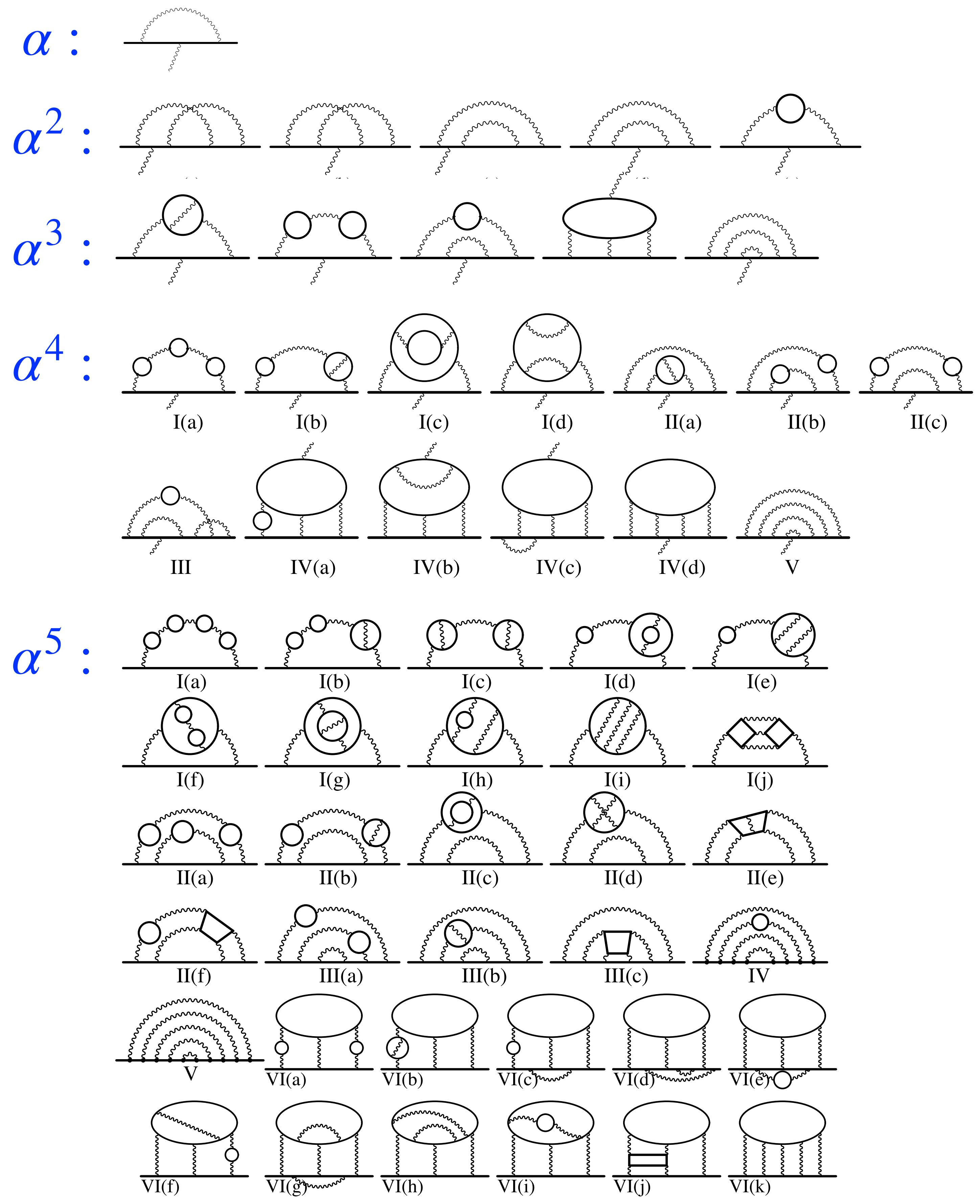
$$a_\mu(\text{QED}) = A_1 + A_2\left(\frac{m_\mu}{m_e}\right) + A_2\left(\frac{m_\mu}{m_\tau}\right) + A_3\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right)$$

$$A_i = \sum_{n=0}\left(\frac{\alpha}{\pi}\right)^n A_i^{2n}$$

n	# of diagrams	Contribution x 10^{11}
1	1	116140973.32
2	7	413 217.63
3	71	30141.90
4	891	381.00
5	12672	5.08

$$a_\mu(\text{QED}) = 116\,584\,718.9\,(1) \times 10^{-11}$$

[T. Aoyama et al, arXiv:1205.5370, PRL;
T. Aoyama, T. Kinoshita, M. Nio, Atoms 7 (1) (2019) 28]

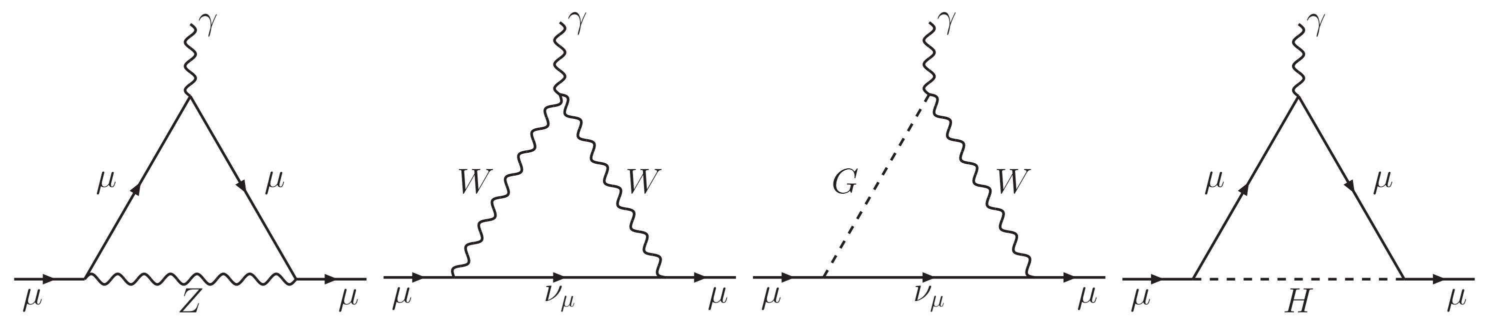


Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

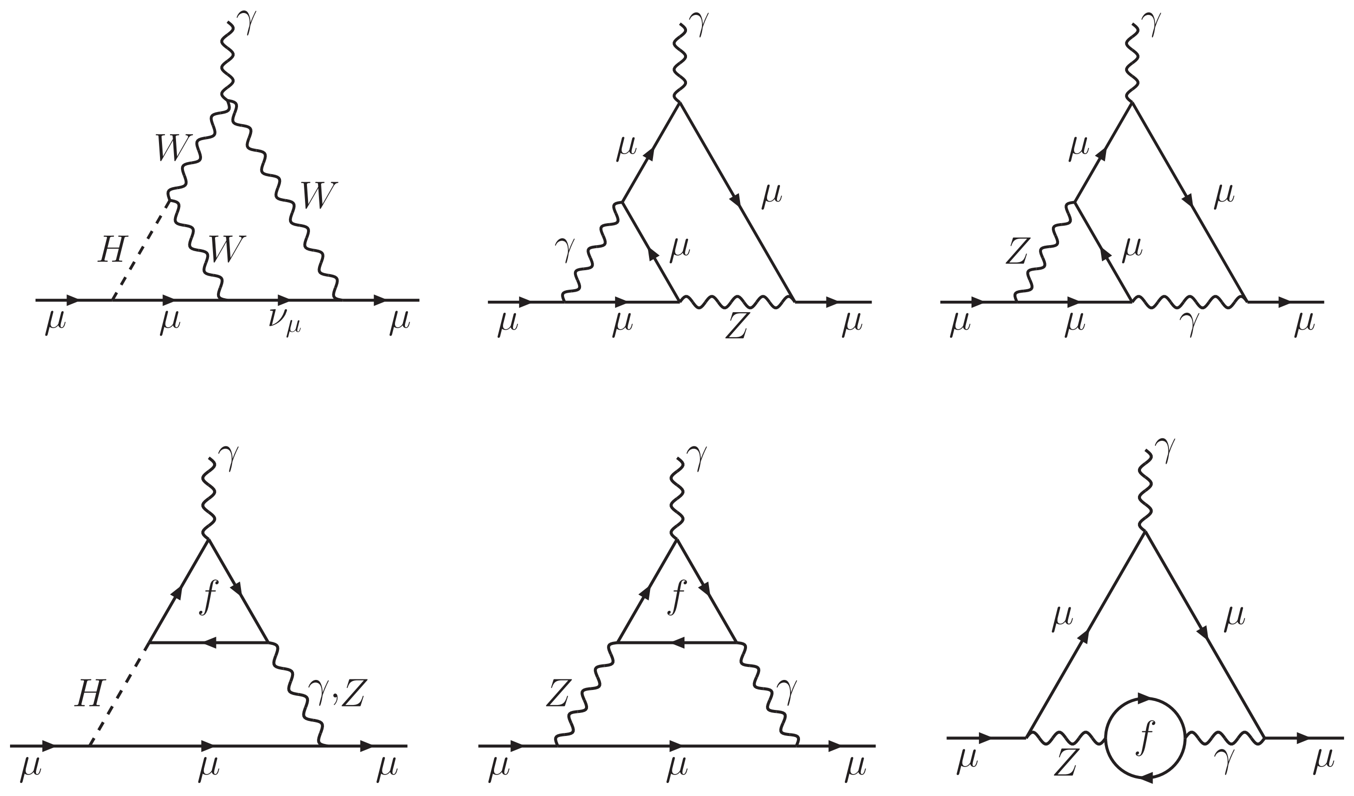
Electroweak
(contributions from W,Z,H bosons)

1-loop



Compared to QED, suppressed by $\sim \frac{m_\mu^2}{M_W^2} \sim 10^{-6}$

2-loop



$$a_\mu(\text{EW}) = 153.6\,(1.0) \times 10^{-11}$$

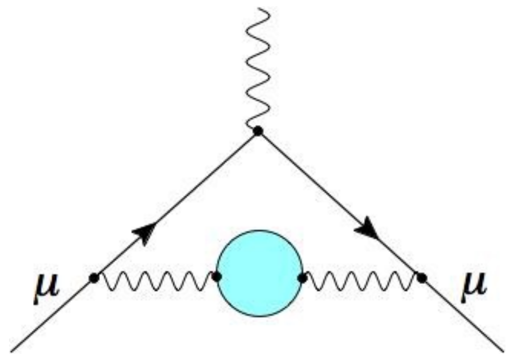
[A. Czarnecki et al, hep-ph/0212229, PRD;
C. Gnendinger et al, arXiv:1306.5546, PRD]

Muon g-2: SM contributions

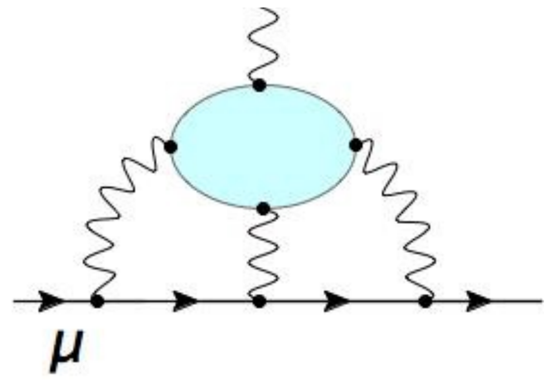
$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

leading hadronic

α^2



α^3

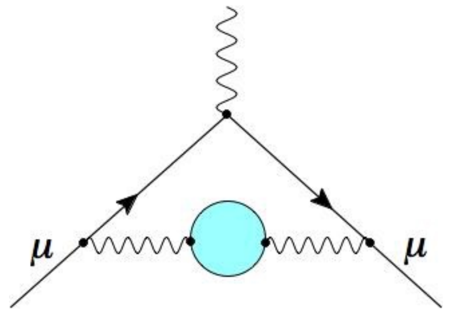


✦ The hadronic contributions are written as:

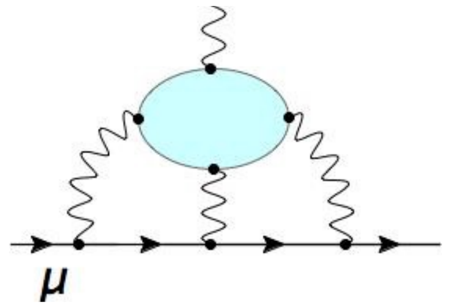
$$a_\ell(\text{hadronic}) = a_\ell^{\text{HVP, LO}} + a_\ell^{\text{HVP, NLO}} + a_\ell^{\text{HVP, NNLO}} + \ldots$$
$$+ a_\ell^{\text{HLbL}} + a_\ell^{\text{HLbL, NLO}} + \ldots$$

α^2 α^3 α^4

$\sim 10^{-7}$



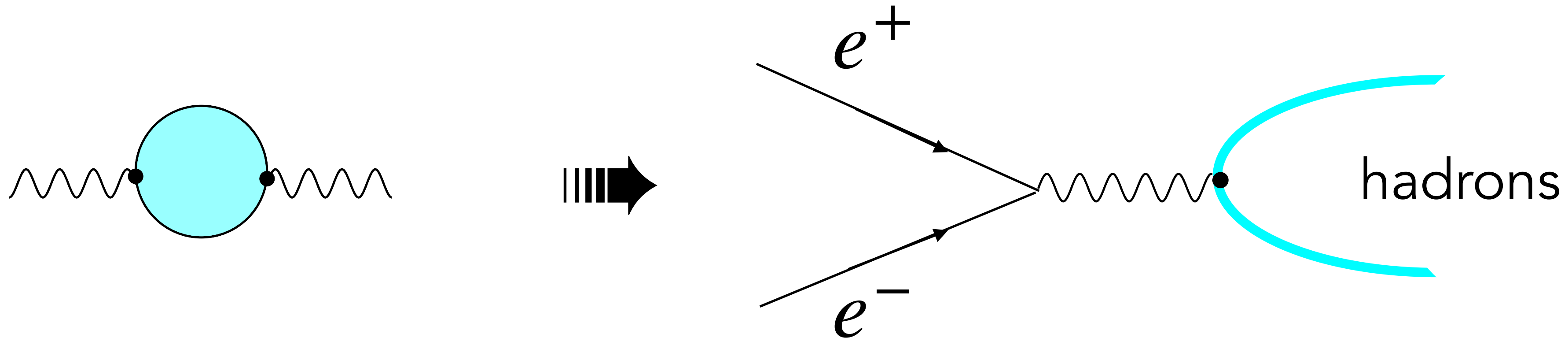
Muon g-2: hadronic corrections



1. Dispersive data-driven approach:

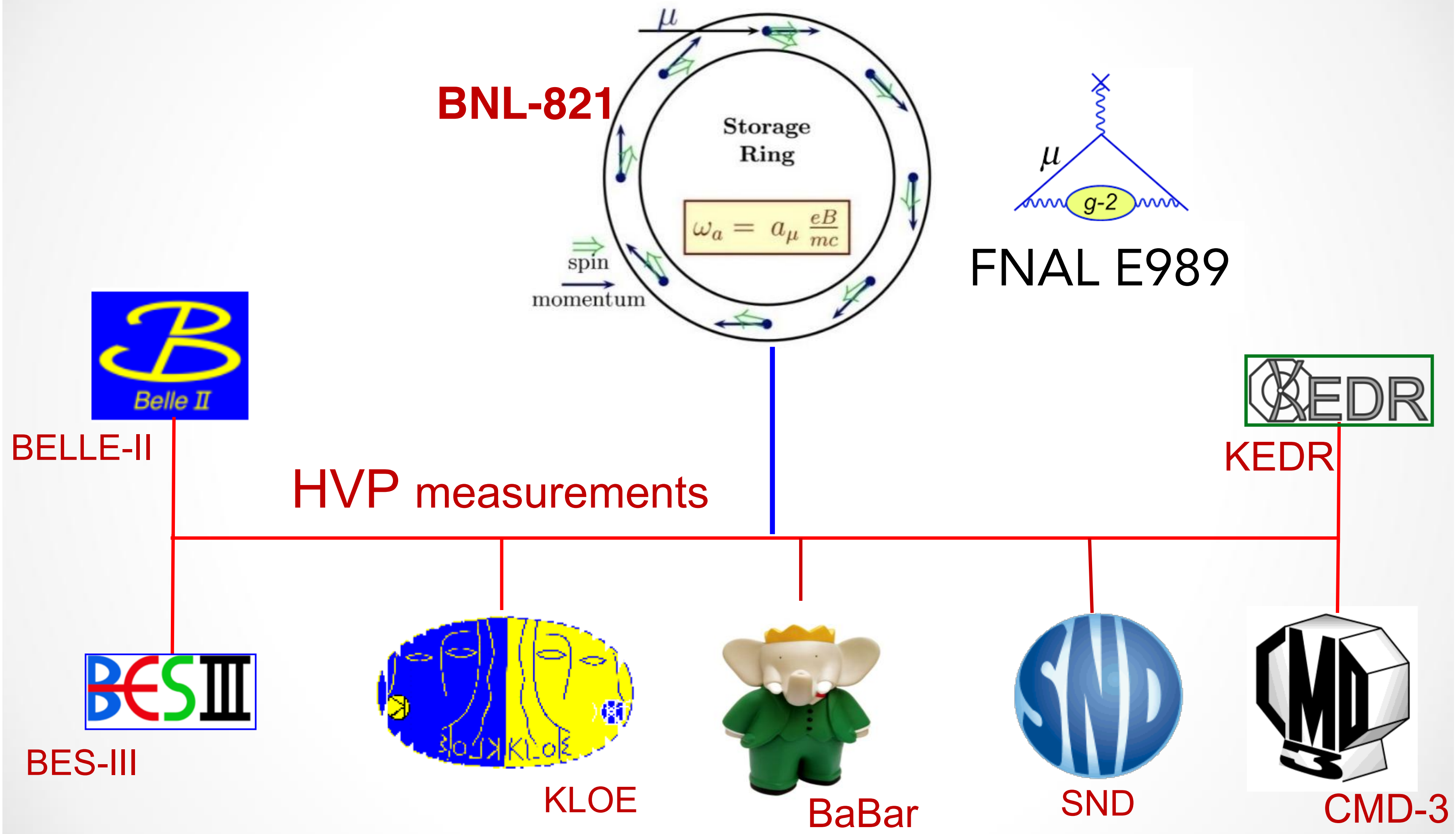
Use experimental data together with dispersion theory. For example:

HVP:

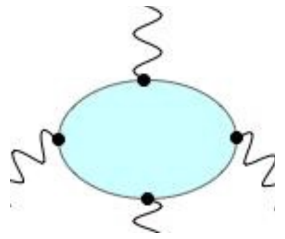


Many experiments (over 20+ years) have measured the e^+e^-cross sections for the different channels over the needed energy range with increasing precision. The combined data + dispersion theory yield HVP with a current error ~ 0.6%.

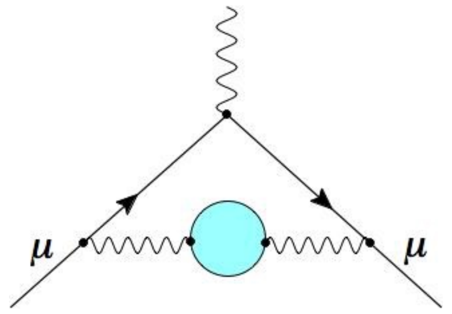
S. Serednyakov (for SND)
@ HVP KEK workshop



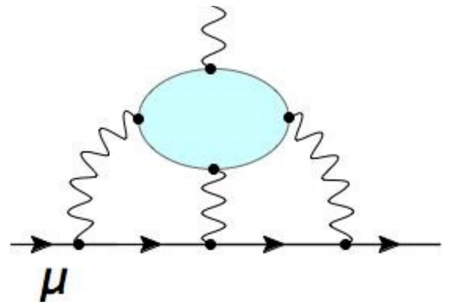
HLbL:



New dispersive approach now also allows for data-driven evaluations of HLbL, currently ~20% error ➠ theory error is (almost) completely quantified. Replaces previous results obtained using simplified models of QCD.



Muon g-2: hadronic corrections



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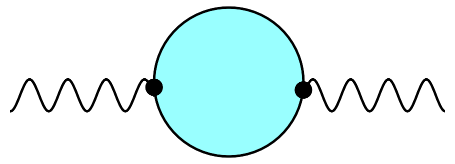
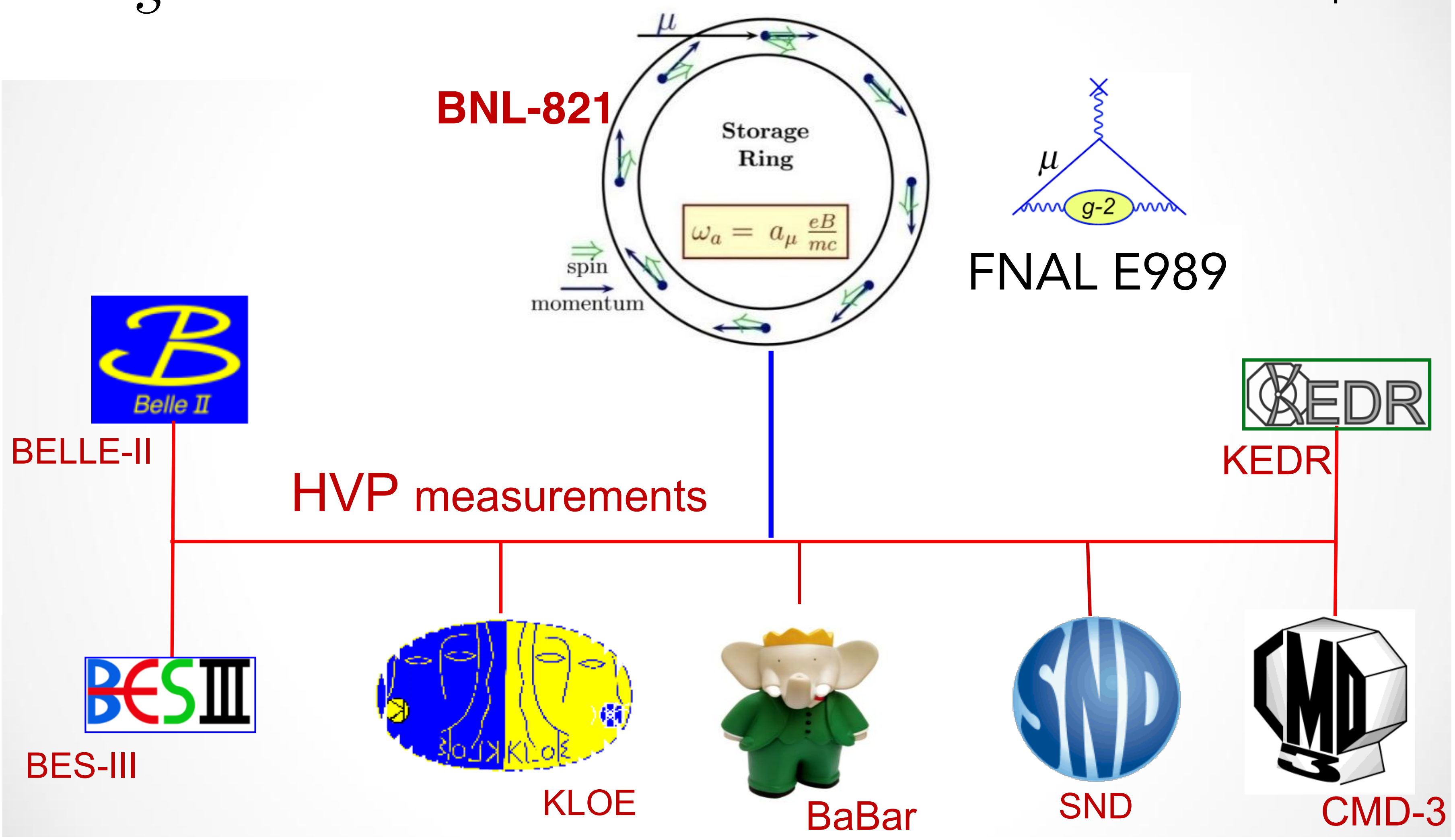
HVP:  ➠

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\,\hat{\Pi}(q^2) = \frac{m_\mu^2}{12\pi^3}\int ds\,\frac{\hat{K}(s)}{s}\,\sigma_{\mathrm{exp}}(s)$$

S. Serednyakov (for SND)
@ HVP KEK workshop



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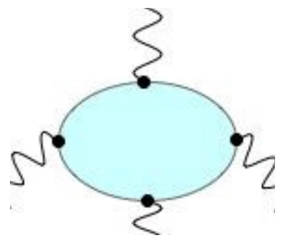
HLbL: 

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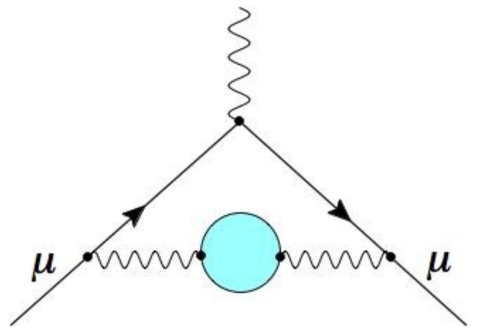
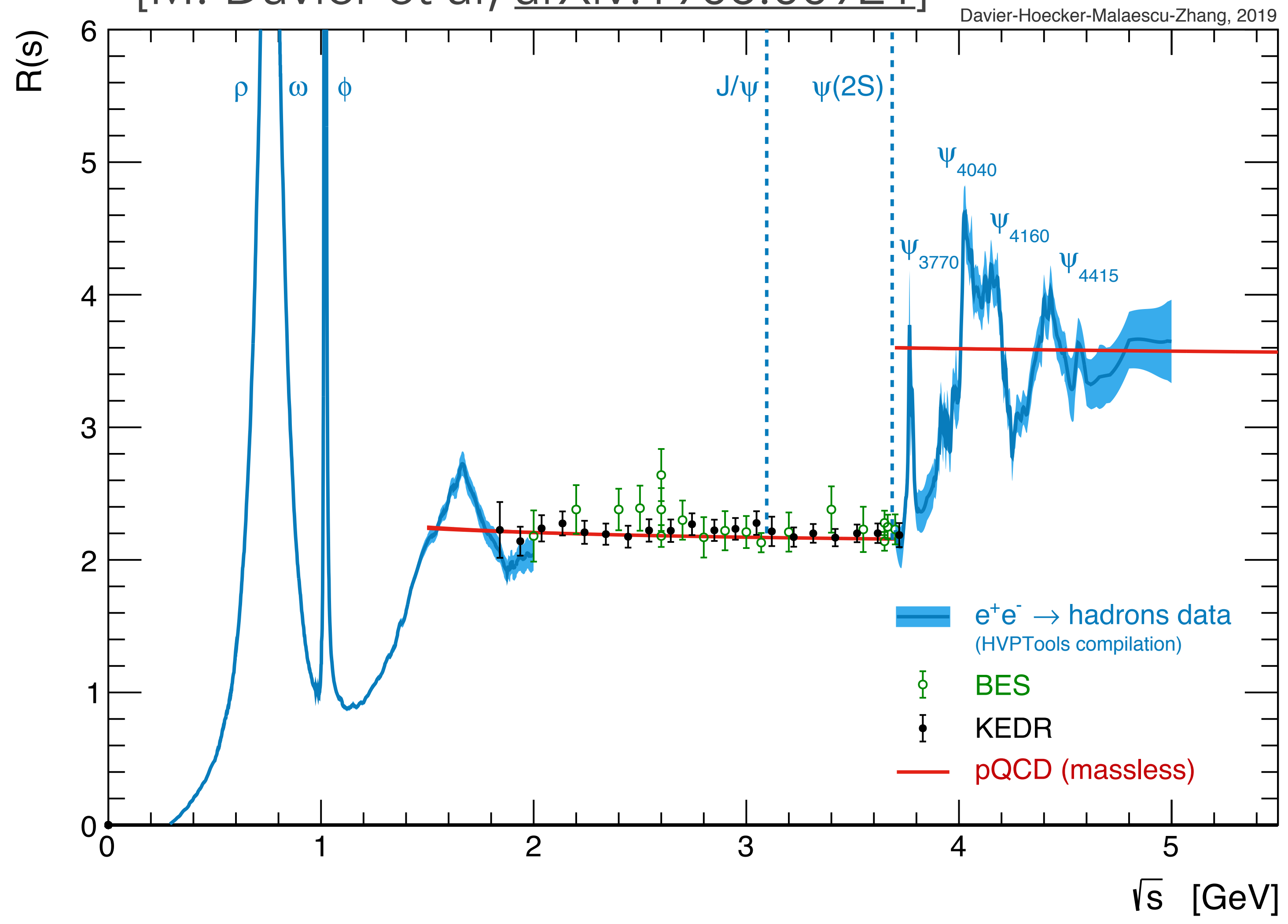
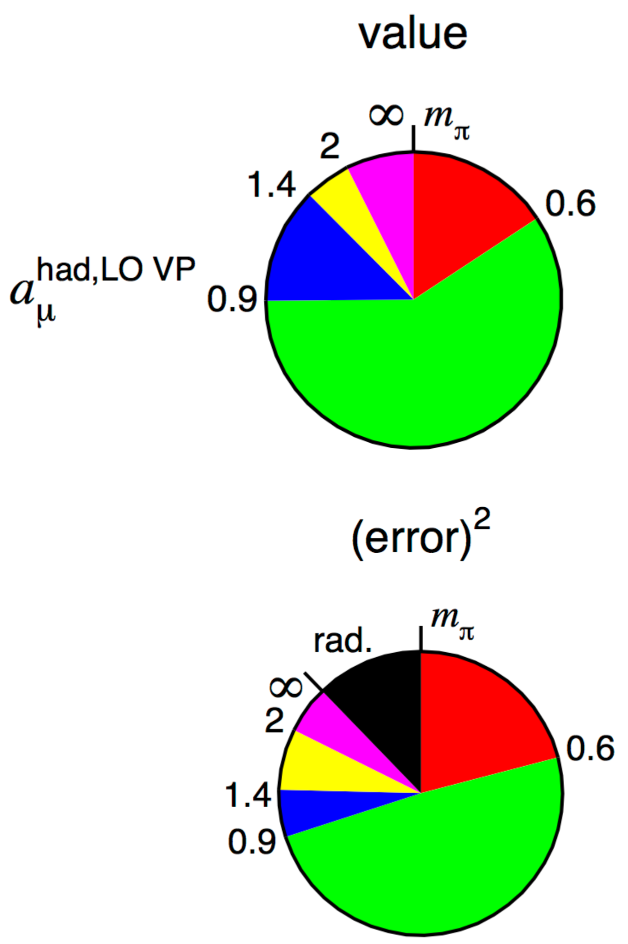


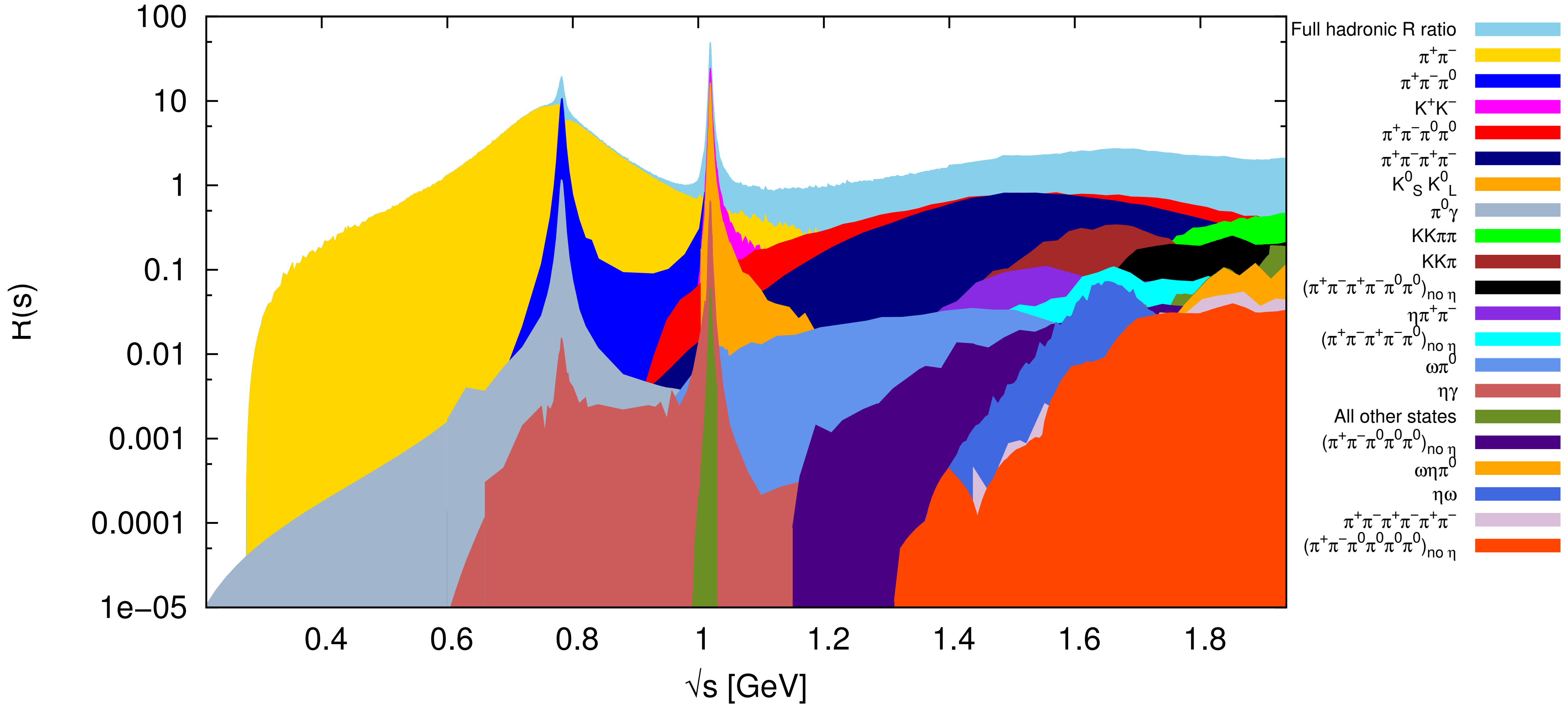
HVP: data-driven

[M. Davier et al, arXiv:1908.00921]





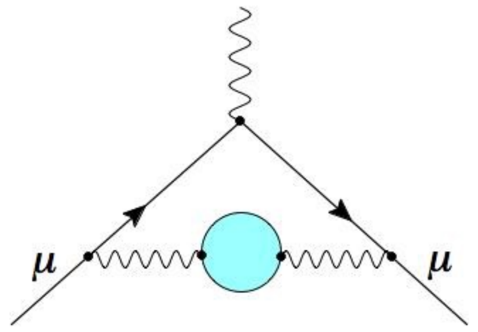
[A. Keshavarzi et al, arXiv:1802.02995]



- total hadronic cross section $\sigma_{\rm had}$ from > 100 data sets in more than 35 channels summed up to ~ 2GeV
- For > 2 GeV: inclusive data + pQCD + narrow resonances
- $\sigma_{\rm had}$ defined to include real & virtual photons
- direct integration method: no need to specify resonances $(\rho, \omega, \ldots)$
- two independent compilations (DHMZ, KNT)

Tensions between BaBar and KLOE data sets:

- Cross checks using analyticity and unitarity relating pion form factor to $\pi\pi$ scattering
- Combinations of data sets affected by tensions
  ➠ conservative merging procedure



HVP: data-driven

Conservative merging procedure to obtain a realistic assessment of the underlying uncertainties:

[B. Malaescu @ INT g-2 workshop]

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

Energy range	ACD18	CHS18	DHMZ19	DHMZ19′	KNT19
≤ 0.6 GeV		110.1(9)	110.4(4)(5)	110.3(4)	108.7(9)
≤ 0.7 GeV		214.8(1.7)	214.7(0.8)(1.1)	214.8(8)	213.1(1.2)
≤ 0.8 GeV		413.2(2.3)	414.4(1.5)(2.3)	414.2(1.5)	412.0(1.7)
≤ 0.9 GeV		479.8(2.6)	481.9(1.8)(2.9)	481.4(1.8)	478.5(1.8)
≤ 1.0 GeV		495.0(2.6)	497.4(1.8)(3.1)	496.8(1.9)	493.8(1.9)
[0.6, 0.7] GeV		104.7(7)	104.2(5)(5)	104.5(5)	104.4(5)
[0.7, 0.8] GeV		198.3(9)	199.8(0.9)(1.2)	199.3(9)	198.9(7)
[0.8, 0.9] GeV		66.6(4)	67.5(4)(6)	67.2(4)	66.6(3)
[0.9, 1.0] GeV		15.3(1)	15.5(1)(2)	15.5(1)	15.3(1)
≤ 0.63 GeV	132.9(8)	132.8(1.1)	132.9(5)(6)	132.9(5)	131.2(1.0)
[0.6, 0.9] GeV		369.6(1.7)	371.5(1.5)(2.3)	371.0(1.6)	369.8(1.3)
$[\sqrt{0.1}, \sqrt{0.95}]$ GeV		490.7(2.6)	493.1(1.8)(3.1)	492.5(1.9)	489.5(1.9)

Include constraints using unitarity & analyticity for $\pi\pi$ and $\pi\pi\pi$ channels

[CHS 2018, Colangelo et al, arXiv:1810.00007; HHKS19, Hoferichter et al, arXiv:1907.01556]

$$\Rightarrow a_\mu^{\text{HVP,LO}} = 693.1\,(2.8)_{\text{exp}}\,(2.8)_{\text{sys}}\,(0.7)_{\text{DV+pQCD}} \times 10^{-10} = 693.1\,(4.0) \times 10^{-10}$$

Efforts on Radiative Corrections for low energy $e^+e^- \to \text{hadrons}$

Workstop+Conference in Zurich 5-9 June 2023 (LOC: A. Signer, G. Stagnitto, Y. Ulrich)

STRONG 2020

RADIO MONTECARLOW

Three-day in-person (Workstop) +
a three half day hybrid conference

5 Working Groups:

- WP1: Leptonic processes at NNLO
- WP2: Form factor contributions at N3LO
- WP3: Processes with hadrons
- WP4: Parton showers
- WP5: Experimental input

Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions

5–9 Jun 2023
University of Zurich
Europe/Zurich timezone

Enter your search term

- Overview
- Timetable
- Contribution List
- Code of Conduct

Contact
yannick.ulrich@durham....

In this workstop, we will discuss radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions. This is to be seen as part of the Strong 2020 effort. We will cover

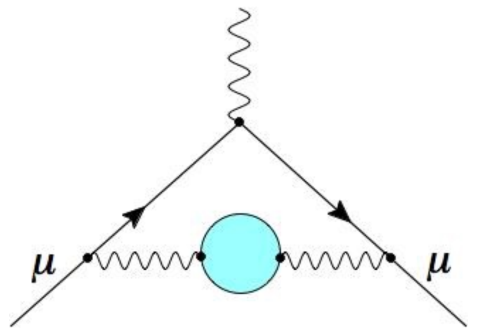
- leptonic processes at NNLO and beyond
- processes with hadrons
- parton shower
- experimental input

Each area will be given at least half a day, starting with an open 1h seminar followed by a lengthy discussion.

Just like previous workstops, we try to gather a small number of theorists who actively work on this topic to make very concrete progress. It should not just be about giving talks, but to actually learn from each other and put together the jigsaw pieces.

Additionally to the workstop that is only by-invite only, there is a broader conference directly following the workstop.

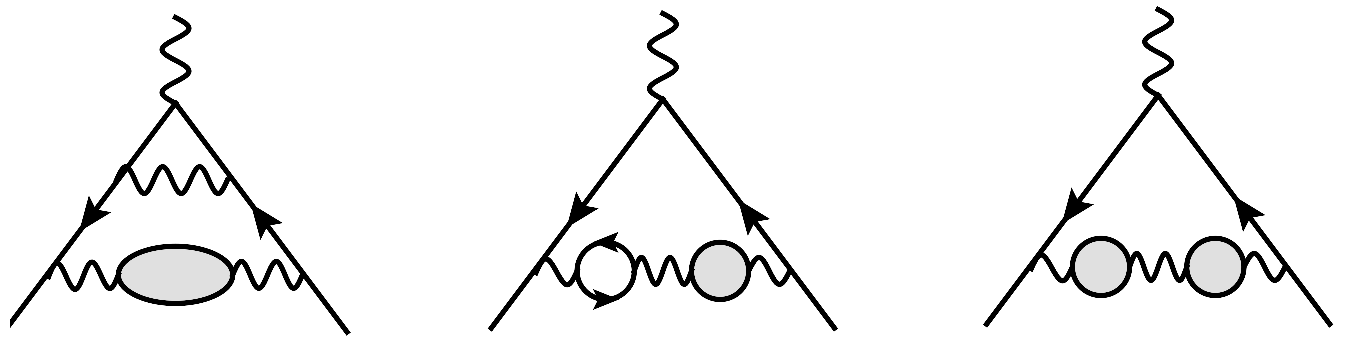
Final goal: full NNLO MC. Aim to write a report by Autumn 2023



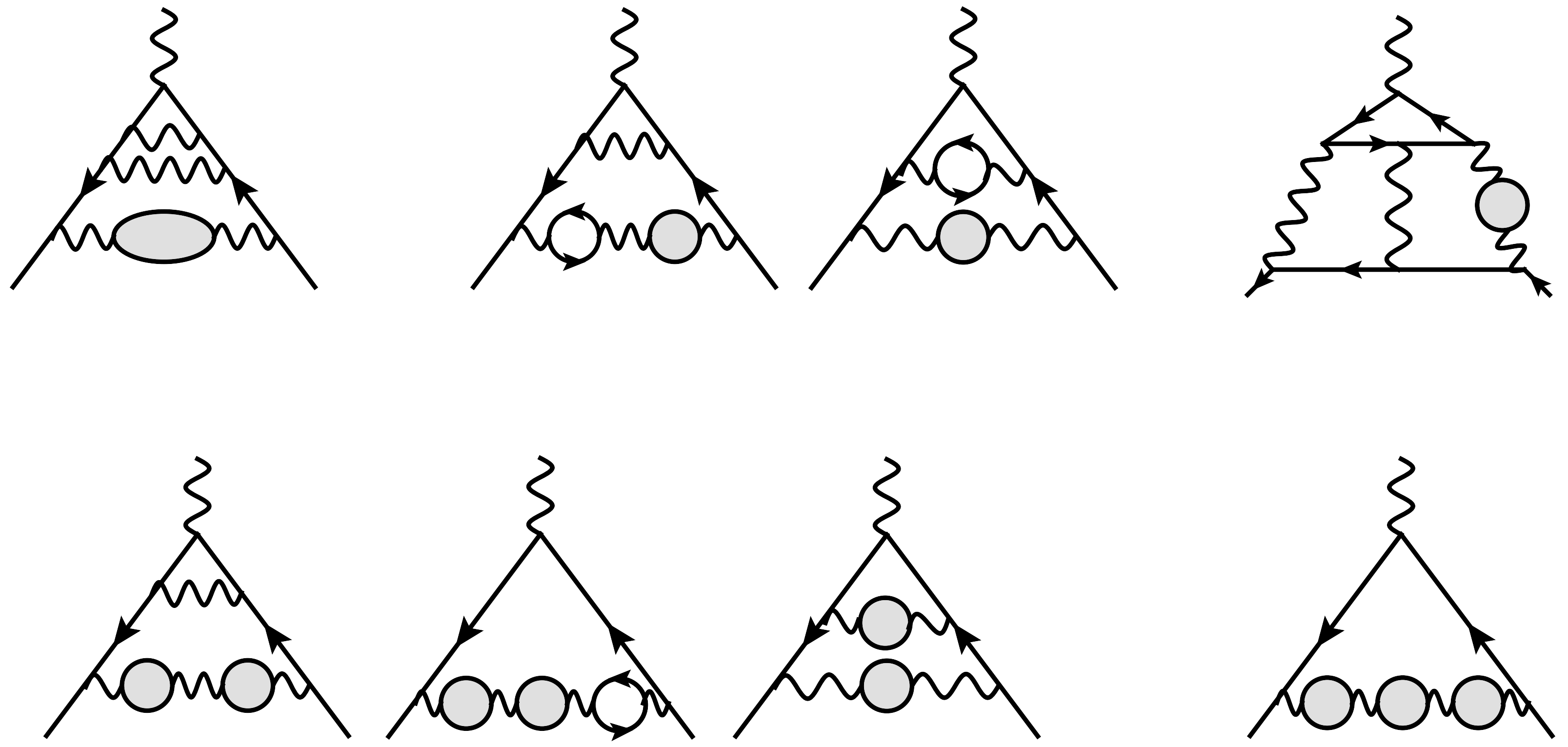
HVP: higher order (NLO, NNLO)

Figures by T. Teubner

$a_\mu^{\mathrm{HVP,NLO}} = -9.83(7) \times 10^{-10}$ [based on KNT 2019]



$a_\mu^{\mathrm{HVP,NNLO}} = 1.24(1) \times 10^{-10}$ [Kurz et al, arXiv:1403.6400, PLB 2014]

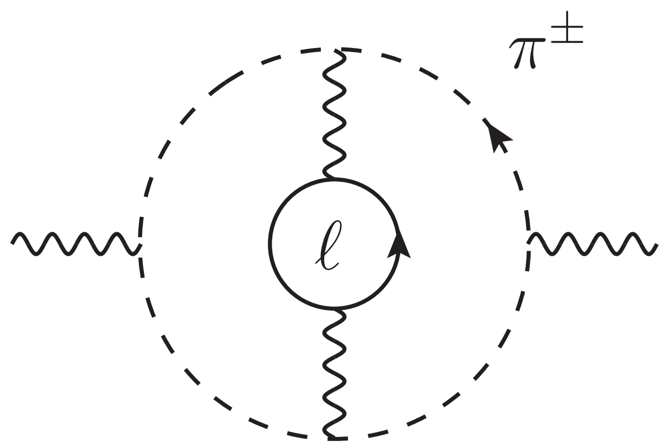


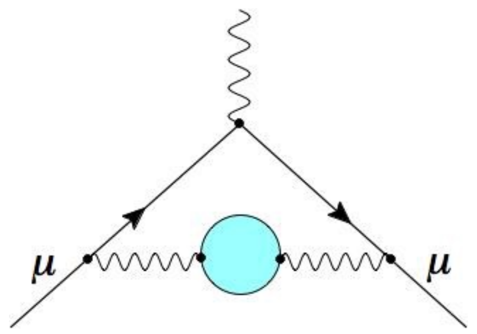
space-like NLO and NNLO HVP kernels for LQCD evaluations and MUonE

[Balsani et al, arXiv:2112.05704; Nesterenko, arXiv:2209.03217, arXiv: 2112.05009]

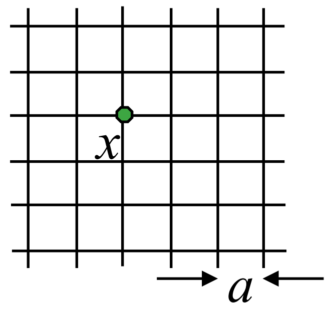
mixed leptonic, hadronic (double bubble) contributions to a_μ are $< 10^{-11}$

[Hoferichter + Teubner, arXiv:2112.06929]

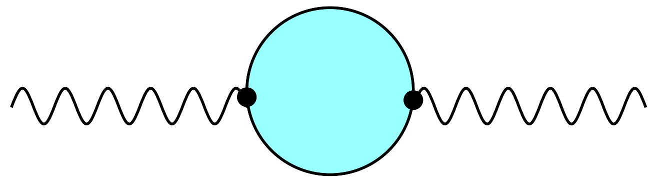




Lattice HVP: Introduction



[B. Lautrup, A. Peterman, E. de Rafael, Phys. Rep 1972; E. de Rafael, Phys. Let. B 1994; T. Blum, PRL 2002]



$\hat{\Pi}(q^2)$

Leading order HVP correction:

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2)\, \hat{\Pi}(q^2)$$

- Calculate $a_\mu^{\mathrm{HVP,LO}}$ in Lattice QCD

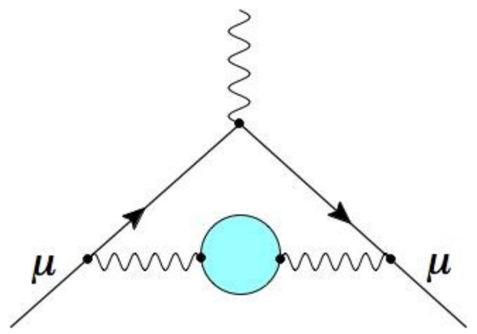
Compute correlation function: $$C(t) = \frac{1}{3}\sum_{i,x}\langle j_i(x,t) j_i(0,0)\rangle$$

and $$\hat{\Pi}(Q^2) = 4\pi^2 \int_0^\infty dt\, C(t)\left[t^2 - \frac{4}{Q^2}\sin^2\left(\frac{Qt}{2}\right)\right]$$

[D. Bernecker and H. Meyer, arXiv:1107.4388, EPJA 2011]

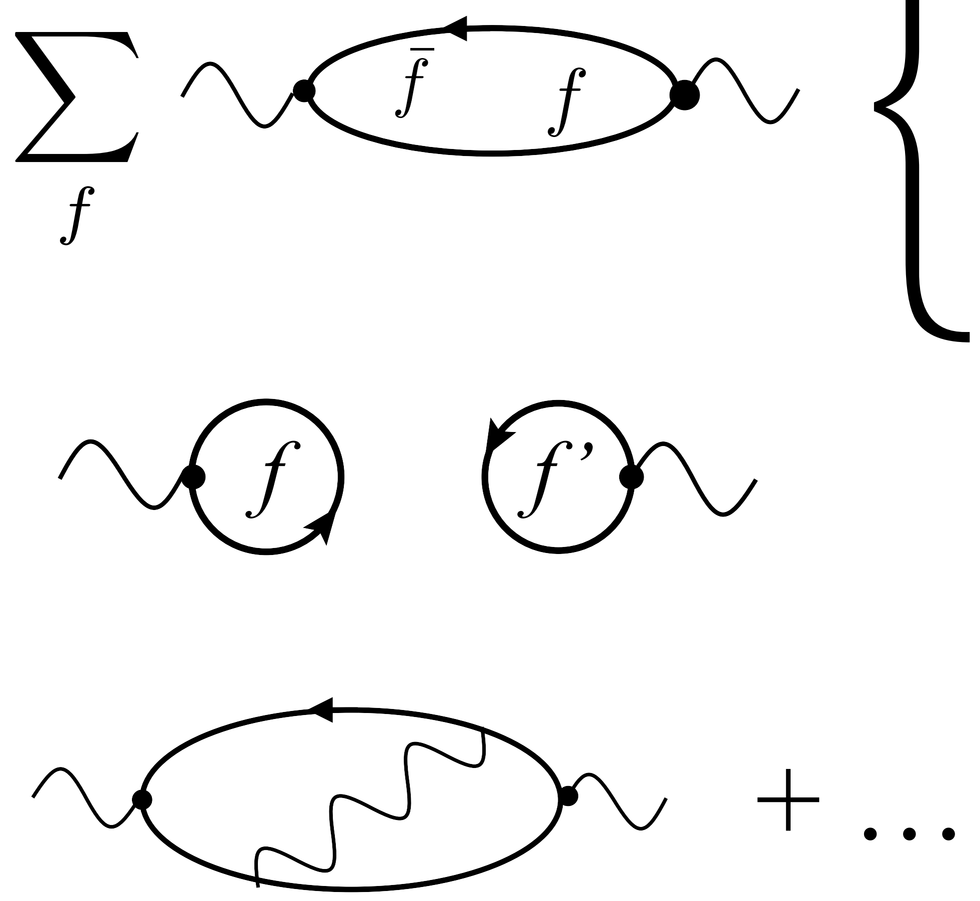
Obtain $a_\mu^{\mathrm{HVP,LO}}$ from an integral over Euclidean time:

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$



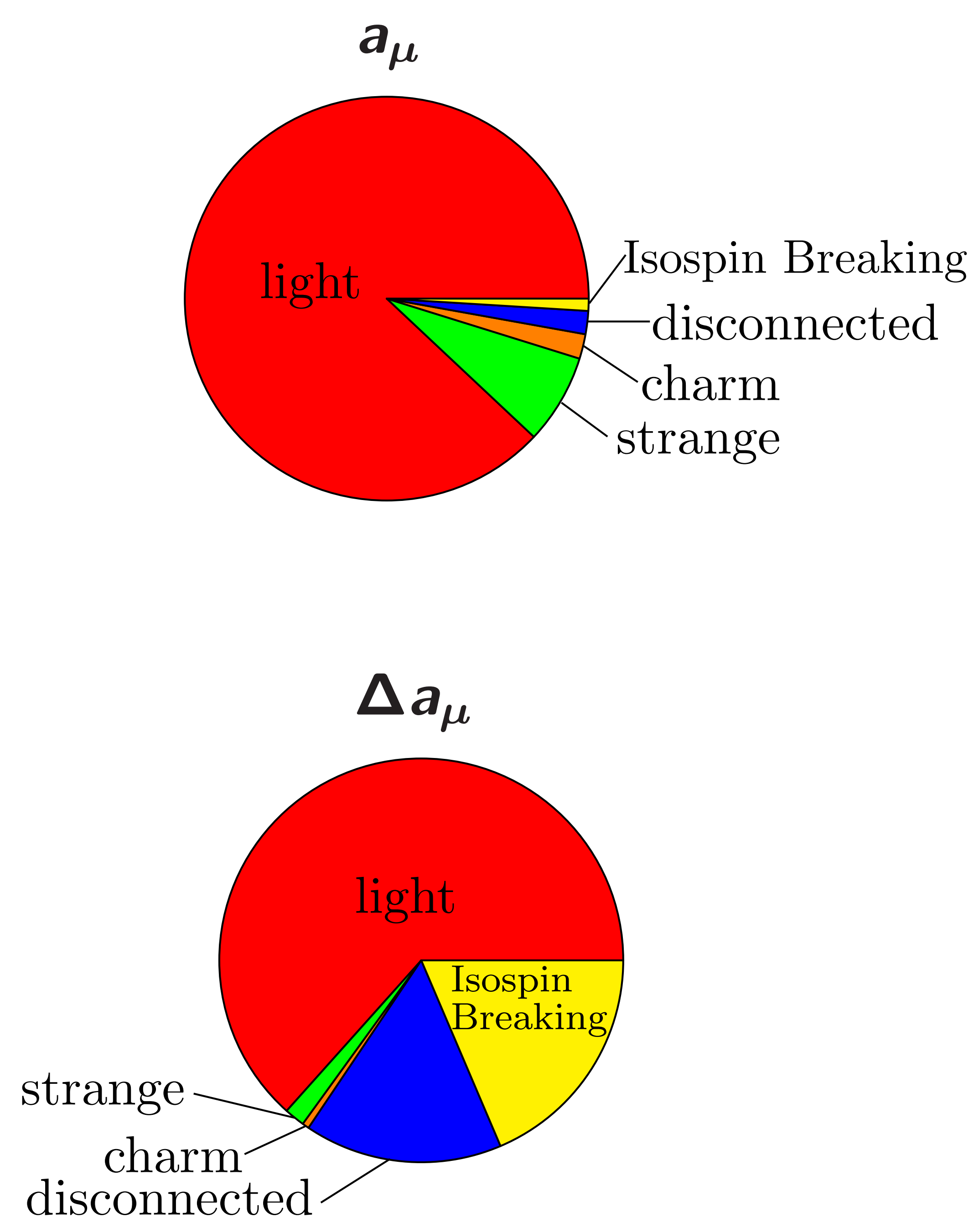
Lattice HVP: Introduction

- Target: ~ 0.2% total error

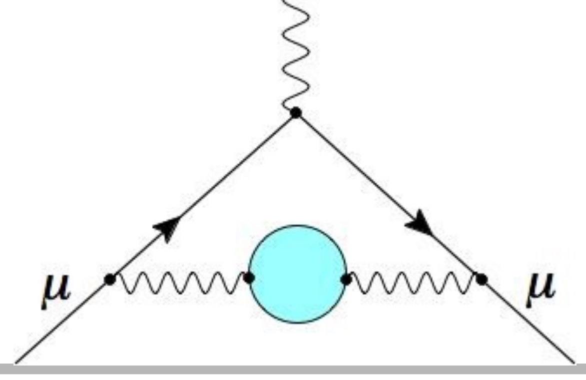


- light-quark connected contribution:
  $a_\mu^{\mathrm{HVP,LO}}(ud)$ ~90% of total
- *s*,*c*,*b*-quark contributions
  $a_\mu^{\mathrm{HVP,LO}}(s,c,b)$ ~8%, 2%, 0.05% of total
- disconnected contribution:
  $a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}}$ ~2% of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
  $\delta a_\mu^{\mathrm{HVP,LO}}$ ~1% of total

V. Gülpers @ Lattice HVP workshop

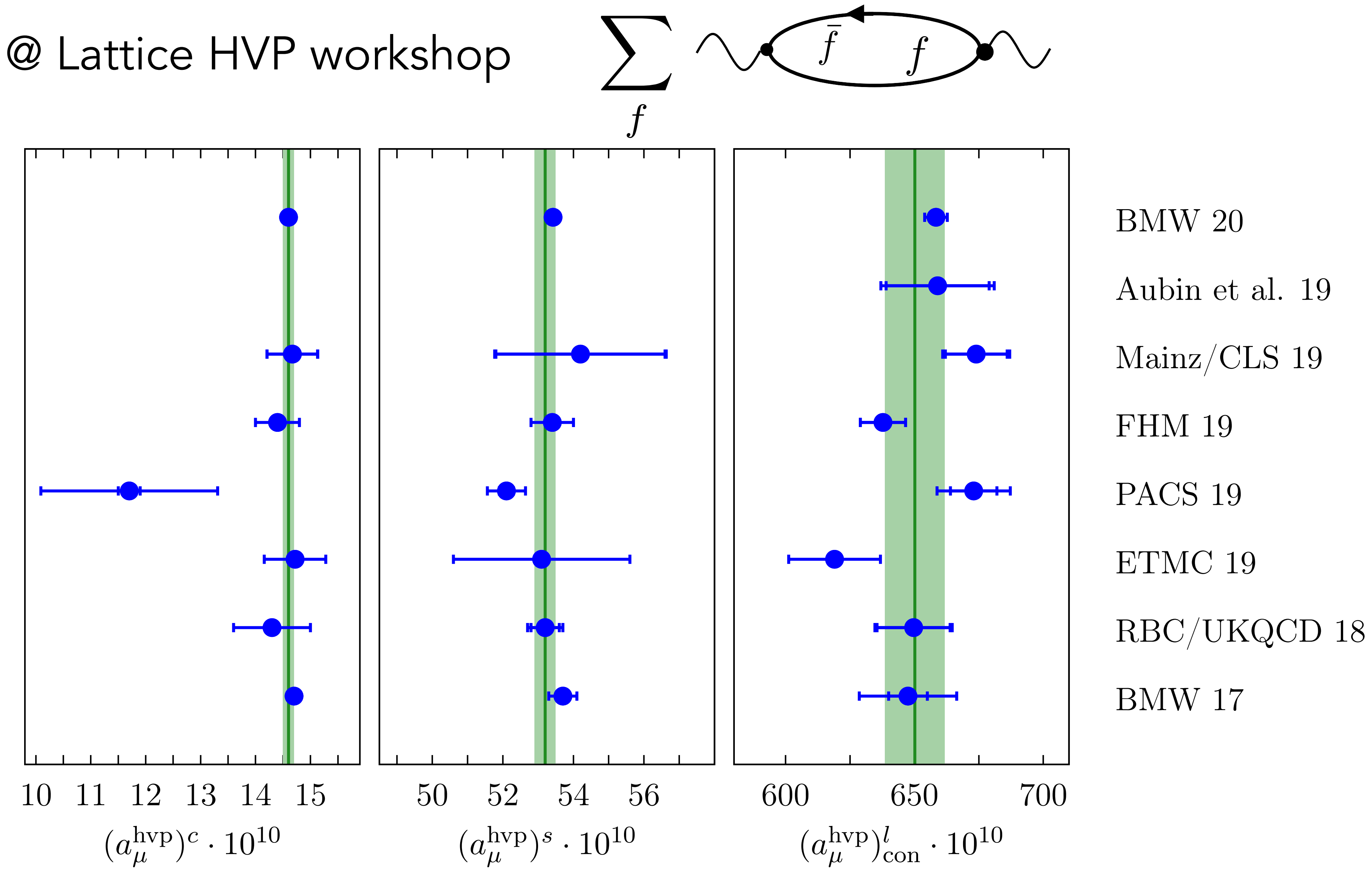


$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{\mathrm{HVP,LO}}(ud) + a_\mu^{\mathrm{HVP,LO}}(s) + a_\mu^{\mathrm{HVP,LO}}(c) + a_{\mu,\mathrm{disc}}^{\mathrm{HVP,LO}} + \delta a_\mu^{\mathrm{HVP,LO}}$$



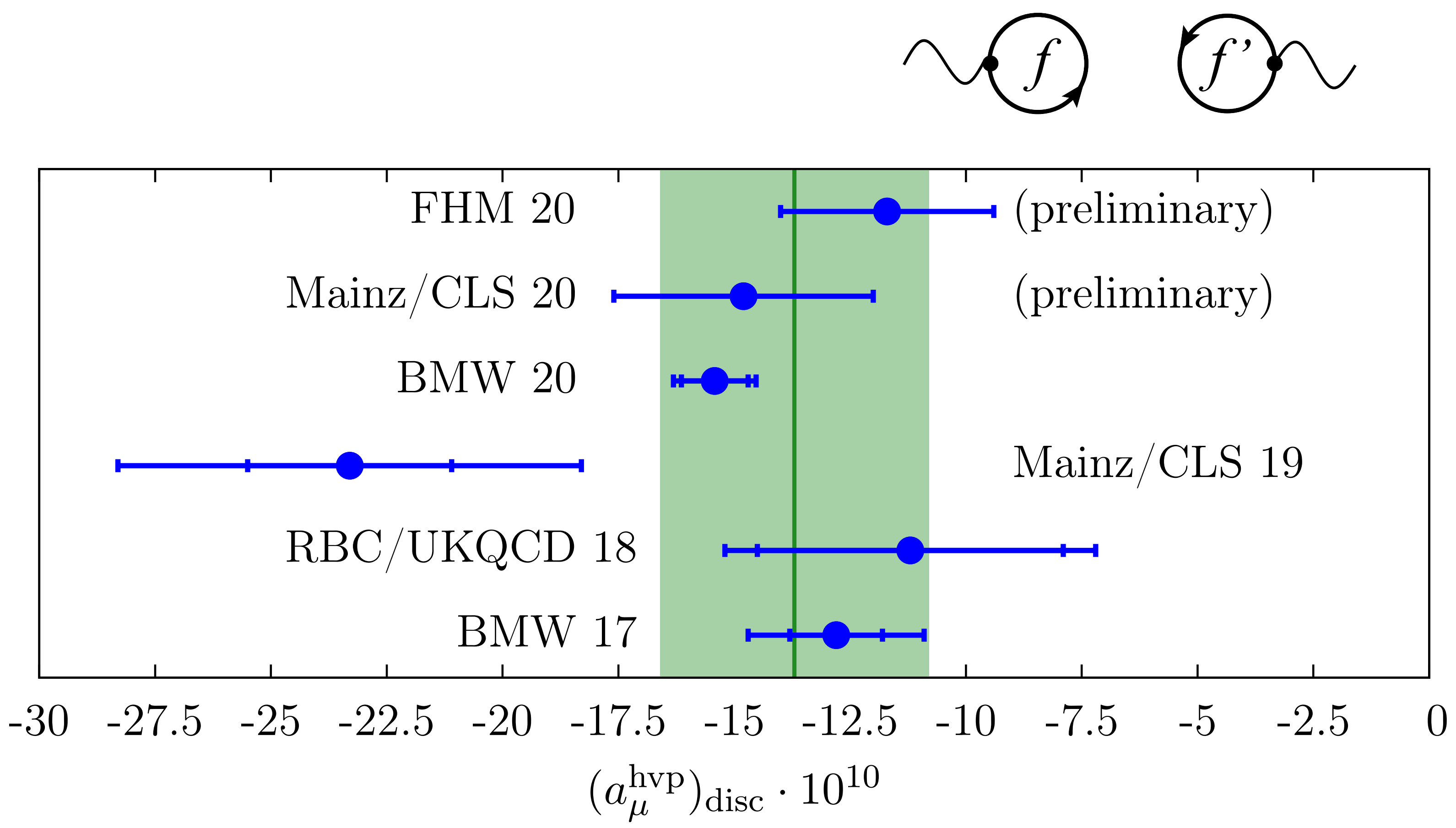
Lattice HVP: comparisons

H. Wittig @ Lattice HVP workshop



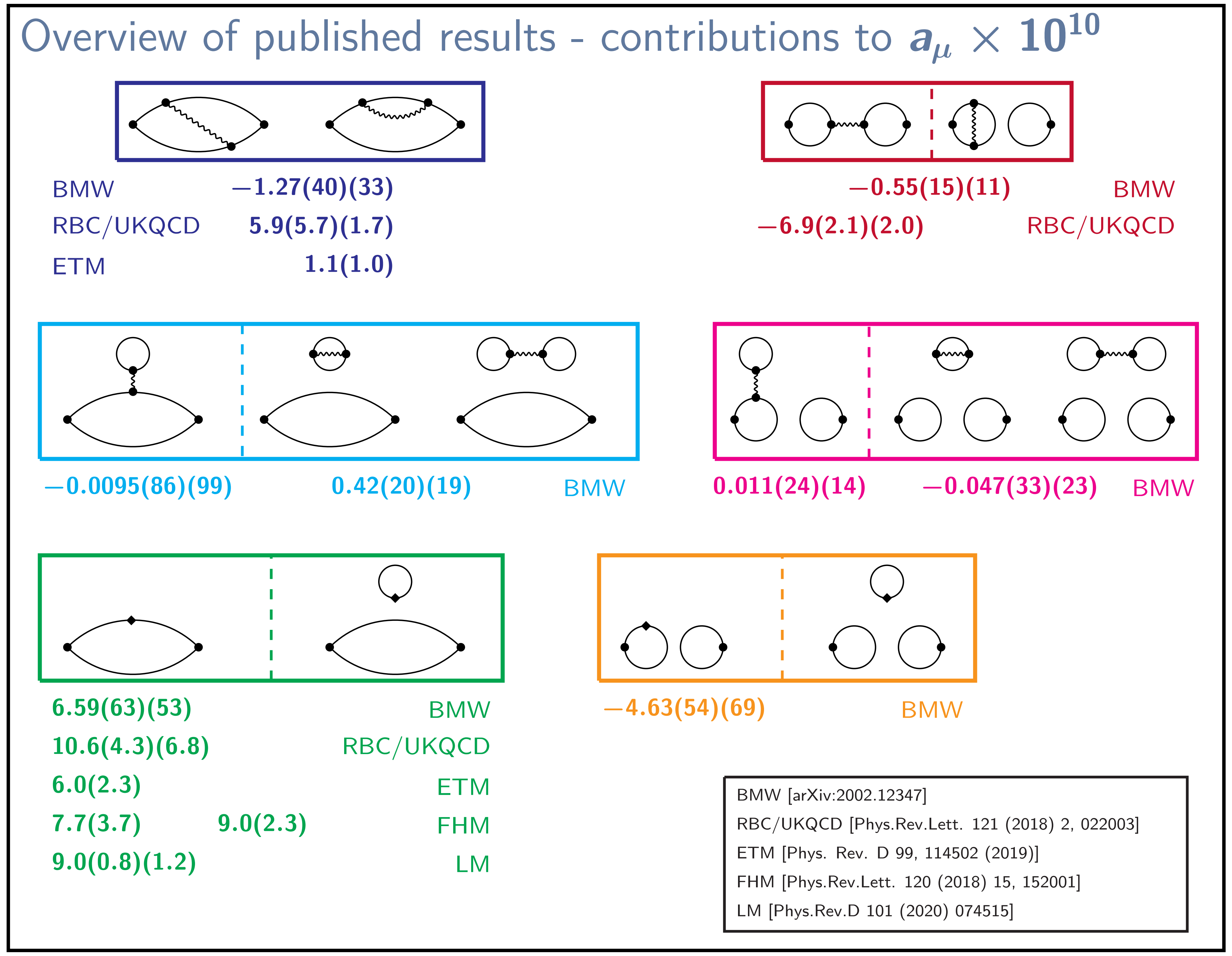
- Charm, strange contributions already well determined.
- Mild tensions for light contribution

Ongoing efforts by FNAL-HPQCD-MILC RBC/UKQCD, Mainz

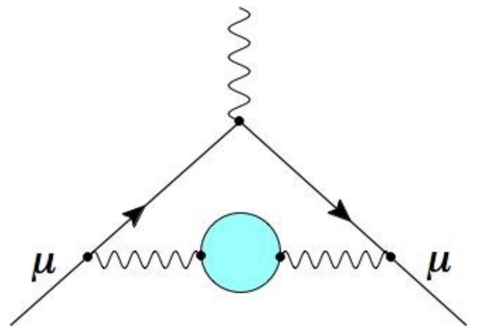


Consistent results with increasing precision

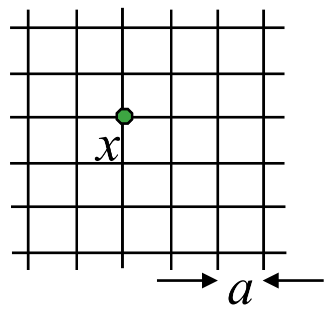
V. Gülpers @ Lattice HVP workshop



- Some tensions between lattice results for individual contributions.
- Large cancellations between individual contributions: $\delta a_\mu^{\rm IB} \lesssim 1\,\%$



HVP: lattice



In 2020 WP:

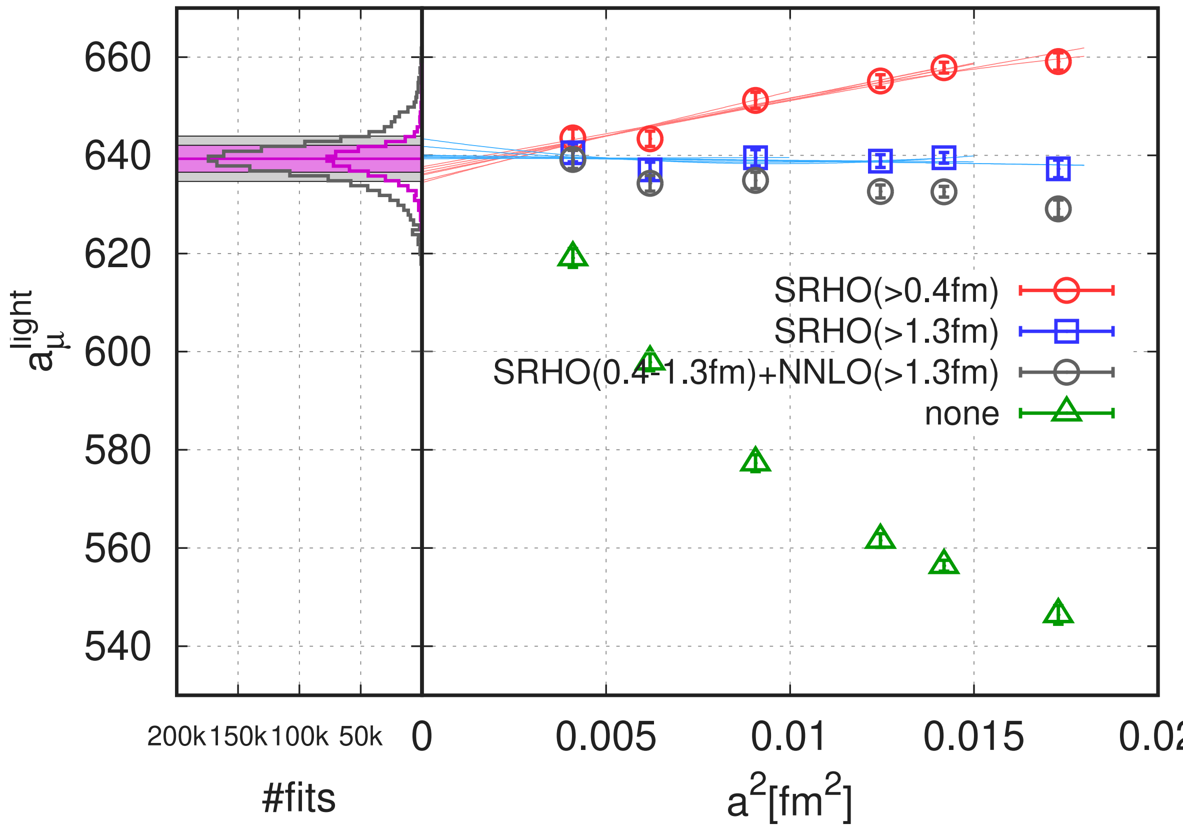
- Lattice HVP average at $2.6\,\%$ total uncertainty: $a_\mu^{\mathrm{HVP,LO}} = 711.6\,(18.4) \times 10^{10}$
- BMW 20 (published in 2021)
  first LQCD calculation with sub-percent $(0.8\,\%)$ error
  in tension with data-driven HVP (2.1σ)
- total error in BMW calculation is dominated by light-quark connected contribution.
- Large taste-breaking effects with BMW set-up
  ➠ uncorrected data not easily fit to power series, i.e.

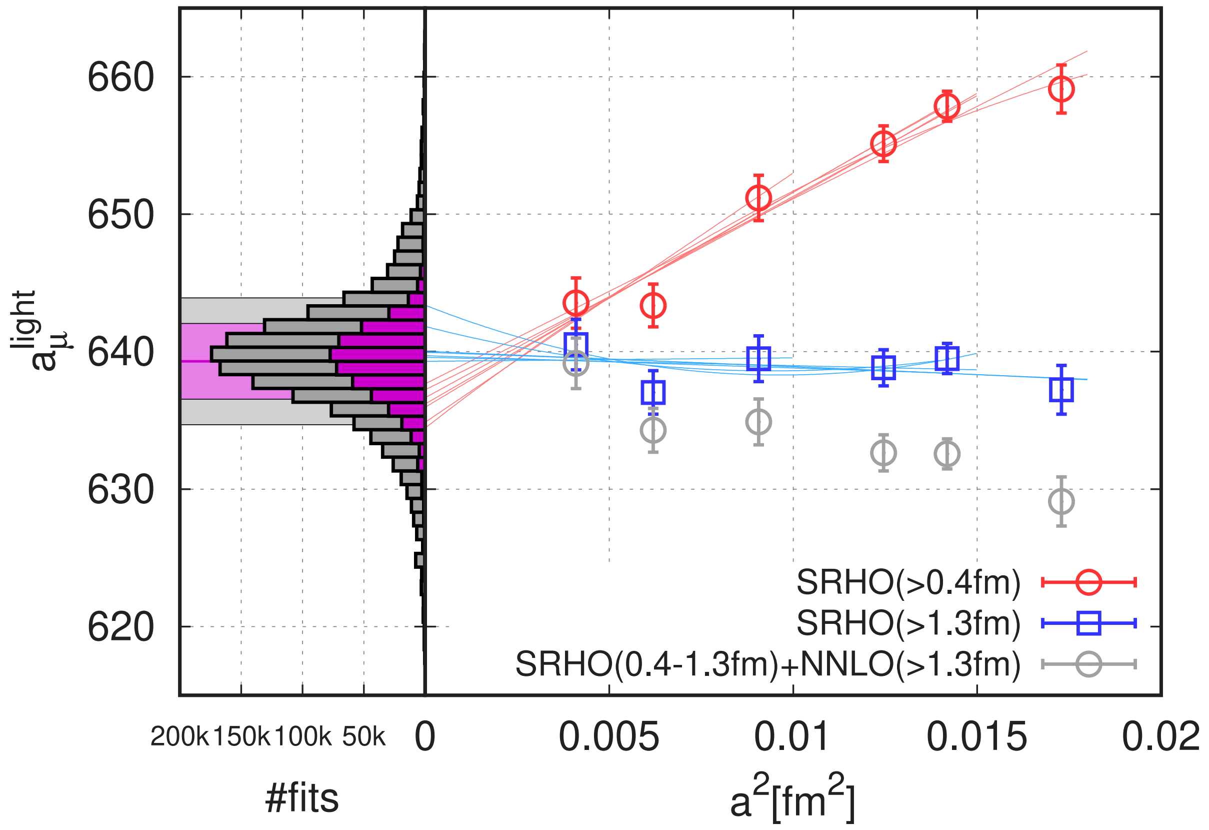
1. $A_0 + A_1\,[a^2] + A_2\,[a^2]^2$
2. $A_0 + A_1\,[a^2\alpha_s^3(\tfrac{1}{a})] + A_2\,[a^2\alpha_s^3(\tfrac{1}{a})]^2$

Kalman Szabo (BMW) @ Lattice 2021

Taste improvement II

- $a_\mu(a) \to a_\mu(a) - a_\mu^{\mathrm{SRHO}}(a) + a_\mu^{\mathrm{RHO}}$
- reduces lattice artefact, also makes a^2 dependence linear





SRHO improvement gives central value. Systematic errors by:

1. change starting point of improvement $t = 0.4 \to 1.3$ fm
2. skip coarse lattices
3. change $\Gamma = 0$ and $\Gamma = 3$
4. replace SRHO by NNLO SXPT above 1.3 fm

Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \to \text{hadrons})$, but weighted towards higher energies.
- a shift in a_μ^{HVP} also changes $\Delta\alpha_{\text{had}}(M_Z^2)$: ➠ EW fits
  [Passera, et al, 2008, Crivellin et al 2020, Keshavarsi et al 2020, Malaescu & Scott 2020]
  If the shift in a_μ^{HVP} is in the low-energy region ($\lesssim 1\,\text{GeV}$), the impact on $\Delta\alpha_{\text{had}}(M_Z^2)$ and EW fits is small.
- A shift in a_μ^{HVP} from low ($\lesssim 2\,\text{GeV}$) energies
  ➠ $\sigma(e^+e^- \to \pi\pi)$
  must satisfy unitarity & analyticity constraints ➠ $F_\pi^V(s)$
  can be tested with lattice calculations
  [Colangelo, Hoferichter, Stoffer, arXiv:2010.07943]

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Martin Hoferichter @ Lattice HVP workshop

Hadronic running of α and global EW fit

	e^+e^- KNT, DHMZ	EW fit HEPFit	EW fit GFitter	guess based on BMWc
$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) \times 10^4$	276.1(1.1)	270.2(3.0)	271.6(3.9)	277.8(1.3)
difference to e^+e^-		-1.8σ	-1.1σ	$+1.0\sigma$

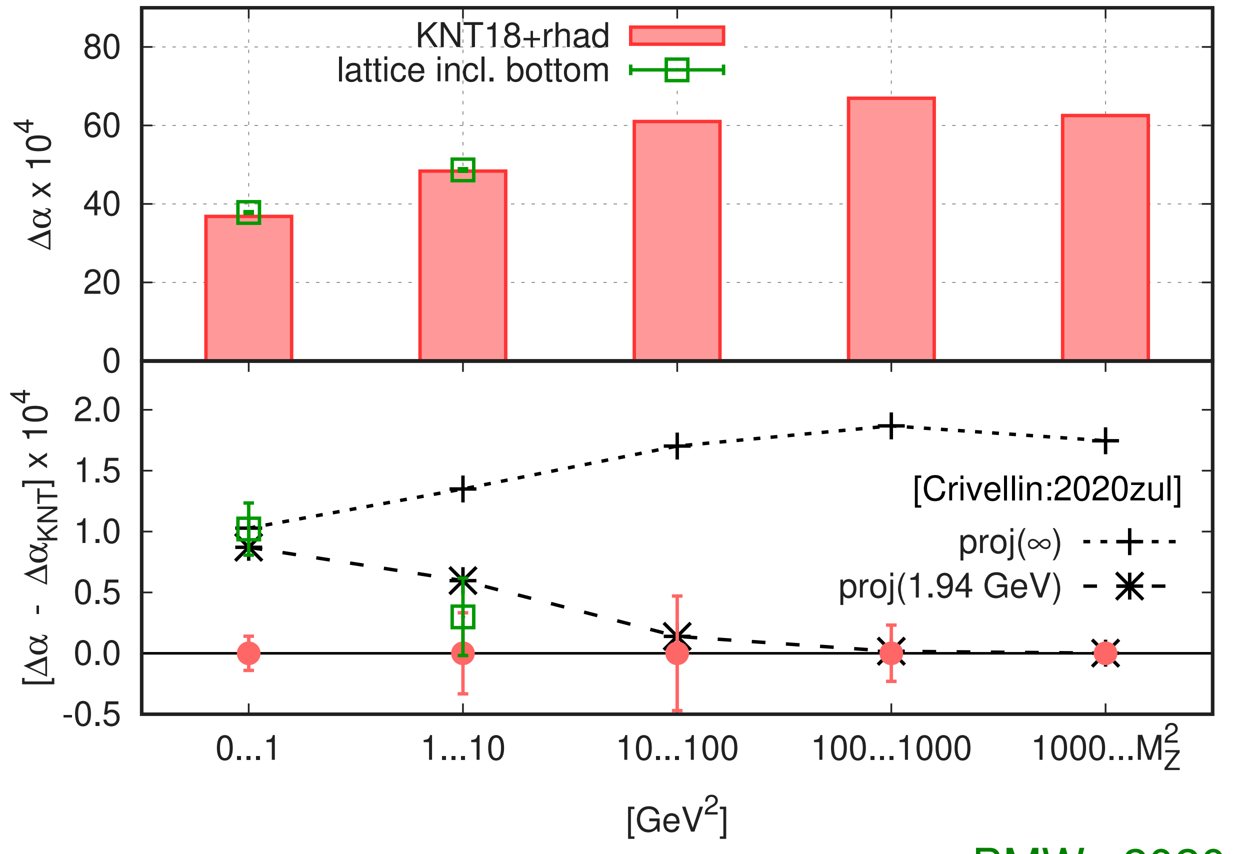
- **Time-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)}$$

- **Space-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha}{\pi}\hat{\Pi}(-M_Z^2) + \frac{\alpha}{\pi}\left(\hat{\Pi}(M_Z^2) - \hat{\Pi}(-M_Z^2)\right)$$

- Global EW fit
  - Difference between HEPFit and GFitter implementation mainly treatment of M_W
  - Pull goes into **opposite direction**



BMWc 2020

More in talks by M. Passera, B. Malaescu (phenomenology) and K. Miura, T. San José (lattice)

Connections

$$\sigma(e^+e^- \to \text{hadrons}) \quad \Leftrightarrow \quad a_\mu^{\text{HVP}} \quad \Leftrightarrow \quad \Delta\alpha_{\text{had}}(M_Z^2)$$

- $\Delta\alpha_{\text{had}}(M_Z^2)$ also depends on the hadronic vacuum polarization function, and can be written as an integral over $\sigma(e^+e^- \to \text{hadrons})$, but weighted towards higher energies.
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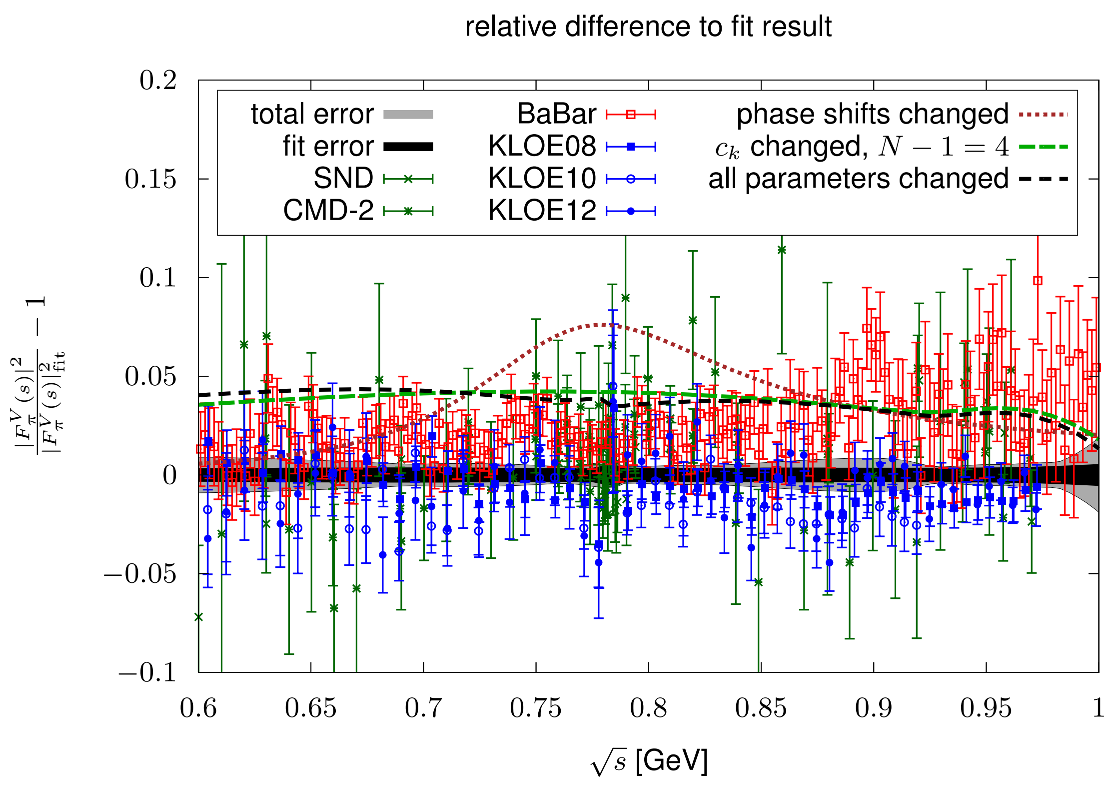
  can be tested with lattice calculations
  [Colangelo, Hoferichter, Stoffer, arXiv:2010.07943]

Peter Stoffer @ Lattice HVP workshop

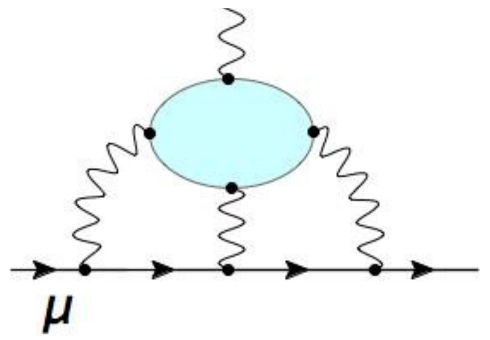
Constraints on the two-pion contribution to HVP arXiv:2010.07943 [hep-ph]

Modifying $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$

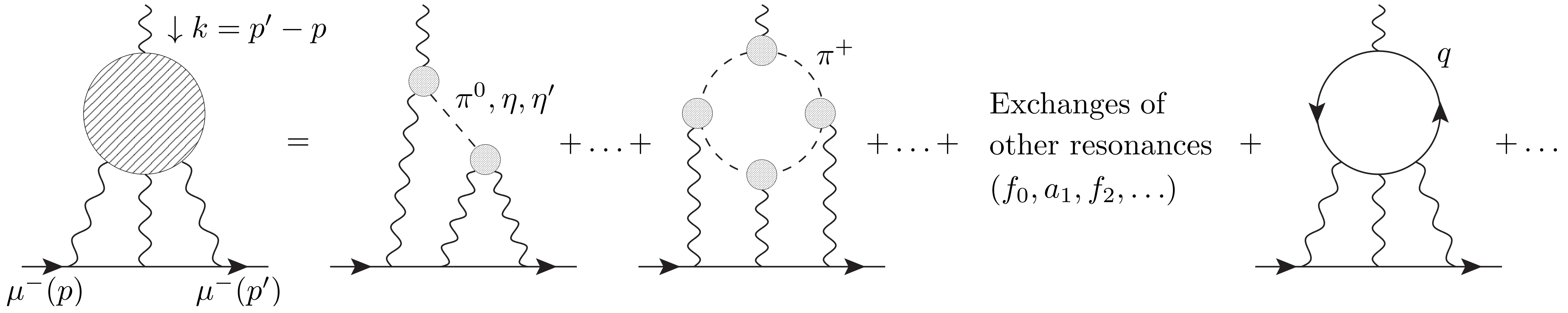
- "low-energy" scenario: local changes in cross section of $\sim 8\%$ **around ρ**
- "high-energy" scenario: impact on **pion charge radius** and space-like VFF $\Rightarrow$ chance for **independent lattice-QCD checks**
- requires **factor ~ 3 improvement** over χQCD result:
  $\langle r_\pi^2 \rangle = 0.433(9)(13)\,\text{fm}^2$
  → arXiv:2006.05431 [hep-ph]



- Can new physics hide in the low-energy $\sigma(e^+e^- \to \pi\pi)$ cross section? ➠ No [Luzio, et al, arXiv:2112.08312]
- Neutral, long-lived hadrons, heretofore undetected? [Farrar, arXiv:2206.13460]



Hadronic Light-by-light

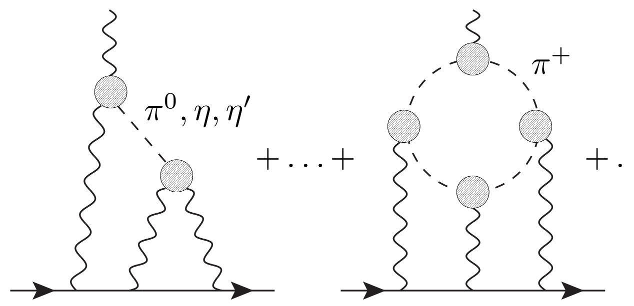


Dispersive approach:

[Colangelo at al, 2014; Pauk & Vanderhaegen 2014; …]

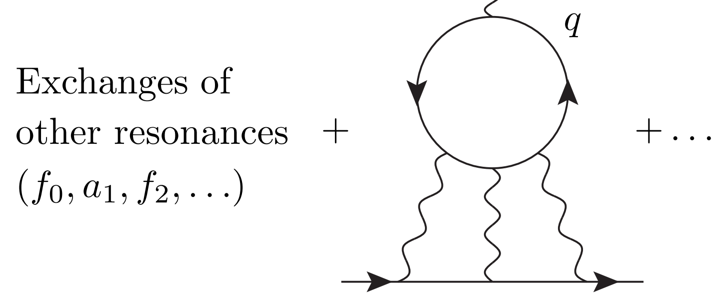
- ✦ model independent
- ✦ significantly more complicated than for HVP
- ✦ provides a framework for data-driven evaluations
- ✦ can also use lattice results as inputs

Dominant contributions ($\approx 75\,\%$ of total):



- ✦ Well quantified with $\approx 6\,\%$ uncertainty
- ✦ η, η' pole contributions: Canterbury approximants only
- ✦ Ongoing work: consolidation of η, η' pole contributions using disp. relations and LQCD

Subleading contributions ($\approx 25\,\%$ of total):



- ✦ Not yet well known
  ➠ dominant contribution to total uncertainty
- ✦ Ongoing work:
  - Implementation of short-distance constraints (now at 2-loop)
  - DR implementation for axial vector contributions
  - new $q_4 = 0$ DR program for higher spin intermediate states
    [Luedtke @ Higgscentre workshop with Procura and Stoffer, in progress]
  - Mainz and BESIII ramping up $\gamma^{(*)}\gamma^*$ programs
    [A. Denig and C. Redmer @ Higgscentre workshop]

Dispersive, data-driven evaluation of HLbL with $\leq 10\,\%$ total uncertainty feasible by ~2025.