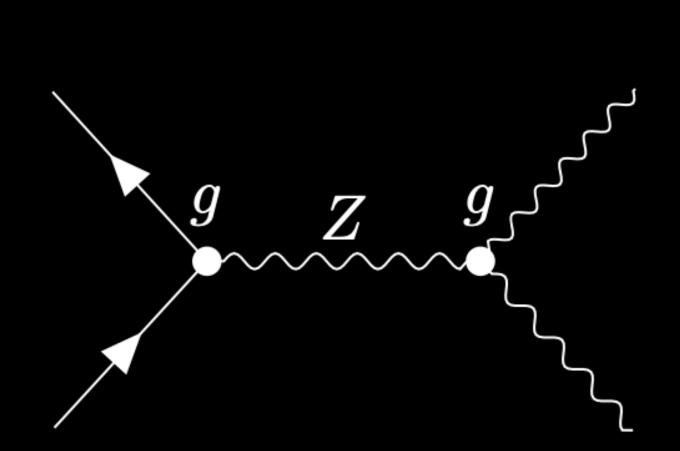
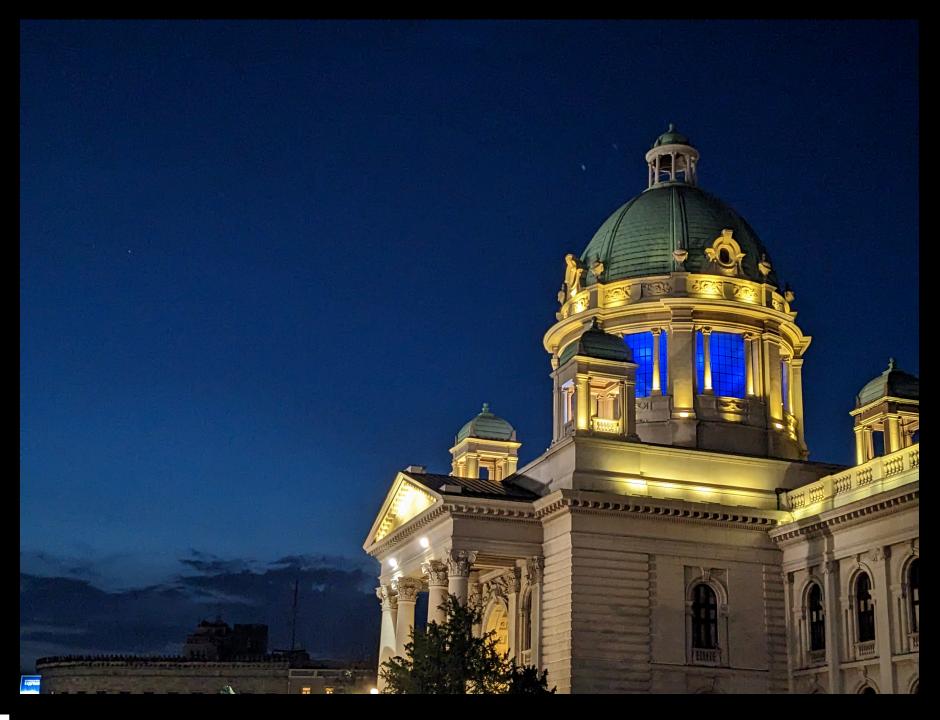
Recent diboson and polarization measurements at CMS







Saptaparna Bhattacharya

Northwestern University, Humboldt Fellow LHCP 2023, Belgrade Serbia





Overview of CMS cross section results 18 pb⁻¹ - 138 fb⁻¹ (7,8,13 TeV) **CMS** preliminary • $\sigma(W) = 9.5e + 07 \text{ fb}$ 36 pb^{-1} 7 TeV JHEP 10 (2011) 132 18 pb^{-1} $\sigma(W) = 1.1e + 08 \text{ fb}$ 8 TeV PRL 112 (2014) 191802 10^5 pb 43 pb^{-1} SMP-15-004 $\sigma(W) = 1.8e + 08 \text{ fb}$ 13 TeV 36 pb^{-1} 7 TeV JHEP 10 (2011) 132 $\sigma(Z) = 2.9e + 07 \text{ fb}$ $18 \; \rm pb^{-1}$ $\sigma(Z) = 3.4e + 07 \text{ fb}$ 8 TeV PRL 112 (2014) 191802 12 fb^{-1} SMP-15-011 $\sigma(Z) = 5.6e + 07 \text{ fb}$ 5 fb^{-1} Wγ $\sigma(W\gamma) = 3.4e + 05 \text{ fb}$ 7 TeV PRD 89 (2014) 092005 $137 \; \text{fb}^{-1}$ $\sigma(W\gamma) = 1.4e + 05 \text{ fb}$ 13 TeV PRL 126 252002 (2021) 5 fb⁻¹ $\sigma(Z\gamma) = 1.6e + 05 \text{ fb}$ 7 TeV PRD 89 (2014) 092005 $\sigma(Z\gamma) = 1.9e + 05 \text{ fb}$ 20 fb^{-1} 8 TeV JHEP 04 (2015) 164 Most papers published in 2020-2021 5 fb⁻¹ $\sigma(WW) = 5.2e + 04 \text{ fb}$ WW 7 TeV EPJC 73 (2013) 2610 $\sigma(WW) = 6e + 04 \text{ fb}$ $19 \; {\rm fb^{-1}}$ ww EPJC 76 (2016) 401 8 TeV $\sigma(WW) = 1.2e + 05 \text{ fb}$ 36 fb^{-1} ww PRD 102 092001 (2020) New results unveiled in Moriond '23 5 fb^{-1} WZ EPJC 77 (2017) 236 $\sigma(WZ) = 2e + 04 \text{ fb}$ 7 TeV 20 fb^{-1} $\sigma(WZ) = 2.4e + 04 \text{ fb}$ WZ 8 TeV EPJC 77 (2017) 236 $\sigma(WZ) = 5.1e + 04 \text{ fb}$ 137 fb^{-1} WZ 13 TeV Submitted to JHEP 5 fb⁻¹ $\sigma(ZZ) = 6.2e - 03 \text{ fb}$ 7 TeV JHEP 01 (2013) 063 $\sigma(ZZ) = 7.7/4 + 03 \text{ fb}$ 20 fb^{-1} ZZ PLB 740 (2015) 250 137 fb^{-1} $\sigma(Z) = 1.7e + 04 \text{ fb}$ ΖZ EPJC 81 (2021) 200 137 fb^{-1} VVV $\sigma(VVV) = 1e+03 \text{ fb}$ 13 TeV PRL 125 151802 (2020) 137 fb⁻¹ $\sigma(WWW) = 5.9e + 02 \text{ fb}$ WWW 137 fb⁻¹ $\sigma(WWZ) = 3e + 02 \text{ fb}$ WWZ PRL 125 151802 (2020) 137 fb⁻¹ $\sigma(WZZ) = 2e + 02 \text{ fb}$ PRL 125 151802 (2020) 13 TeV 137 fb⁻¹ $\sigma(ZZZ) < 2e + 02$ fb ZZZ PRL 125 151802 (2020) $19 \; { m fb}^{-1}$ $WV\gamma$ $\sigma(WV\gamma)$ < 3.1e+02 fb PRD 90 032008 (2014) 8 TeV ection $\sigma(W\gamma\gamma) = 4.9 \text{ fb}$ $19 \; { m fb}^{-1}$ $W\gamma\gamma$ 8 TeV JHEP 10 (2017) 072 pb $19 \; { m fb}^{-1}$ $W\gamma\gamma$ $\sigma(W\gamma\gamma) = 14 \text{ fb}$ 13 TeV JHEP 10 (2021) 174 $\sigma(\mathsf{Z}\gamma\gamma) = 13 \text{ fb}$ $19 \; { m fb}^{-1}$ JHEP 10 (2017) 072 Ζγγ 19 fb^{-1} $\sigma(Z\gamma\gamma) = 5.4 \text{ fb}$ Ζγγ 13 TeV JHEP 10 (2021) 174 $\sigma(VBF W) = 4.2e + 02 \text{ fb}$ 19 fb^{-1} **VBF W** 8 TeV JHEP 11 (2016) 147 $\sigma(VBF W) = 62e + 03 fb$ 36 fb⁻¹ VBF W EPJC 80 (2020) 43 cross 5 fb^{-1} $\sigma(VBF Z) = 1.5e + 02 \text{ fb}$ JHEP 10 (2013) 101 VBF Z 20 fb^{-1} $\sigma(VBF Z) = 1.7e + 02 \text{ fb}$ EPJC 75 (2015) 66 $\sigma(VBF\ Z) = 5.3e + 02\ fb$ 36 fb^{-1} VBF Z EPJC 78 (2018) 589 138 fb^{-1} Submitted to PLB $\sigma(EW WV) = 1.9e + 03 \text{ fb}$ 47 pb EW WV ex. $\gamma\gamma \rightarrow WW8 \text{ TeV}$ $\sigma(\text{ex. }\gamma\gamma\rightarrow\text{WW})=22\text{ fb}$ 20 fb^{-1} JHEP 08 (2016) 119 Higgs 20 fb⁻¹ $\sigma(EW qqW\gamma) = 11 fb$ JHEP 06 (2017) 106 36 fb^{-1} $\sigma(EW qqW\gamma) = 20 \text{ fb}$ EW qqW γ 13 TeV PLB 811 (2020) 135988 138 fb^{-1} $\sigma(EW \text{ os } WW) = 10 \text{ fb}$ EW os WW 13 TeV SMP-21-001 $19 \; { m fb}^{-1}$ EW ss WW 8 TeV PRL 114 051801 (2015) 137 fb⁻¹ $\sigma(EW \text{ ss WW}) = 4 \text{ fb}$ PRL 120 081801 (2018) $\sigma(EW qqZ\gamma) = 1.9 \text{ fb}$ 20 fb^{-1} EW $qqZ\gamma$ PLB 770 (2017) 380 $\sigma(EW qqZ\gamma) = 5.2 \text{ fb}$ 36 fb^{-1} PRD 104 072001 (2021) EW $qqZ\gamma$ 137 fb⁻¹ $\sigma(EW qqWZ) = 1.8 fb$ PLB 809 (2020) 135710 EW qqWZ 13 TeV 137 fb^{-1} EW qqZZ 13 TeV PLB 812 (2020) 135992 1.0e-01 1.0e + 011.0e+05 1.0e + 09

Span several orders of magnitude!

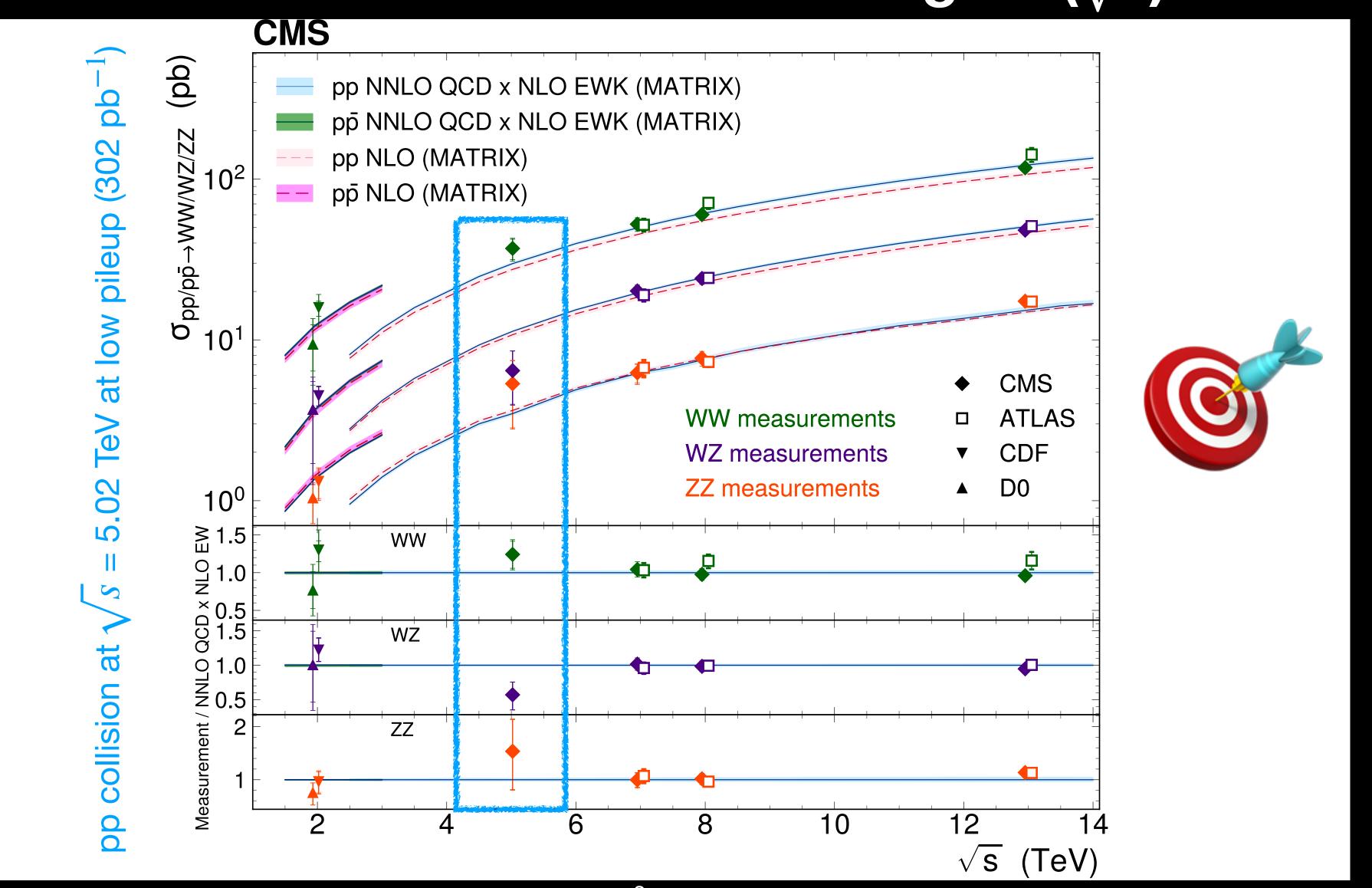
Inner colored bars statistical uncertainty, outer narrow bars statistical+systematic uncertainty Light colored bars: 7 TeV, Medium bars: 8 TeV, Dark bars: 13 TeV, Black bars: theory prediction

Measured cross sections and exclusion limits at 95% C.L.

See here for all cross section summary plots

Jan 2022

Diboson cross section measurements at several center of mass energies (\sqrt{s})



Recent diboson and polarization measurements at CMS

• SMP-22-001: Measurement of the ZZ(4l) + jets

• SMP-21-014: $\gamma\gamma \to W^+W^-$ and $\gamma\gamma \to ZZ$

• SMP-20-014: Measurement of the WZ process

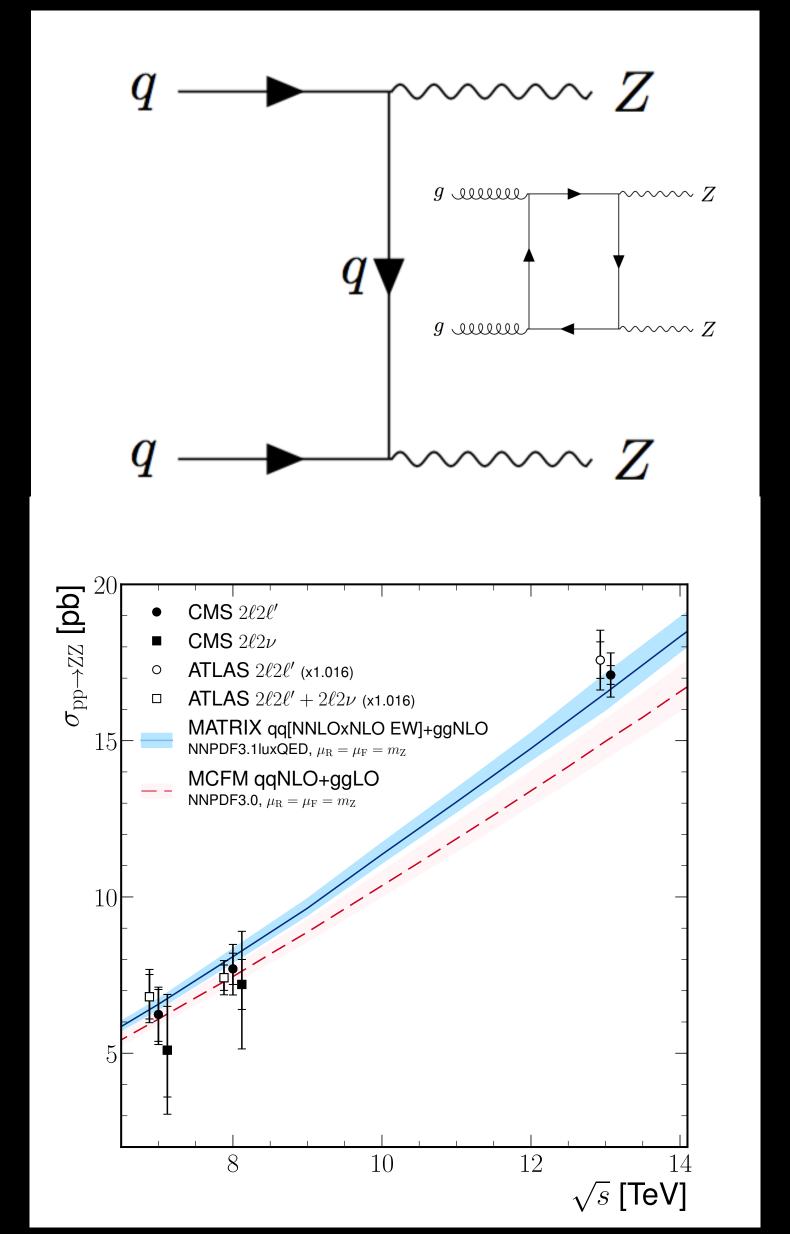
• SMP-18-010: *τ* polarization in Z-boson decays



When an Al thinks about "Dibosons at the precision realm"

Hot off the press! Measurement of the ZZ process

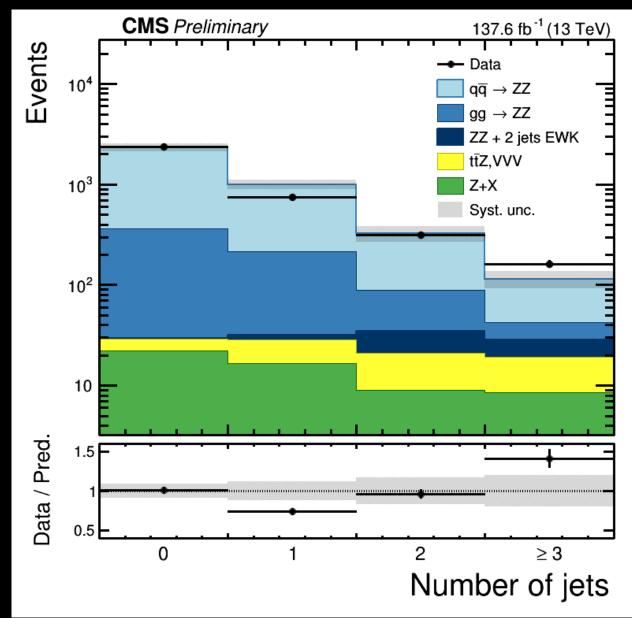
- Measurement of ZZ production in 4 lepton final state
 - No s-channel production forbidden in the SM
 - Loop effects up to 10% contribution
- Inclusive cross section studied in detail previously
- First differential cross sections measured as a function of:
 - number of jets and properties of the jets (p_T and $|\eta|$)
 - invariant mass of the highest $\ensuremath{p_{T}}$ and the second highest $\ensuremath{p_{T}}$ jets
 - $\Delta\eta$ of the highest p_T and the second highest p_T jets
 - m_{4l} as a function of different jet multiplicities
- New results compared with the state-of-the-art next-to-next-to-leading order (NNLO) and parton shower (PS) predictions using MiNNLO_{PS}

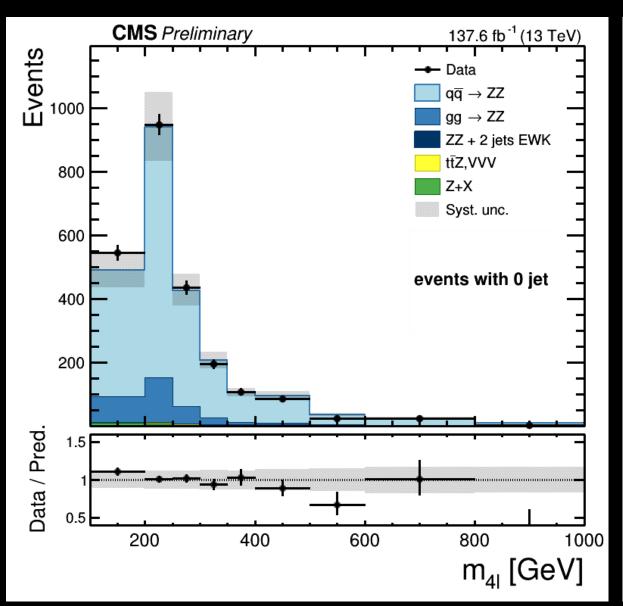


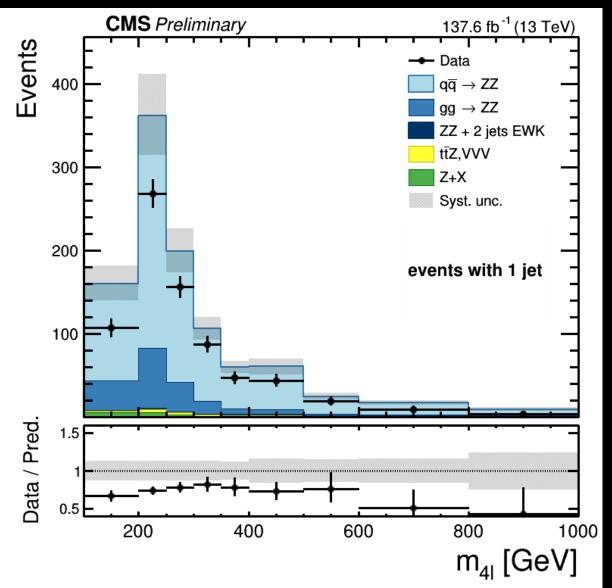
Hot off the press!

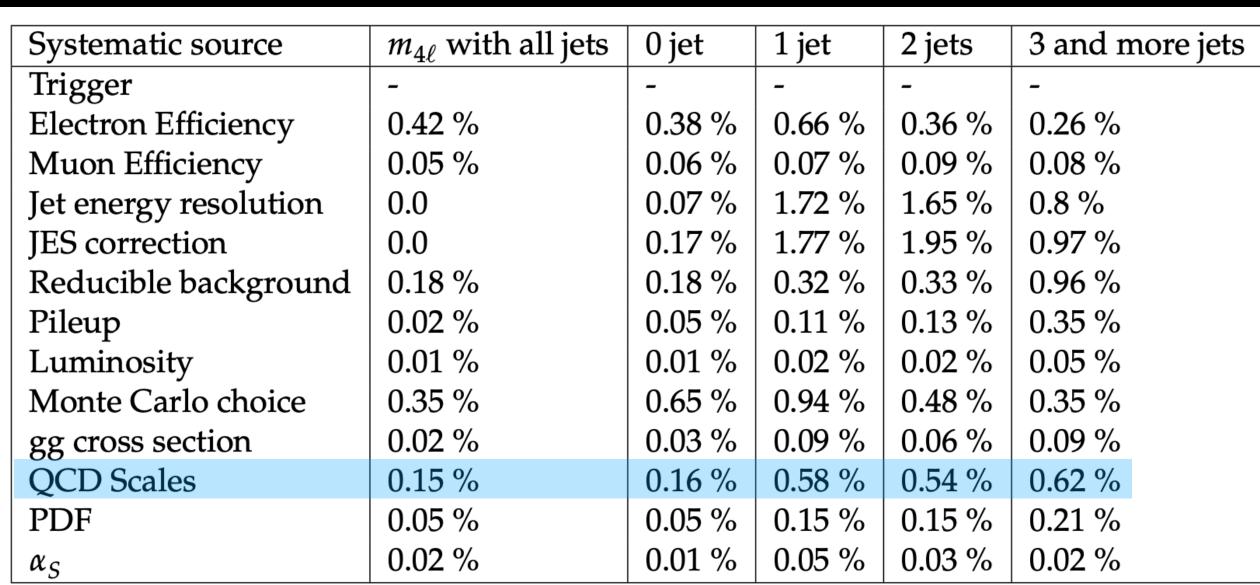
Measurement of the ZZ process

- Predictions over-estimate data \rightarrow largest discrepancy for highest p_T jet with $p_T < 100 \text{ GeV}$
- Shape of distributions described well by predictions → normalization is not
- Large uncertainty from QCD scales envelope of μ_F and μ_R scale variations $1/4 < \mu_R/\mu_F < 4$









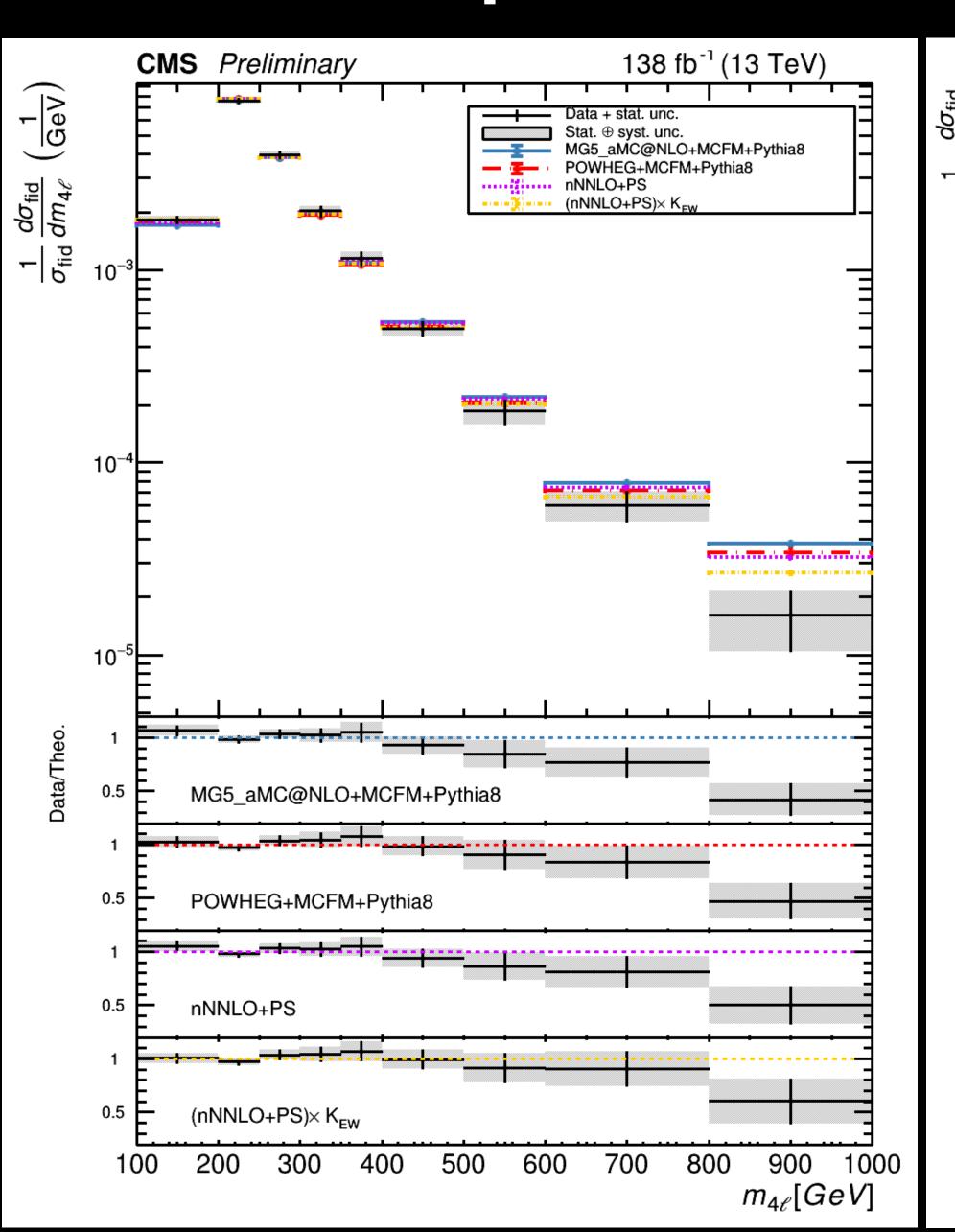
Data agrees with prediction

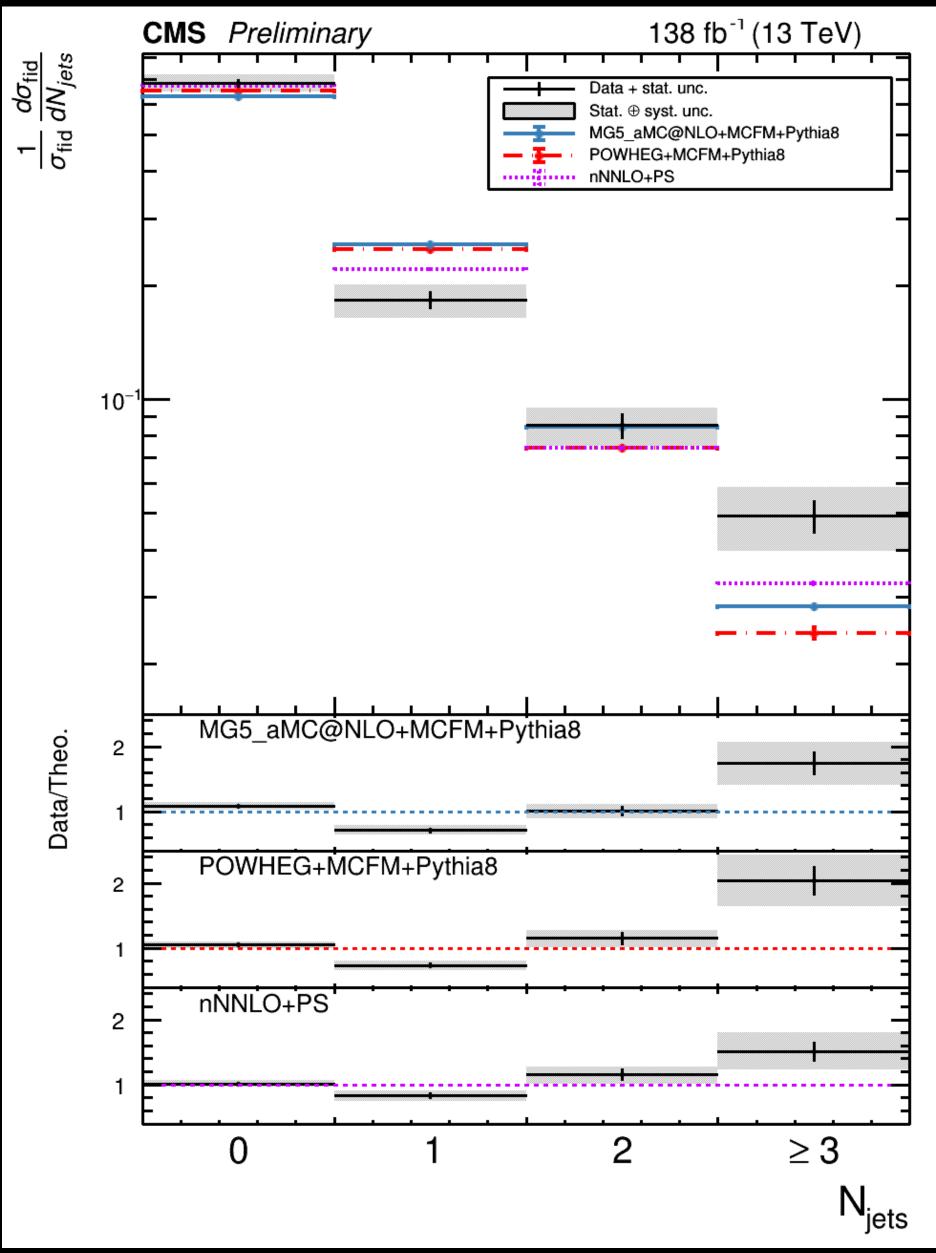
Data disagrees with prediction 6

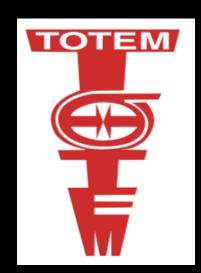
Hot off the press!

Measurement of the ZZ process — Unfolded distributions

- Differential cross sections normalized to the fiducial cross sections
- On-shell Z-boson requirement applied $(60 < m_{Z_1,Z_2} < 120)$
- Comparisons with:
 - Madgraph5_aMC@NLO $(q\bar{q} \to ZZ)$ +MCFM $(gg \to ZZ)$ +POWHEG $(H \to ZZ)$
 - POWHEG $(q\bar{q} \to ZZ)$ +MCFM $(gg \to ZZ)$ +POWHEG $(H \to ZZ)$
 - Both predictions include Madgraph electro-weak corrections
 - NNLO+PS
 - NNLO+PS with electroweak corrections

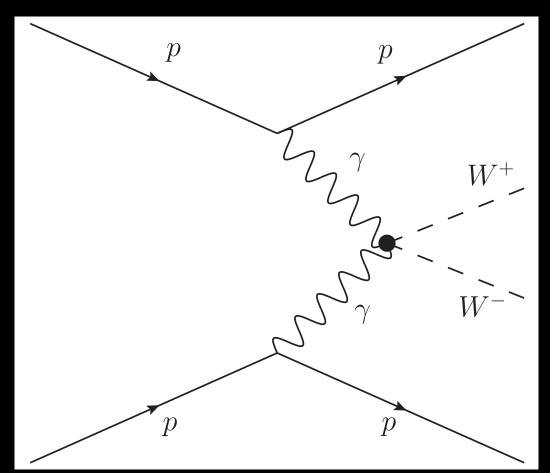


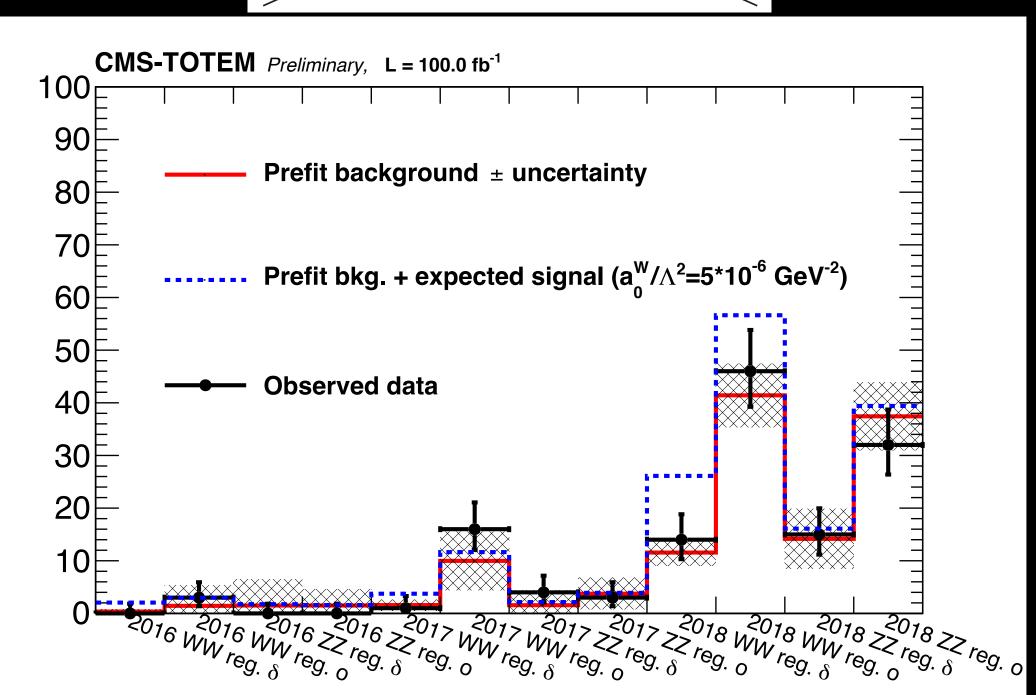




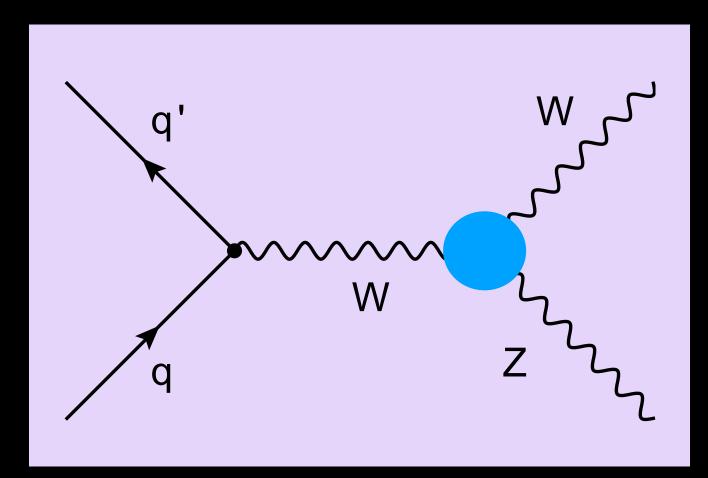
Search for exclusive $\gamma\gamma \to WW$ and $\gamma\gamma \to ZZ$ production in final states with jets and forward protons

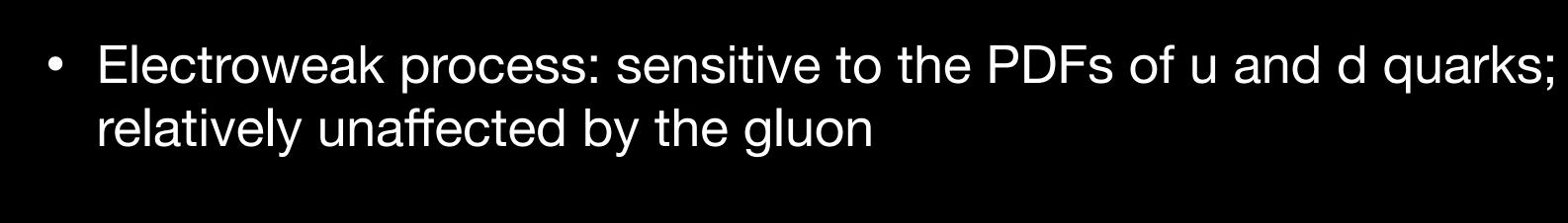
- Both protons tagged by the precision proton spectrometer (PPS)
- The $\gamma\gamma \to WW$ process allows the study of the quartic coupling
- Events selected based on properties of jets, the protons and their correlation
- First search for anomalous high-mass $\gamma\gamma \to WW$ and $\gamma\gamma \to ZZ$ using reconstructed forward protons
 - Limits 15-20x more stringent than previous results



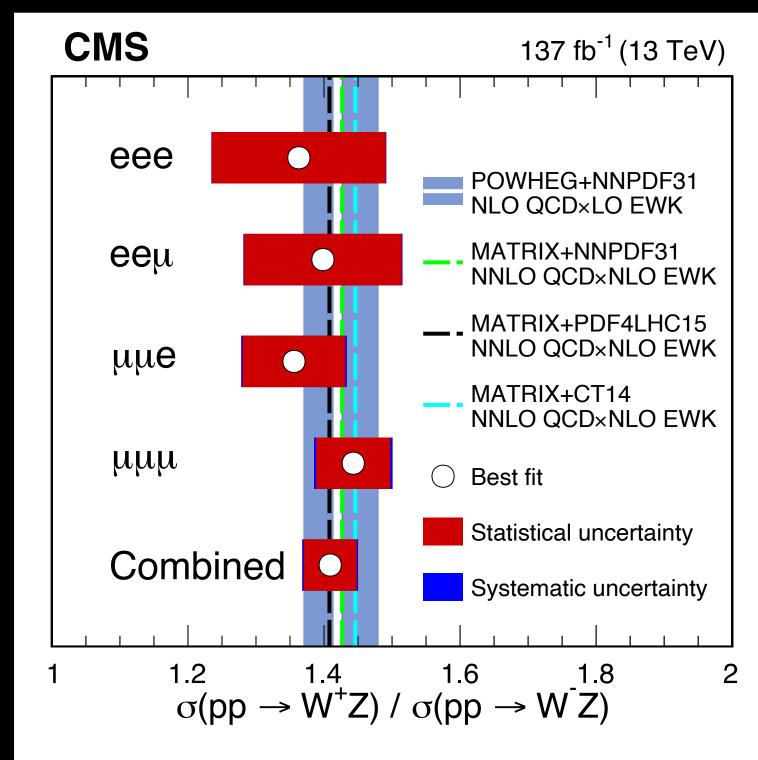


Measurement of the WZ process





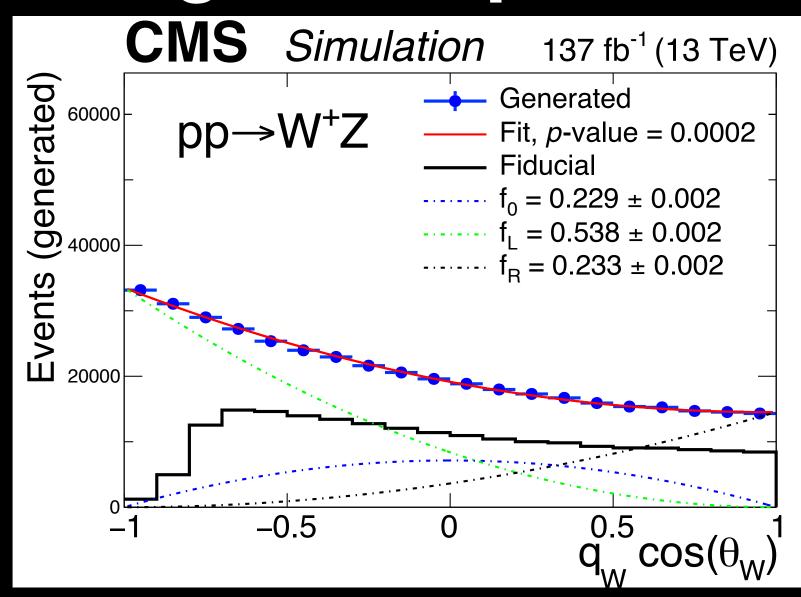
 High WZ cross section makes it the dominant process that can be studied in the trilepton final state

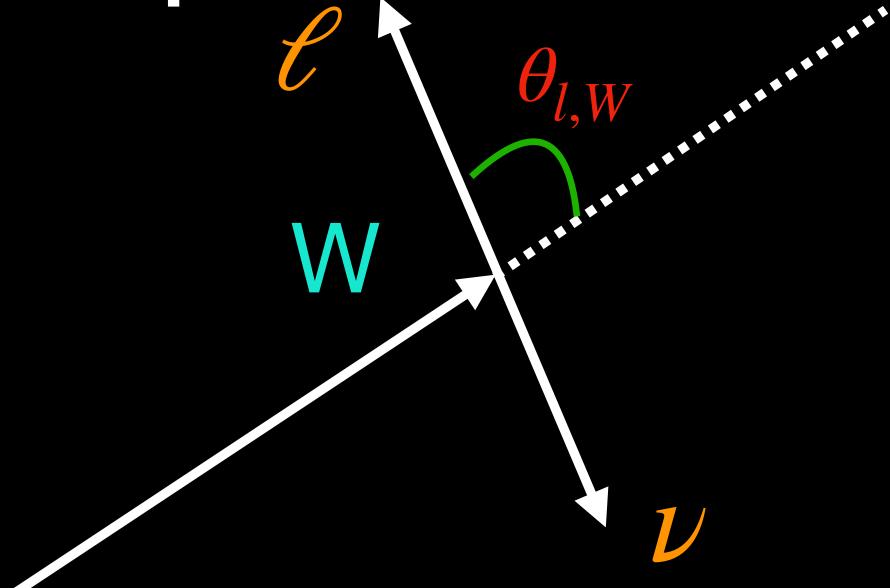


- . Ratio of $\frac{W^+Z}{W^-Z}$ cross section is one of the most precisely measurable quantities
- Constitutes first measurement of longitudinally polarized W-bosons

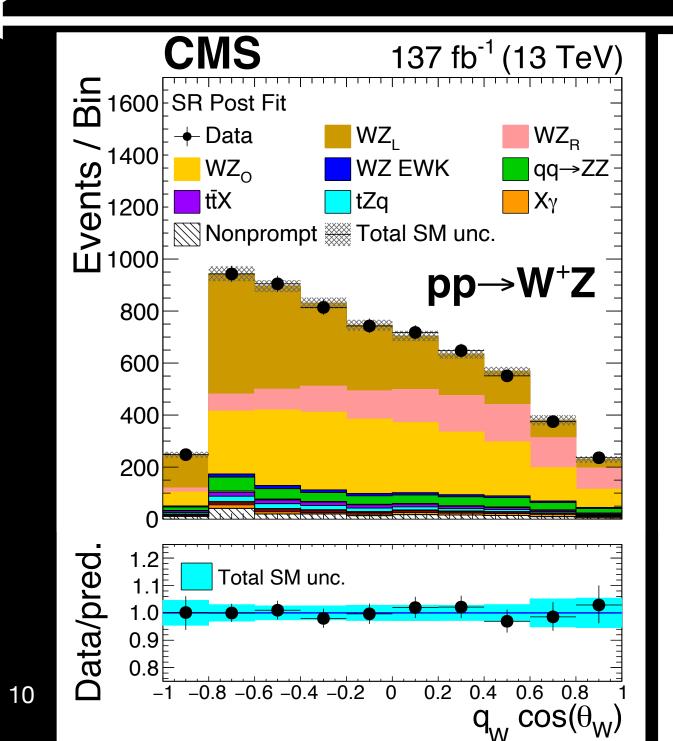
<u>JHEP 07 (2022) 032</u>

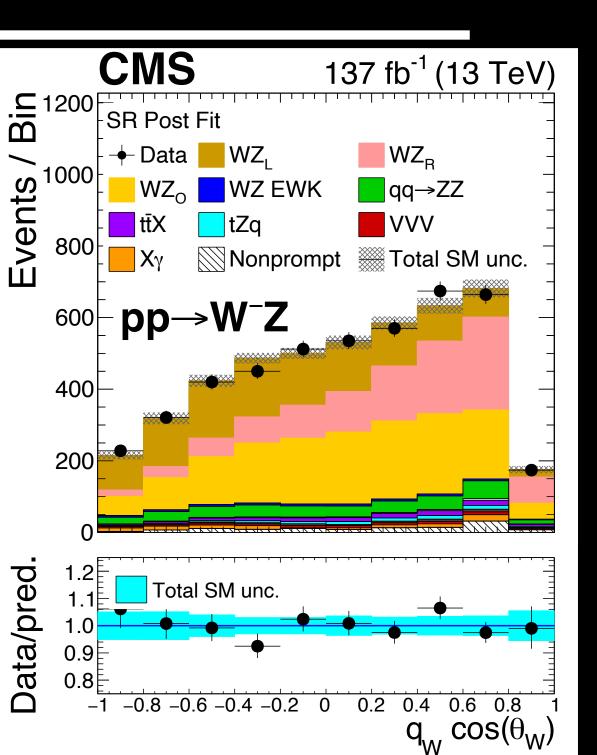
Measuring the W polarization in WZ processes



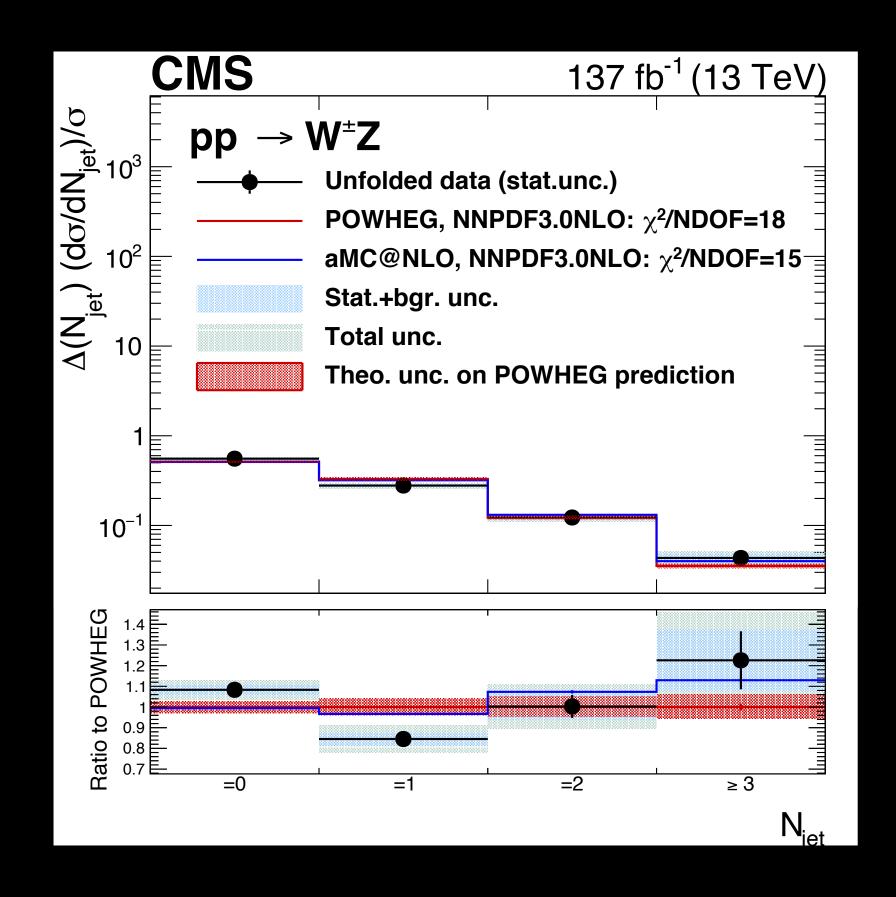


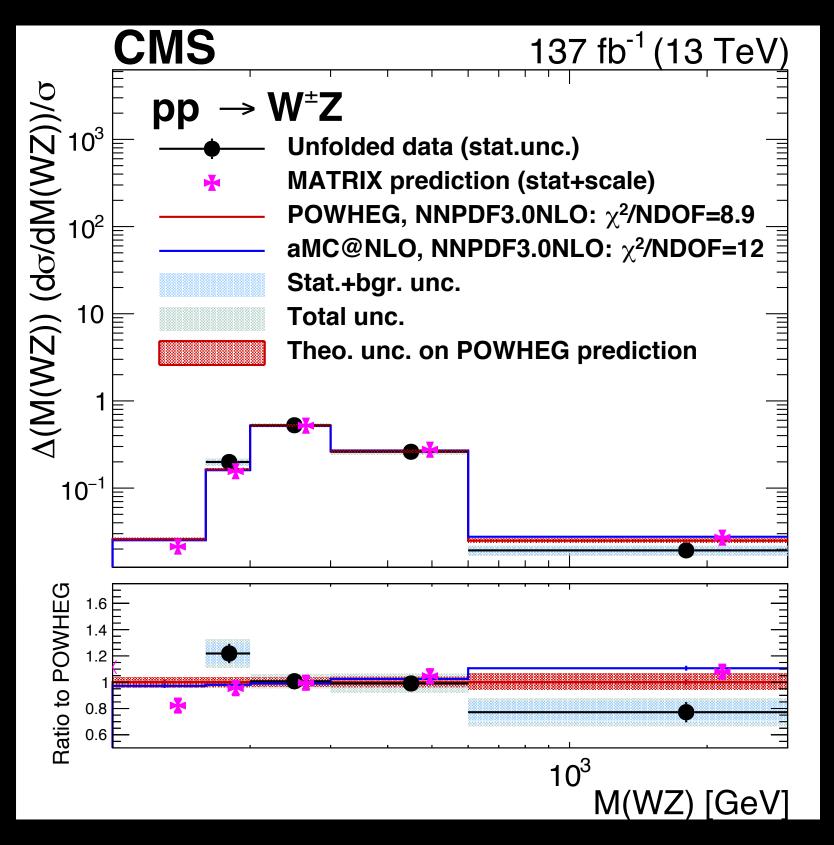
θ^W: angular distance between the momenta of the W boson and the charged lepton from its primary decay

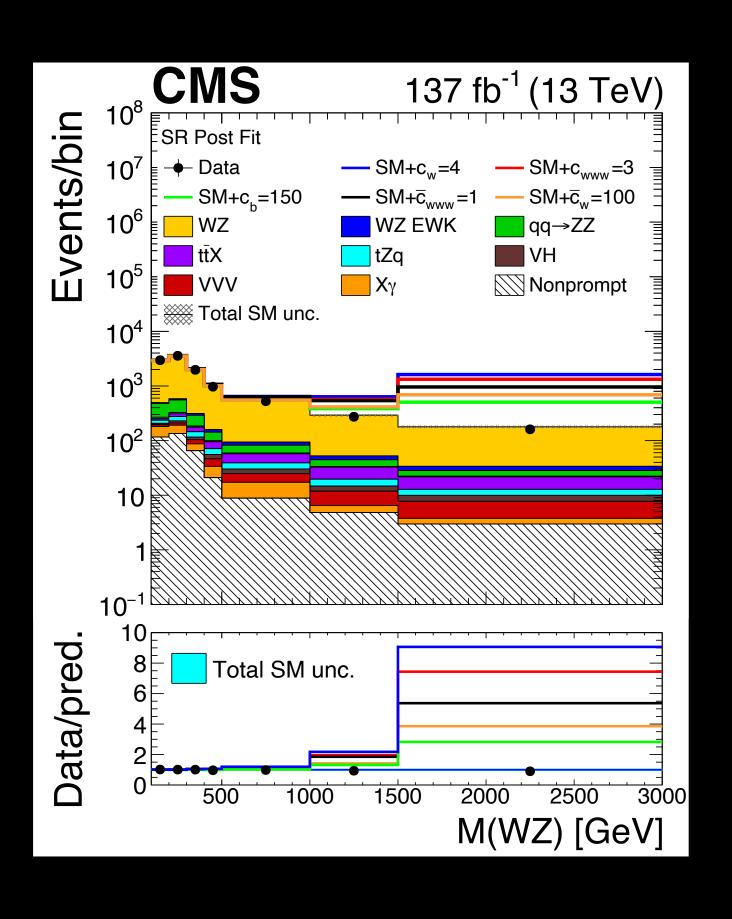




Measurement of the WZ process



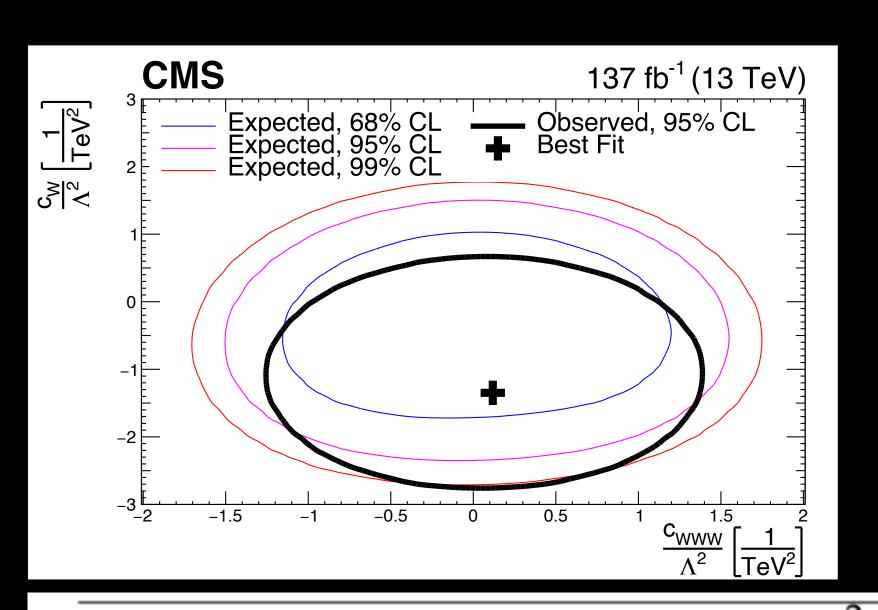


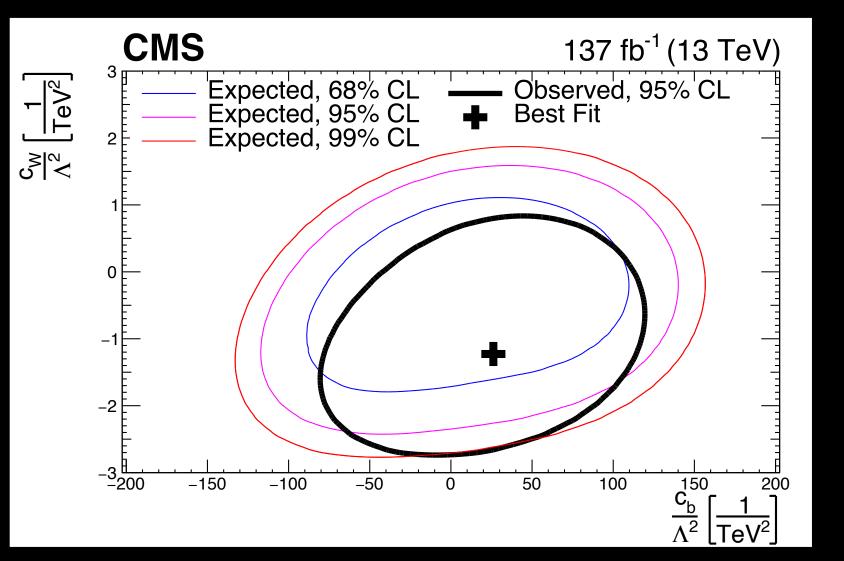


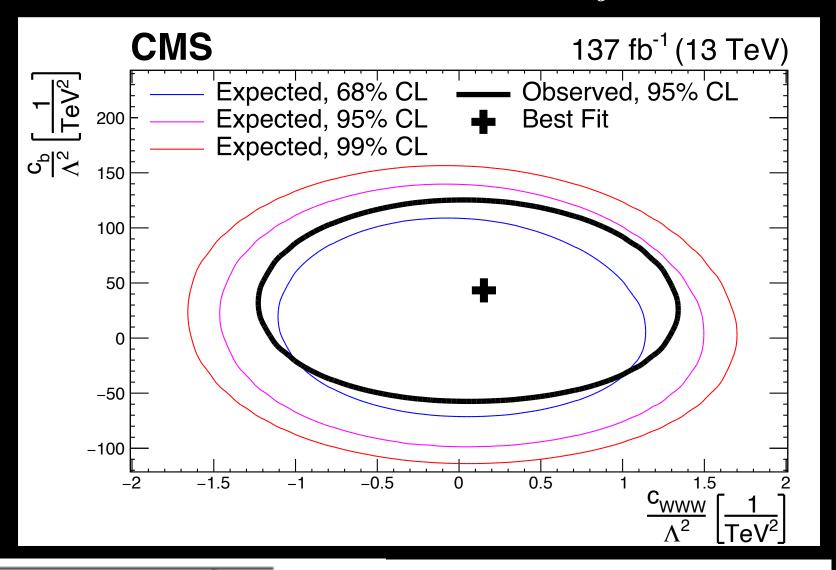
- Unfolded distributions of sensitive variables well modeled
- Simulated signal samples normalized to NNLO cross sections

Effective Field Theory

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i rac{c_i}{\Lambda^2} \mathcal{O}_i + \sum_j rac{f_j}{\Lambda^4} \mathcal{O}_i$$







	eV^{-2}	Best fit, Obs. (T	95% CI, Obs. (TeV ⁻²)	95% CI, Exp. (TeV ⁻²)	Parameter
		-1.34	[-2.52, 0.33]	[-2.05, 1.27]	$c_{\rm w}/\Lambda^2$
CP conserving		0.15	[-1.04, 1.19]	[-1.27, 1.33]	$c_{\mathrm{www}}/\Lambda^2$
CD non concerving		43.6	[-42.7, 113.0]	[-86.0, 125.0]	$c_{\rm b}/\Lambda^2$
CP non-conserving		-0.03	[-0.62, 0.53]	[-0.76, 0.65]	$\tilde{c}_{\mathrm{www}}/\Lambda^2$
		0.0	[-45.9, 45.9]	[-46.1, 46.1]	$\tilde{c}_{\rm w}/\Lambda^2$

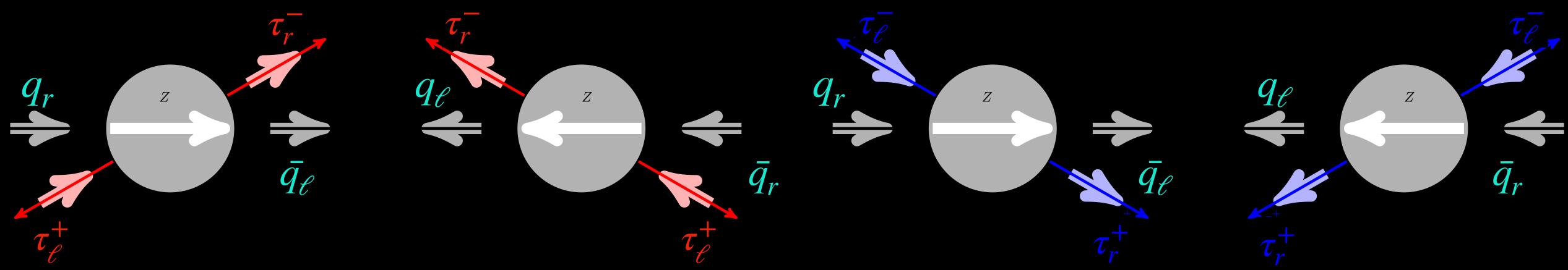
- Limits computed taking interference with SM (Λ-2) + pure BSM (Λ-4) into account
- Improvement over limits from W+W- analysis (taking luminosity scaling into account) for C_w and C_{www}

τ polarization in Z-boson decays

- Polarization of τ leptons is measured in $Z \to \tau \tau$ events
- Differential cross section of the $q\bar{q} \to Z \to \tau^+\tau^-$ in lowest order is expressed as:

$$\frac{d\sigma}{d\cos\theta_{\tau}} = F_0(\hat{s})(1 + \cos^2\theta_{\tau}) + 2F_1(\hat{s})\cos\theta_{\tau} - h_{\tau}[F_2(\hat{s})(1 + \cos^2\theta_{\tau}) + 2F_3(\hat{s})\cos\theta_{\tau}]$$

- Forward-backward asymmetry defined in terms of ${\cal F}_1$ and ${\cal F}_2$
- Polarization defined in terms of ${\cal F}_2$ and ${\cal F}_0$
 - Simplified when $\sqrt{\hat{s}} = M_Z$



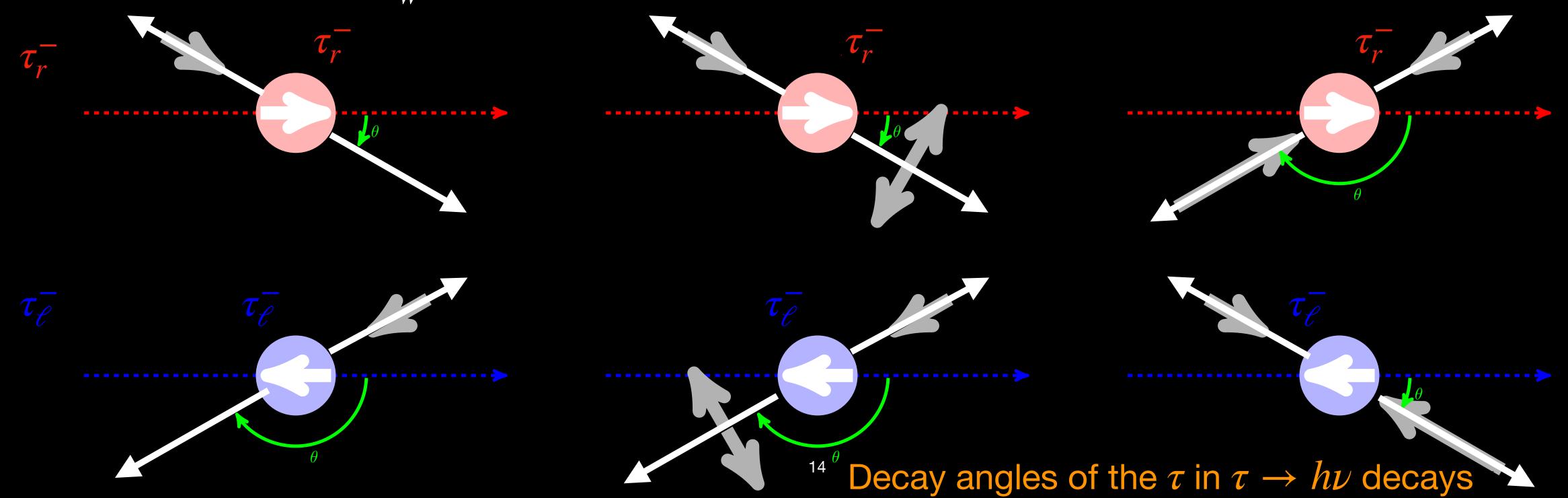
Helicity states of incoming quarks and outgoing τ leptons: thin arrows show direction of movement, thick arrows show helicity

 θ_{τ} : scattering angle of the τ^- with respect to the quark momentum in the rest frame of the Z boson

 $h_{ au}$: helicity

τ polarization in Z-boson decays

- Tau spin observables constructed from several decay angles
- Optimal observable to measure τ helicity → polarimetric vector
 - Final-state dependent variable
 - Largest source of uncertainty from QCD normalization
- Measured value of τ polarization: $P_{\tau}(Z) = -0.144 \pm 0.006$ (stat) \pm 0.014 (syst)
 - In agreement with LEP, SLD, ATLAS
- More precise than the previous ATLAS measurement, almost matches precision of single LEP experiments
- Weak mixing angle $\sin^2 \theta_W^{\text{eff}} = 0.2319 \pm 0.0019$ (0.8% precision)



Outlook

- Several diboson (ZZ, WZ and photon initiated processes) analyses with novel methods presented
- Comprehensive exploration of final states with a au lepton production through the decay of Z bosons
- Diboson physics is being performed at the precision realm
- Enabling stringent tests of the electroweak gauge sector of the Standard Model

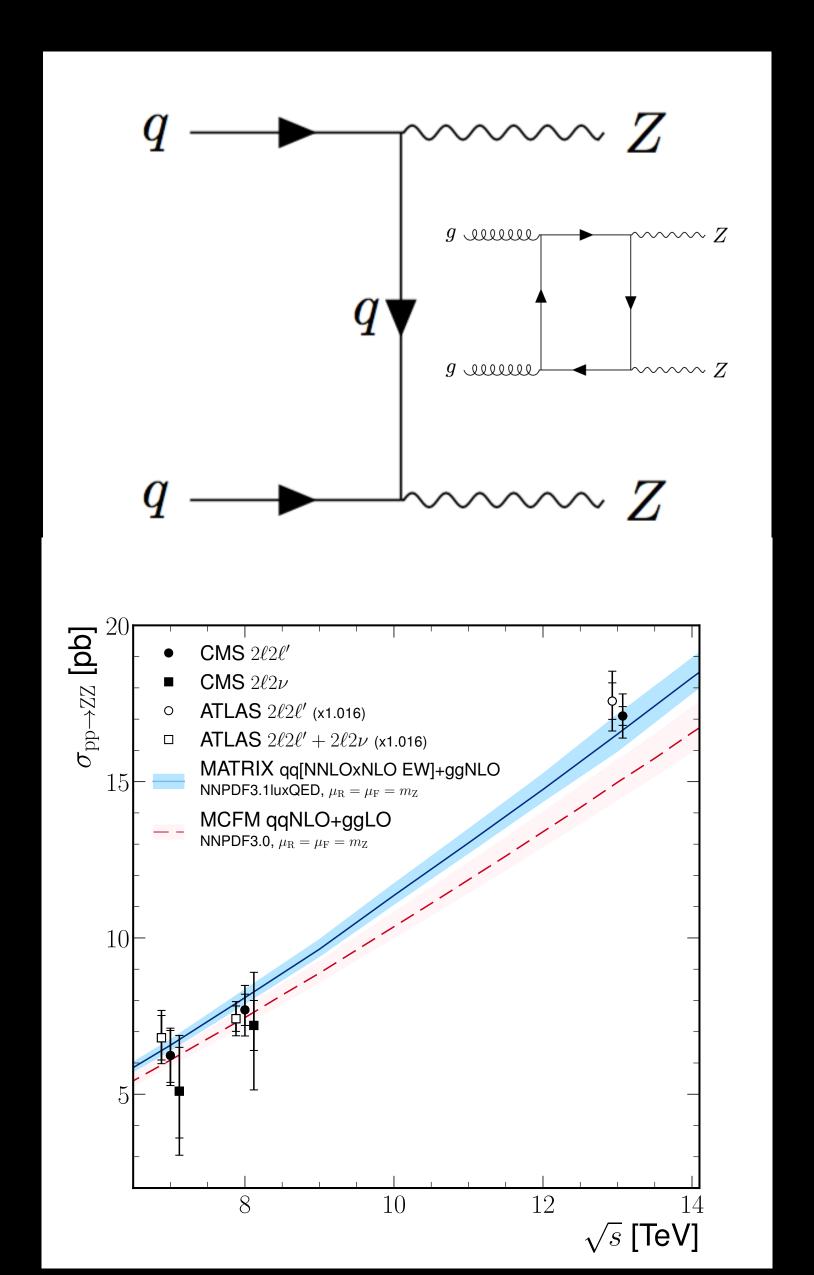
Additional Material

Measurement of the ZZ process

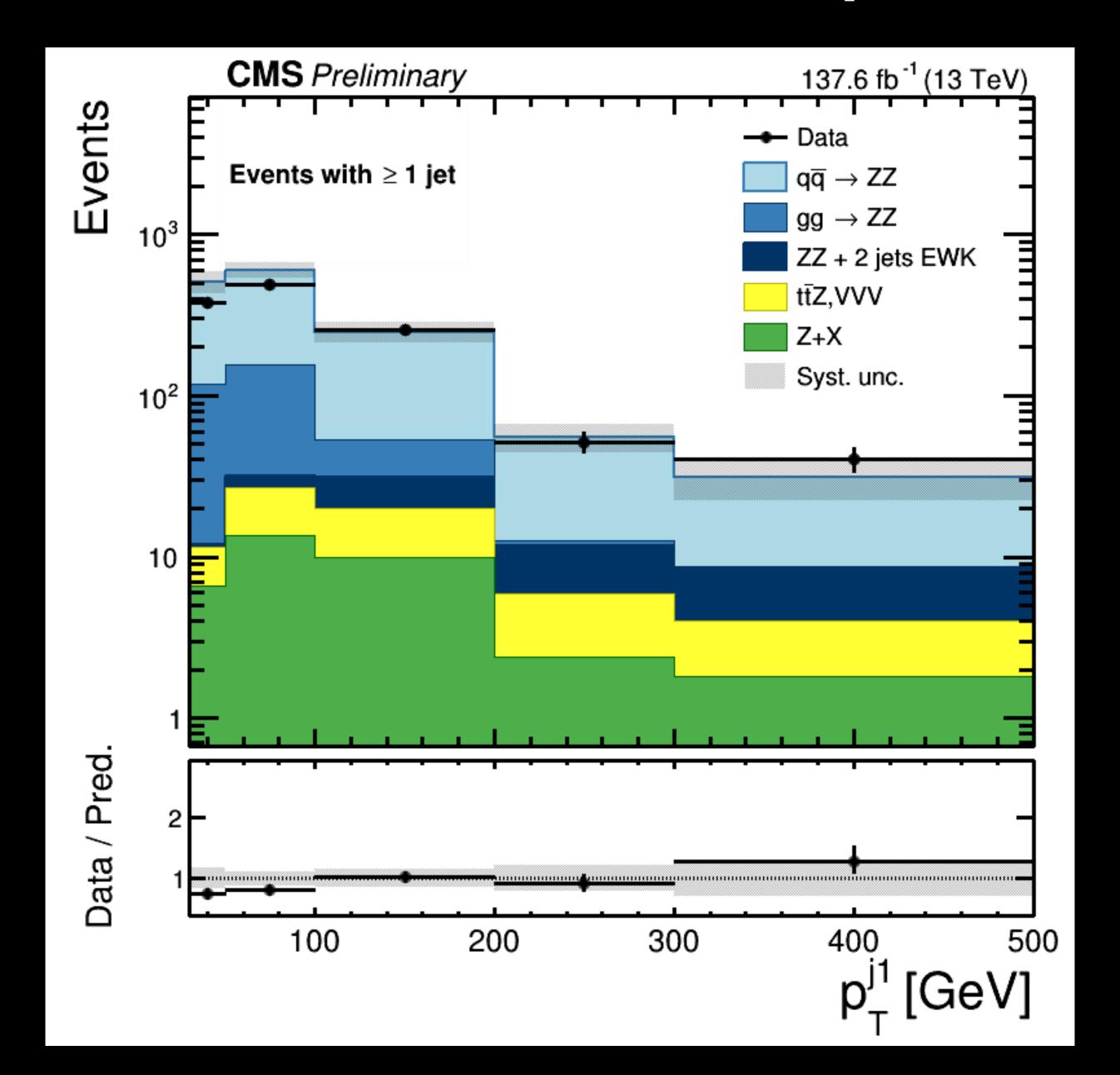
- Monte Carlo samples
 - Samples generated at NLO with Madgraph5_aMC@NLO and POWHEG
 - $gg \rightarrow ZZ$ simulated at LO with MCFM
 - Cross sections of these samples \rightarrow normalized to the cross section at NNLO for $q\bar{q}\to ZZ$ (k-factor of 1.1)
 - Cross section NLO in QCD for $gg \rightarrow ZZ$ (k-factor of 1.7)
 - nNNLO+PS: NNLO predictions for quark initiated channel combined with parton showers using the MiNNLO_{PS} method
 - NLO predictions for loop induced gluon fusion channel matched to parton showers
 - Spin correlations, interferences and off-shell effects are included by calculating the full process $pp \to \ell^+ \ell^- \ell^{'+} \ell^{'-}$
- Unfolding
 - Used D'agostini's method (RooUnfold toolkit)

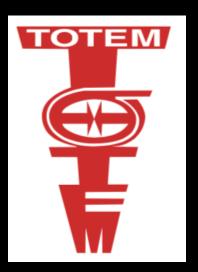
Hot off the press! Measurement of the ZZ process

- Measurement of ZZ production in 4 lepton final state
 - No s-channel production forbidden in the SM
 - Loop effects up to 10% contribution
- On-shell requirement on both Z-boson candidates
- Inclusive cross section studied in detail
- First differential cross sections measured as a function of:
 - number of jets
 - transverse momentum (p_T)
 - pseudorapidity (η)
 - invariant mass of the highest $\ensuremath{p_{T}}$ and the second highest $\ensuremath{p_{T}}$ jets
 - $\Delta \eta$ of the highest p_T and the second highest p_T jets
 - m_{4l} as a function of different jet multiplicities



Measurement of the ZZ process





Search for exclusive $\gamma\gamma \to WW$ and $\gamma\gamma \to ZZ$ production in final states with jets and forward protons

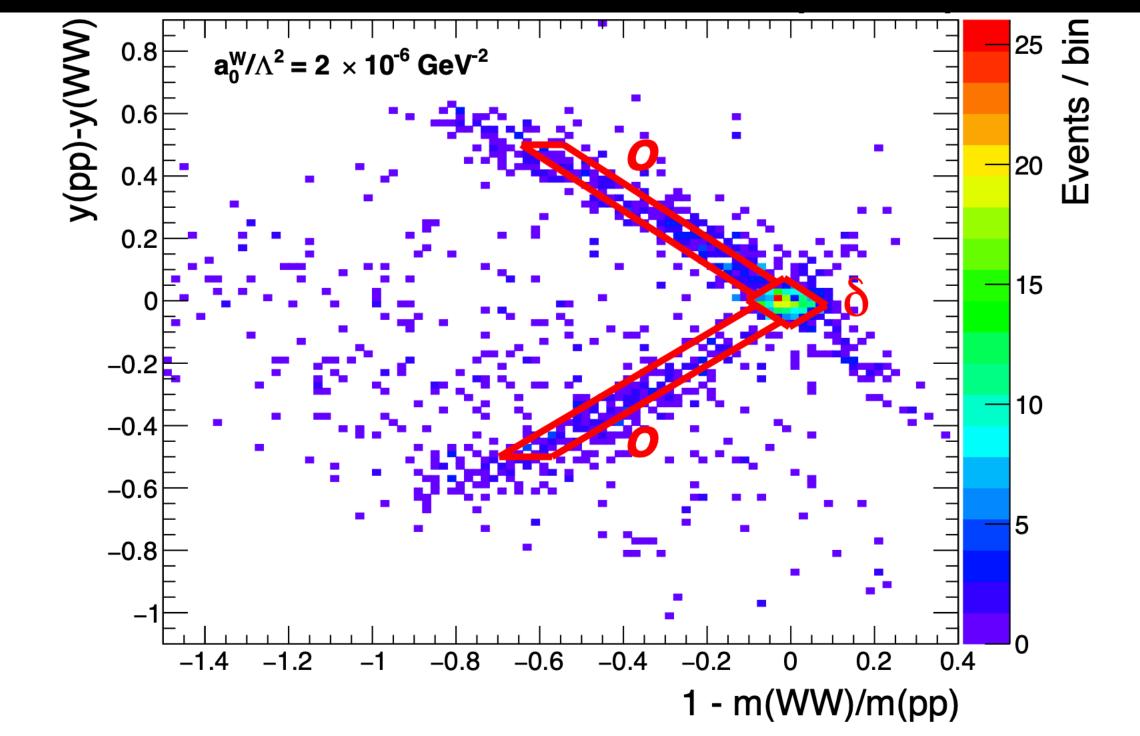
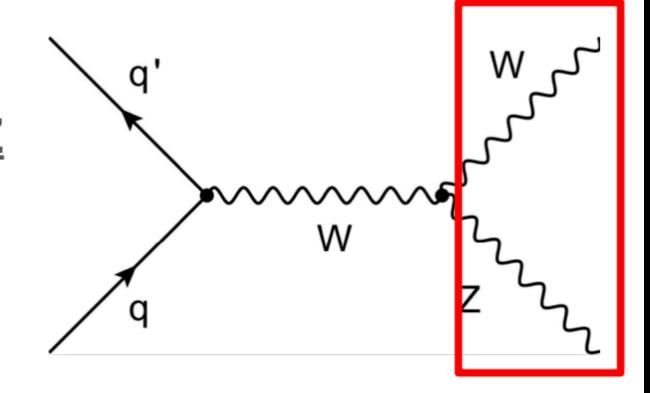


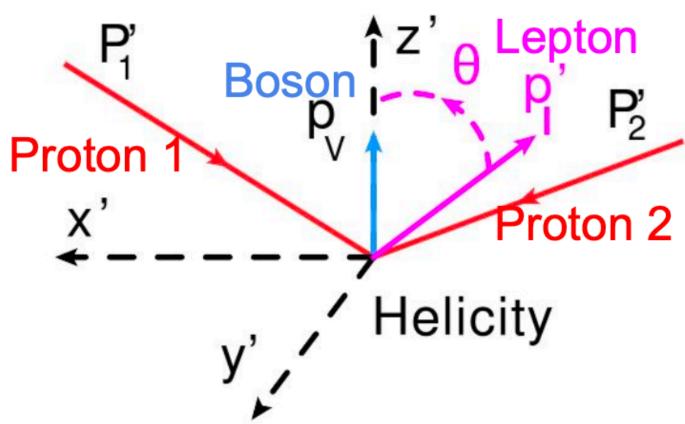
Figure 4: Matching between the jets and the protons, in invariant mass and rapidity, for simulated WW exclusive signal events. The red diamond-shaped area near the origin (signal region δ) corresponds to the case where both protons are correctly matched to the jets. The areas within the red diagonal bands (signal region δ) correspond to the case where one proton is correctly matched, and the second proton originates from a pileup interaction.

Boson polarization studies - Description

→ We also look at the "final" WZ state. We study the **polarization of the W and Z bosons**, a measurement sensitive to possible gauge/higgs couplings anomalies.



 \rightarrow The polarization is studied in terms of the polarization angle θ :



Helicity frame: the lab frame is rotated so the V boson is in the Z axis, then it is boosted such that the V boson is at rest.

--> At LO in EW the following quadratic relations hold for θ:

$$\frac{d\sigma}{\sigma d \cos \theta^{Z}} = \frac{3}{8} \left[(1 + \cos^{2}(\theta^{Z}) + 2c \cos(\theta^{Z})) f_{L}^{Z} + (1 + \cos^{2}(\theta^{Z}) - 2c \cos(\theta^{Z})) f_{R}^{Z} + 2 \sin^{2}(\theta^{Z}) f_{0}^{Z} \right]$$

Where f_L, f_R, f_O are the "polarization fractions", quantities that are related to the boson spin density matrix, and are <u>well defined but frame-dependant</u> as they relate to particle helicities. By construction they fulfill: $f_L + f_R + f_O = 1$. A separate set appears for W and Z production.

→ We aim to measure the W/Z polarization fractions into the <u>helicity frame</u>
 (technically also prepared for Collins-Sopper).

τ polarization in Z-boson decays

Considering the decay $\tau^- \to \pi^- \nu$, there is no angular momentum in the two-body τ decay and since the pion carries no spin, angular momentum conservation requires the neutrino to adopt the spin of the τ lepton. In the τ lepton rest frame, the neutrino and pion are emitted back-to-back. Since the neutrino is always left-handed the π^- prefers small values for the angle θ in the decay of a right-handed τ_R and large values in case the τ lepton is left-handed. In this case the angle θ carries all spin information.

The decay into a spin-one resonance, ρ or a_1 , offers the kinematic simplicity of a two-body decay, like the $\tau^- \to \pi^- \nu$ decay, but with more complicated dynamics since the ρ and a_1 resonances can have longitudinal and transverse helicity states. Conservation of angular momentum allows for the ρ and a_1 resonances helicities to be equal to $\lambda_V = 0$ or -1.

If the τ lepton is in the right-handed state, the V ($V = \rho, a_1$) resonance tends to be in a longitudinally polarized state ($\lambda_V = 0$). Conversely, if the τ is left-handed, the V is preferably transversely polarized, $\lambda_V = -1$. Combining the spin amplitudes for all possible configurations of the V resonance and τ helicities, one gets [35]:

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta} \propto 1 + \alpha_V h_\tau \cos\theta,\tag{8}$$

where the dilution factor $\alpha_V = (|M_L|^2 - |M_T|^2)/(|M_T|^2 + |M_L|^2) = (m_\tau^2 - 2m_V^2)/(m_\tau^2 + 2m_V^2)$ is a result of the presence of the transverse amplitude M_T of the V resonance in addition to the longitudinal amplitude M_L . The value of the factor α_V characterizes the sensitivity of the $\cos\theta$ observable. For comparison, in the τ lepton decay to the a_1 resonance, $\alpha_{a_1} = 0.021$, in the ρ decay, $\alpha_\rho = 0.46$ and in the pion decay, $\alpha_\pi = 1$. Consequently, the sensitivity to the τ lepton helicity in $\tau \to V \nu$ decays is strongly reduced if only the $\cos\theta$ angle is considered. The spin of V is transformed into the total angular momentum of the decay products and thus can be retrieved by analyzing the subsequent V decay.

τ polarization in Z-boson decays

There are two additional angles to be considered in the decay of $\tau^- \to \rho^- \nu$ ($\rho^- \to \pi^- \pi^0$) : β denotes the angle between the direction of the charged pion in the ρ rest frame and the direction of the ρ , given by:

$$\cos \beta = \mathbf{q} \cdot \mathbf{n}_{\rho} = \frac{m_{\rho}}{\sqrt{m_{\rho}^2 - 4m_{\pi}^2}} \cdot \frac{E_{\pi^-} - E_{\pi^0}}{|p_{\pi^-} - p_{\pi^0}|}, \tag{9}$$

where **q** is a unit vector along the direction of the charged pion in the ρ rest frame and n_{ρ} is the direction of the movement of the ρ projected into its rest frame. The rotation plane of two

https://cds.cern.ch/record/2848665/files/SMP-18-010-pas.pdf

pions is aligned correspondingly to the spin of the ρ resonance. In the case of $\lambda_{\rho}=0$, the angle β tends to small values, and to large values if $\lambda_{\rho}=-1$. The angle β in $\tau^-\to\rho^-\nu$ is sketched in Fig. 3b.This angle contains only quantities that can be measured directly and is a very powerful variable to discriminate left- and right-handed helicity states.

The similar, but not identical, angle β can be defined for the $\tau^- \to a_1^- \nu$. In this case β is defined as the angle between the normal to the 3π decay plane and the a_1 flight direction. The angle is shown in Fig. 3d.

A further angle α is defined by the two planes spanned by vectors (\mathbf{n}_V , \mathbf{n}_{τ}) and (\mathbf{n}_V , \mathbf{q}), respectively:

$$\cos \alpha = \frac{(\mathbf{n}_V \times \mathbf{n}_\tau) \cdot (\mathbf{n}_V \times \mathbf{q})}{|\mathbf{n}_V \times \mathbf{n}_\tau| \cdot |\mathbf{n}_V \times \mathbf{q}|}.$$
 (10)

Here all vectors are defined in the resonance rest frame. The vector \mathbf{n}_V denotes the direction of ρ or a_1 resonance and \mathbf{n}_{τ} is the direction of the τ lepton. The angle α describes the correlation between the helicity of the τ lepton and the decay products of the ρ or a_1 resonance. The vector \mathbf{q} denotes the direction of the π^- in case of $\tau^- \to \rho^- \nu$ decay and π^+ in $\tau^- \to a_1^- \nu$ decay. The angle α is illustrated in Fig. 3a.

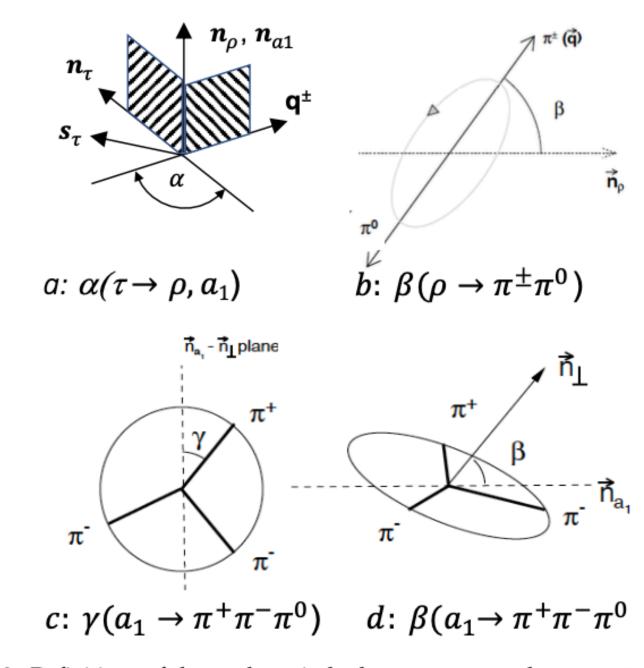


Figure 3: Definitions of the angles α in both $\tau^- \to \rho^- \nu$ and $\tau^- \to a_1^- \nu$ (a)), the angle β in $\tau^- \to \rho^- \nu$ (b)) and for the decay $\tau^- \to a_1^- \nu$ (d)) and finally the angle (γ (c)) for the decay of $a_1^- \to \pi^- \pi^+ \pi^-$

7.2 Optimal observable

Optimal variables to measure the helicity of τ leptons have been widely explored at LEP and are explained, for example, in Ref. [36]. The idea is to replace multi-dimensional fits over all relevant decay angles by a one dimensional fit to a unique variable $\omega(\theta, \alpha, \beta, \gamma, ...)$, which can be calculated using the squared matrix elements of the decay for positive and negative helicity. This procedure is equivalent to the use of the concept of the polarimetric vector.

In general the differential widths of any decay of a polarized τ lepton can be described using the polarimetric vector and has then the simplified form [14, 35]:

$$d\Gamma = \frac{|\overline{M}|^2}{2m_{\tau}} (1 - h_{\mu}s^{\mu}) dLIPS, \tag{11}$$

where *d*LIPS is the element of Lorentz-invariant phase space, *s* is the four-vector of the spin of the τ lepton and *h* is the polarimetric vector. In the rest frame of the τ lepton, assuming that the τ lepton spin is aligned to the τ lepton flight direction (all transverse τ spin components are zero), this reads:

Table 2: Final choice of discriminators in the different event categories

Channel	Category	Discriminator			
$ au_e au_\mu$	$e + \mu$	$m_{\rm vis}(e,\mu)$	visible mass		
$ au_e au_{ m h}$	$e + a_1$	$\omega(a_1)$	optimal observable with SVfit		
	$e + \rho$	$\omega_{ m vis}(ho)$	visible optimal observable		
	$e + \pi$	$\omega(\pi)$	optimal observable with SVfit		
$\tau_{\mu}\tau_{\rm h}$	$\mu + a_1$	$\omega(a_1)$	optimal observable with SVfit		
•	$\mu + \rho$	$\omega_{ m vis}(ho)$	visible optimal observable		
	$\mu + \pi$	$\omega(\pi)$	optimal observable with SVfit		
$ au_{ m h} au_{ m h}$	$a_1 + a_1$	$m_{\rm vis}(a_1,a_1)$	visible mass		
	$a_1 + \pi$	$\Omega(a_1,\pi)$	combined optimal observable with SVfit		
	$ ho + au_{ m h}$	$\omega_{ m vis}(ho)$	visible optimal observable (for leading ρ)		
	$\pi + \pi$	$m_{\mathrm{vis}}(\pi,\pi)$	visible mass		

τ polarization in Z-boson decays

$$w = \frac{1}{4} \sum_{i,j=0}^{4} h_i^+ h_j^- R_{ij}$$
 (7.1)

where h^+ and h^- are the so-called polarimeter vectors for the τ^+ and τ^- decays, respectively, and the matrix R_{ij} describing the spin orientation of the system, in the case of a Z decay, takes the form

$$R = \begin{pmatrix} 1 & 0 & 0 & -P_{\tau} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -P_{\tau} & 0 & 0 & 1 \end{pmatrix}$$
 (7.2)

Here P_{τ} is the tau polarization given by Equation 3.10. Equations 7.1 and 7.2 are expressed in the rest frame of the Z with the z-axis in the direction of flight of the τ^- . By inserting Equation 7.2 into Equation 7.1 one obtains

$$w = \frac{1}{4} \left[1 + h_z^+ h_z^- - P_\tau \left(h_z^+ + h_z^- \right) \right] \tag{7.3}$$