TWO-PARTICLE FEMTOSCOPIC CORRELATION MEASUREMENTS

MÁTÉ CSANÁD (EÖTVÖS U) FOR THE ALICE, ATLAS, CMS COLLABORATIONS

LHCP 2023

ALICE

ATLAS EXPERIMENT

CMS
Compact Muon Solenoid

CONTENTS OF THIS TALK

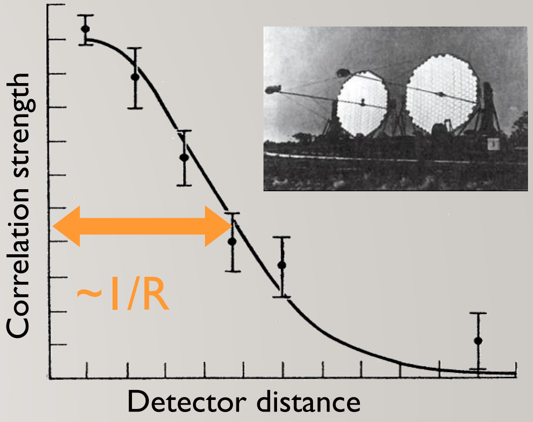
- Basics of femtoscopy and Lévy sources
- Two-particle femtoscopy in PbPb collisions with CMS
- Event-by-event shape analysis in EPOS PbPb collisions
- Non-identical femtoscopy with ALICE
- Two-particle femtoscopy in pp collisions with ATLAS

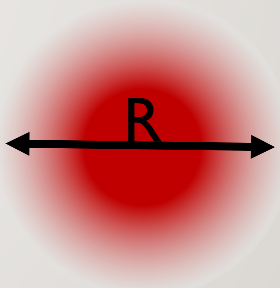
FEMTOSCOPY IN HIGH ENERGY PHYSICS

- R. Hanbury Brown, R. Q. Twiss - observing Sirius with radio telescopes
  - Intensity correlations vs detector distance ⇒ source size
  - Measure the sizes of apparently point-like sources!
- Goldhaber et al: applicable in high energy physics
- Understanding: Glauber, Fano, Baym, …
  Phys. Rev. Lett. 10, 84; Rev. Mod. Phys. 78 1267, …
  - Momentum correlation $C(q)$ related to particle emitting source $S(r)$

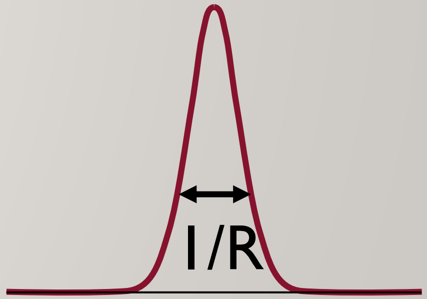
    $C(q) \cong 1 + \left|\int S(r)e^{iqr}dr\right|^2$ (under some assumptions)
  - With distance distribution $D(r)$:

    $C(q) \cong 1 + \int D(r)e^{iqr}dr$
  - Neglected: pair reconstruction, final state interactions,
    N-particle correlations, coherence, …
- Only way to map out source space-time geometry on femtometer scale!





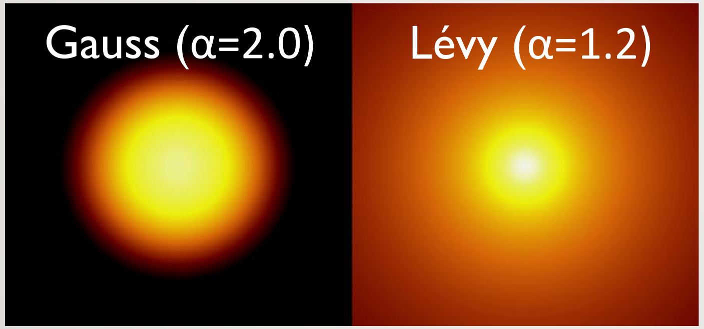
source function S(r)

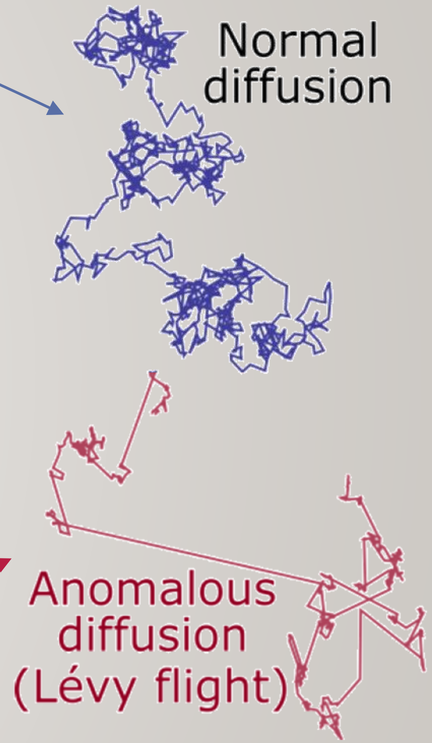


correlation funct. C(q)

SHAPE OF HADRONIC SOURCE AND CORRELATIONS

- Central limit theorem (diffusion) and thermodynamics lead to Gaussians
- Measurements suggest phenomena beyond Gaussian distribution
- Lévy-stable distribution: $\mathcal{L}(\alpha, R; r) = (2\pi)^{-3} \int d^3q e^{iqr} e^{-\frac{1}{2}|qR|^{\alpha}}$
  - From generalized central limit theorem, power-law tail $\sim r^{-(1+\alpha)}$
  - Special cases: $\alpha = 2$ Gaussian, $\alpha = 1$ Cauchy

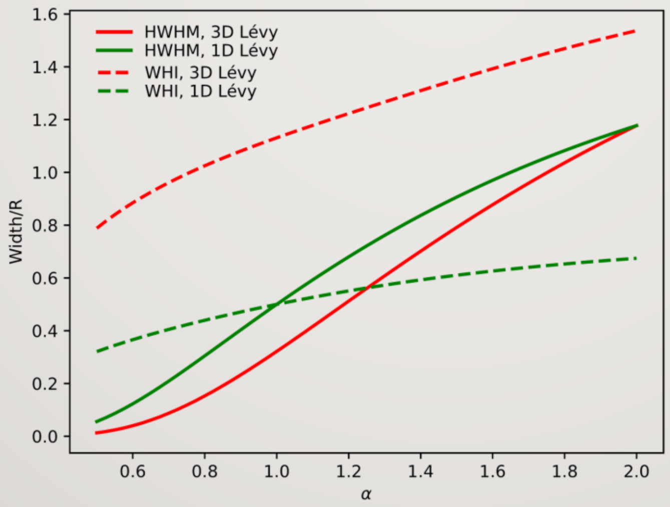


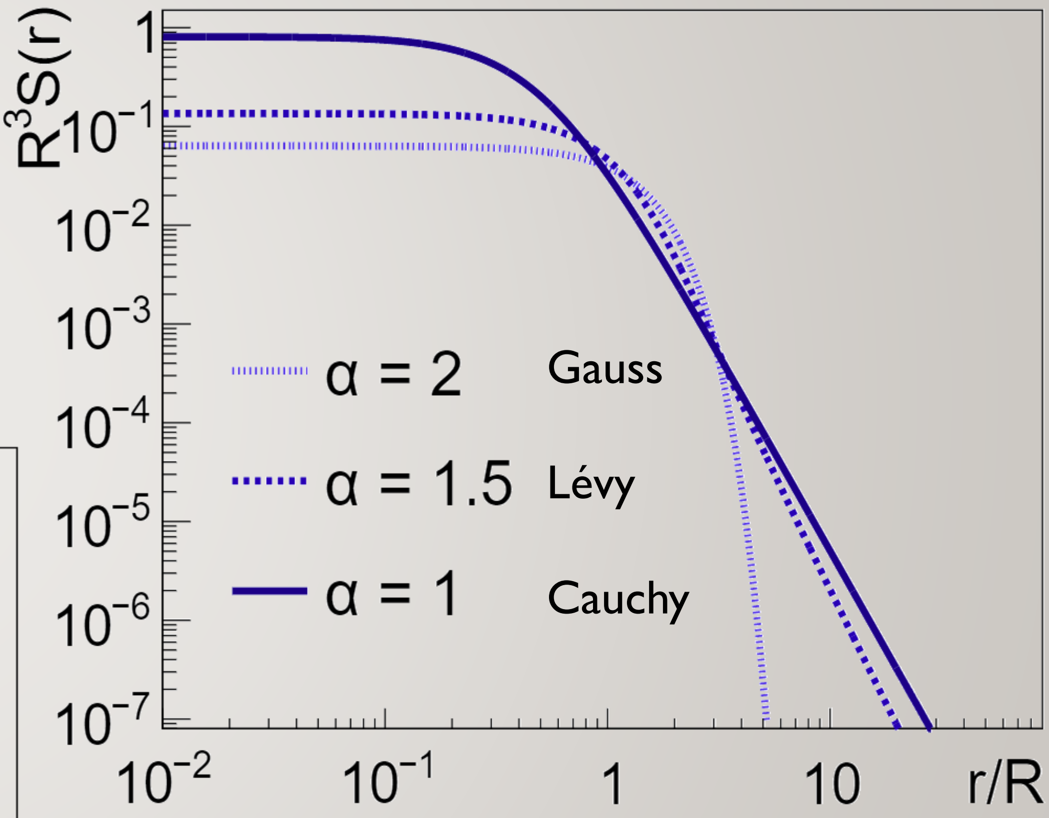


- Shape of the correlation functions with Lévy source:
  - $C_2(q) = 1 + \lambda \cdot e^{-|qR|^{\alpha}}$; $\alpha = 2$: Gaussian; $\alpha = 1$: exponential
    Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67-78
- A possible reason for Levy source: anomalous diffusion, jet fragmentation, critical phenomena, decays, averaging

LÉVY, GAUSS, EXPONENTIAL: MEANING OF HBT SCALE R

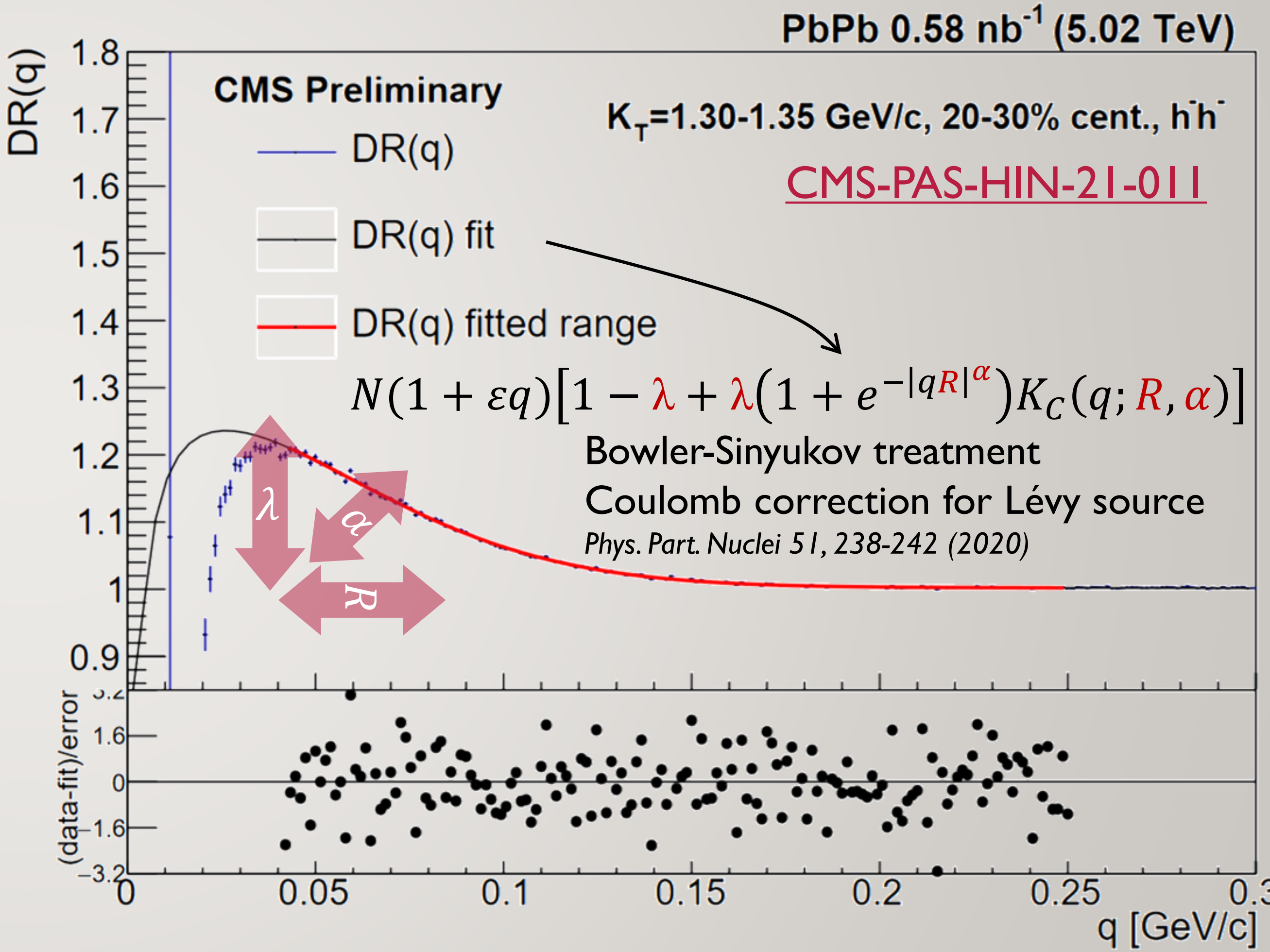
- No tail if $\alpha = 2$, power law if $\alpha < 2$; tail depends on α
- If $S(r)$ Lévy, $D(r)$ Lévy with same α and $R \to 2^{1/\alpha} R$
- In principle, RMS $= \infty$ if $\alpha < 2$, but depends on cutoff
- What do Gaussian HBT radii mean?
- Alternative measures:
  - Width at half integral
  - Width at half maximum
  - Nontrivially connected to α and R





MEASUREMENT AND FITTING CORRELATION FUNCTIONS

- 2018 data: $\sqrt{s_{NN}} = 5.02$ TeV PbPb, 3 billion MinBias events, standard event & track cuts
- Event mixing to measure correlations; pair cuts to reduce splitting and merging
- Long range background fitted; double ratio built to reduce residual effects
- Small q values: pair reconstruction efficiency, studied via MC
- 5 fit parameters, studied in cent. and K_T bins
  - R, α, λ: physical parameters of Lévy source
  - N: normalization; ε: linear background

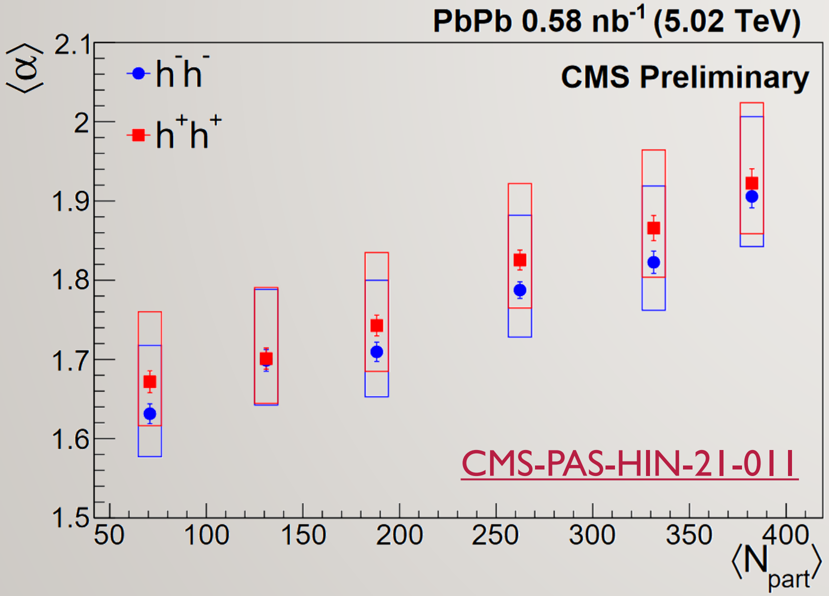


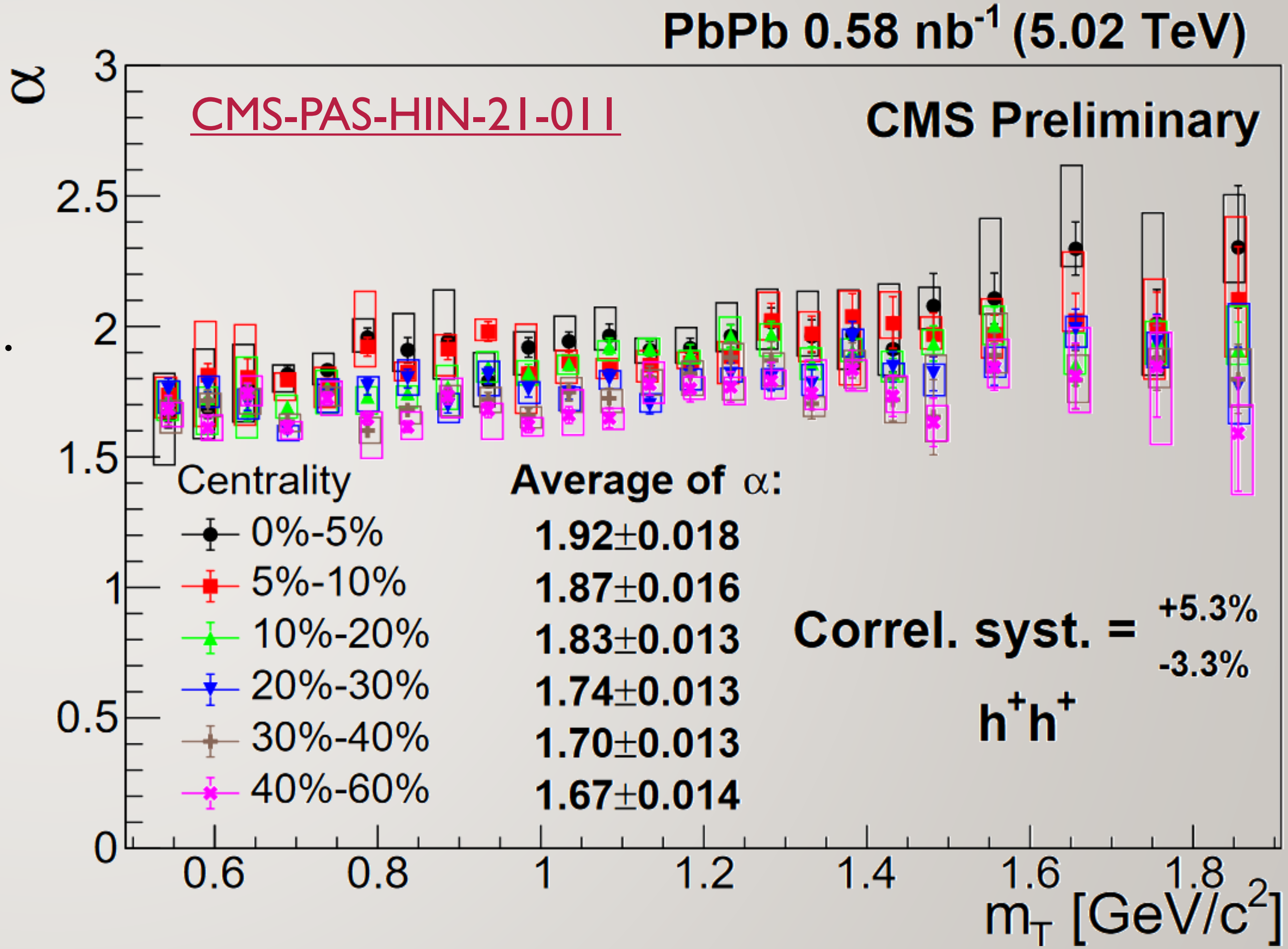
$$N(1+\varepsilon q)\left[1-\lambda+\lambda\left(1+e^{-|qR|^{\alpha}}\right)K_C(q;R,\alpha)\right]$$

Bowler-Sinyukov treatment
Coulomb correction for Lévy source
Phys. Part. Nuclei 51, 238-242 (2020)

LÉVY STABILITY INDEX α

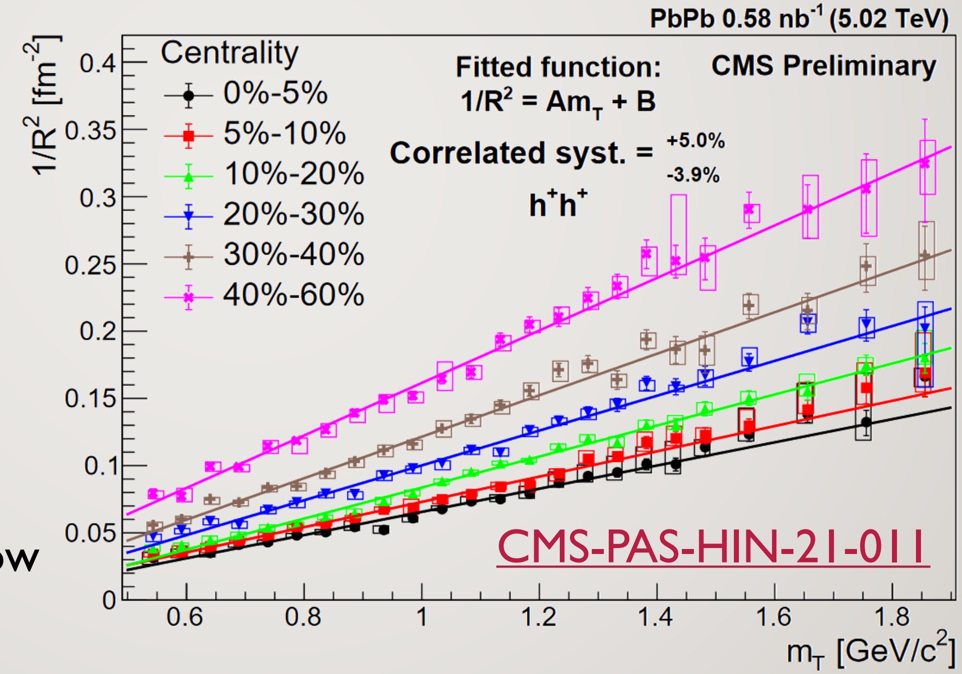
- Source shape not Gaussian ($\alpha \neq 2$)
- Close to constant in a given centrality
  - Average is centrality-dependent
- Average value: 1.6-2.0, largest for central coll.

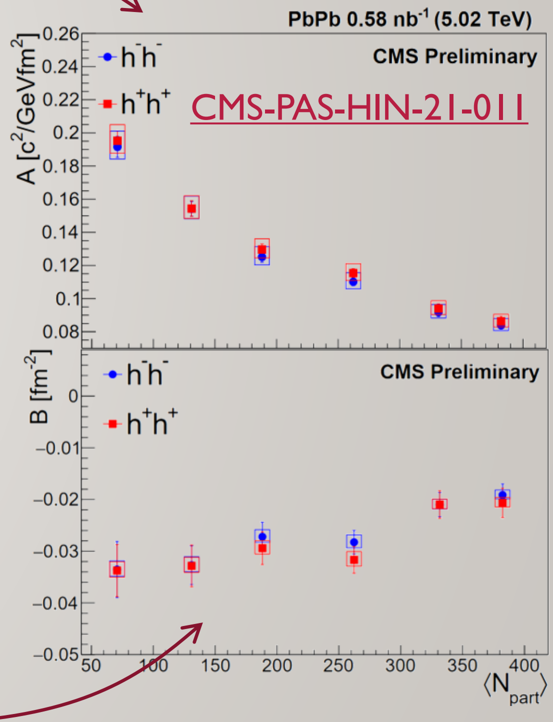




LÉVY SCALE PARAMETER R

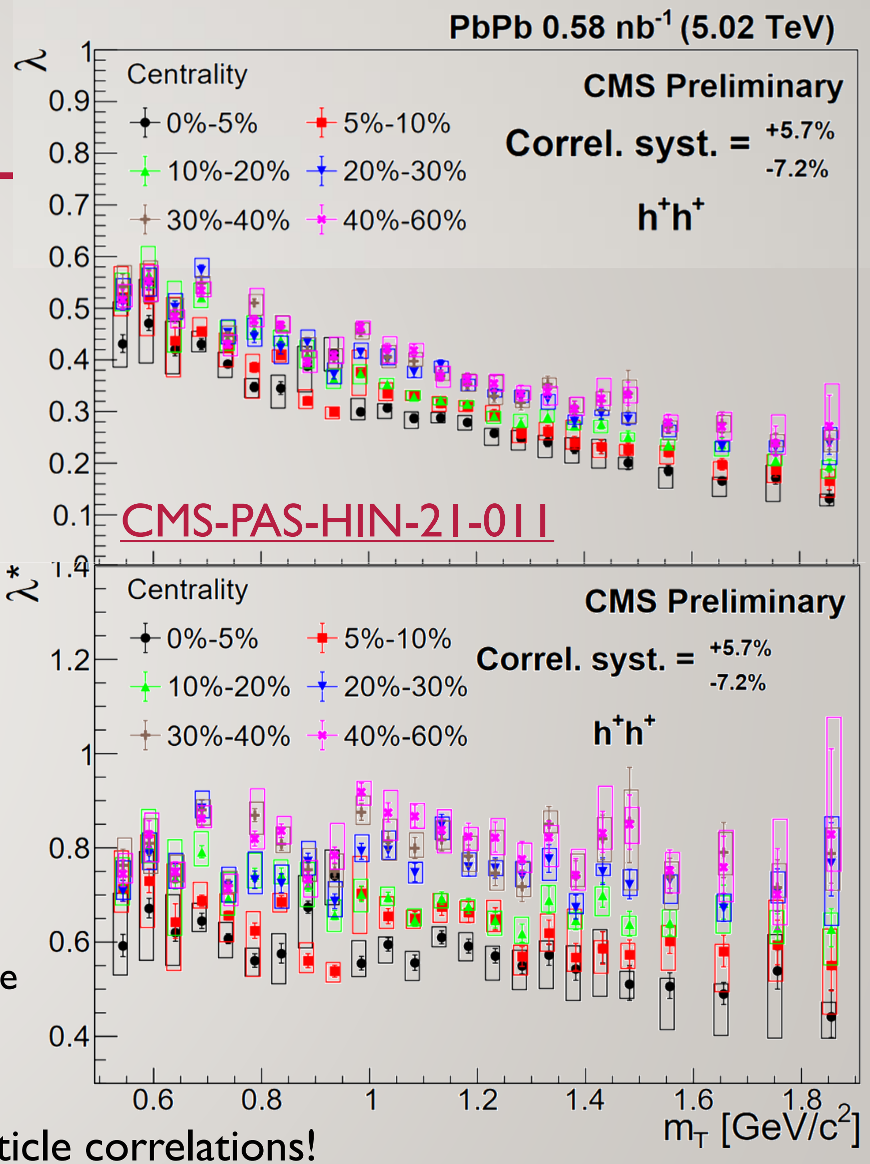
- Spatial scale of the source, nontrivially connected to width at half maximum or half integral
- Hydro model prediction: $1/R^2 \sim m_T$ (for $\alpha = 2$, i.e., Gaussian sources; Sinyukov, Makhlin, Csörgő, …)
  - Slope $A = \frac{H^2}{T_f}$: Hubble constant H?
  - Intercept $B = \frac{1}{R_f^2}$: freeze-out size?
- Linear fits perform well
  - Hydrodynamic scaling works well
  - But source is not Gaussian?!
- Hubble constant: $0.10 - 0.17\ \mathrm{fm}^{-1}$
  - Centrality dependence of radial flow
- Negative intercept!





CORRELATION STRENGTH λ

- Value of λ may be influenced (at least) by:
  - Core fraction (f_c) and partial coherence (p_c):
    $\lambda = f_c^2[(1-p_c)^2 + 2p_c(1-p_c)]$
  - Lack of particle identification: $\lambda \leq (N_\pi / N_{\text{hadron}})^2$
- Results: strongly decreasing trend with m_T
  - Caused by non-pion contamination
- Proton and kaon to pion ratio increases with m_T
  - See ALICE result Phys.Rev.C 101 (2020) 4, 044907
- Can rescale with it: $\lambda^* = \lambda \cdot (N_{\text{hadron}} / N_\pi)^2$
- Close to constant trend vs m_T
  - RHIC observes $U_A(1)$ restoration signal at $m_T \lesssim 300$ MeV/c², not resolvable here
- Average λ^* depends on centrality
- Centrality dependence: more coherence in central collisions? Test with 3-particle correlations!

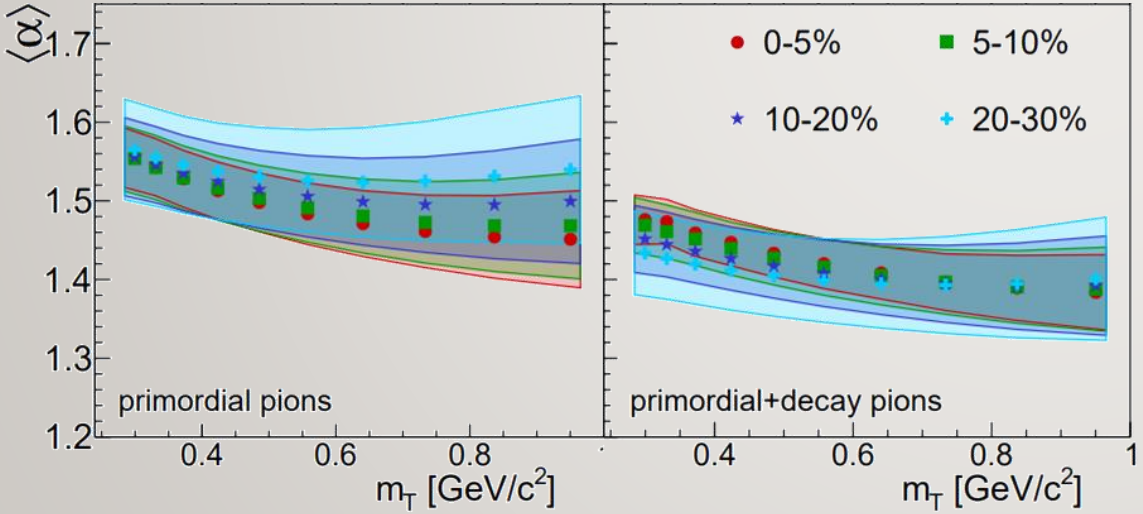


EVENT-BY-EVENT INVESTIGATION WITH EPOS

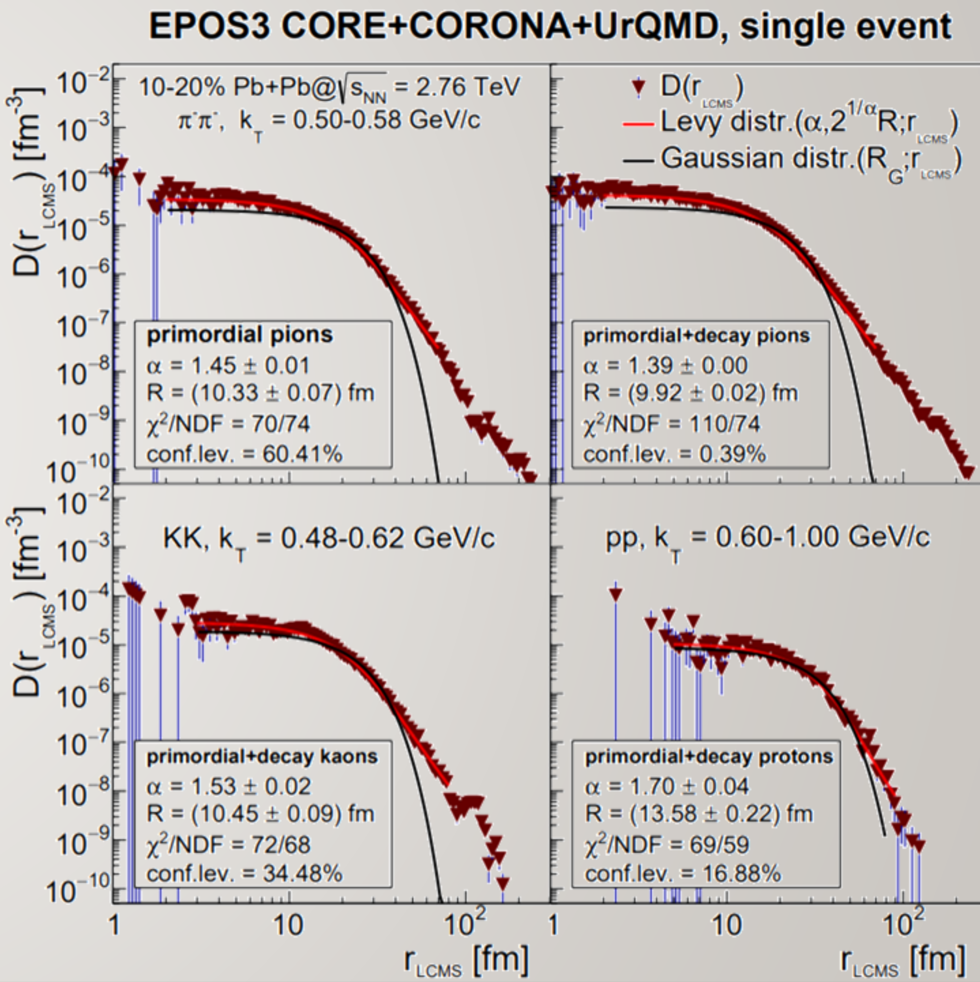
- Can we see Lévy sources in models as well?
- Pion and kaon pair distributions calculated in individual EPOS events

$$D(r_{LCMS}) = \int d\Omega dt D(t, r_x, r_y, r_z)$$

- Lévy source parameters determined for each event separately
  - Fit limits: from 2-5 fm to 70-100 fm, accepted if conf. level > 0.1%
- Strongly non-Gaussian shapes observed



arXiv:
2201.07962
2212.02980



STRANGE PARTICLE FEMTOSCOPY IN PBPB WITH CMS

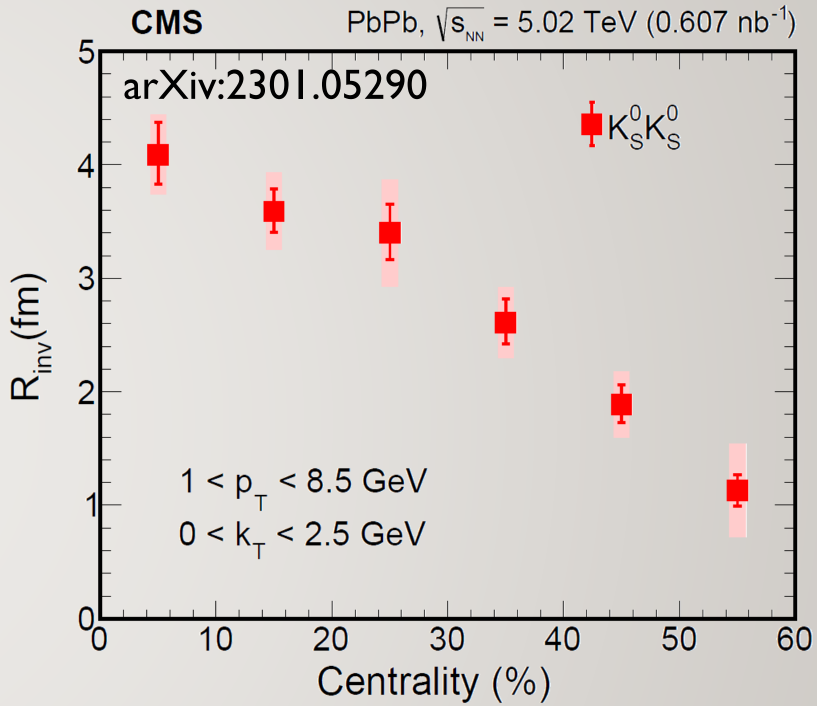
- Neutral kaons and Lambdas investigated
  - Neutral particles: no Coulomb effect
  - Final state strong interaction important
- Identical and non-identical pair source measured
- Lednicky-Lyuboshitz model:

$$N \cdot \{1 + \lambda \cdot [C_{QS}(q) + C_{FSI}(q)]\} \cdot \Omega(q)$$

Model: Sov.J.Nucl.Phys. 35 (1982) 770

Parameters: Phys. Rev. C 74 (2006) 054902 [STAR Coll.]

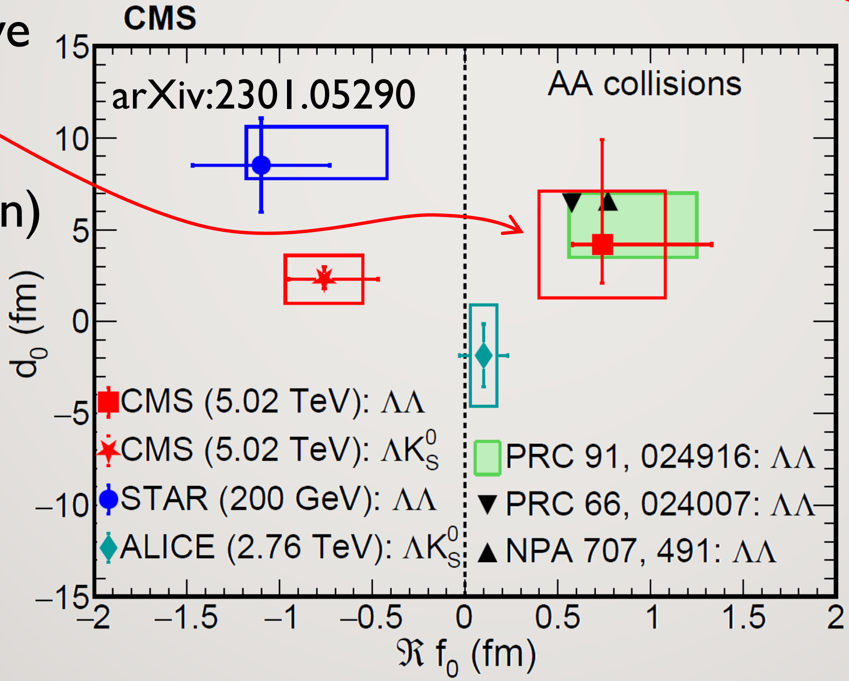
- Geometry dependence of K^0 source as expected
  - Λ source smaller, due to larger m_T
- Results used to measure strong scattering

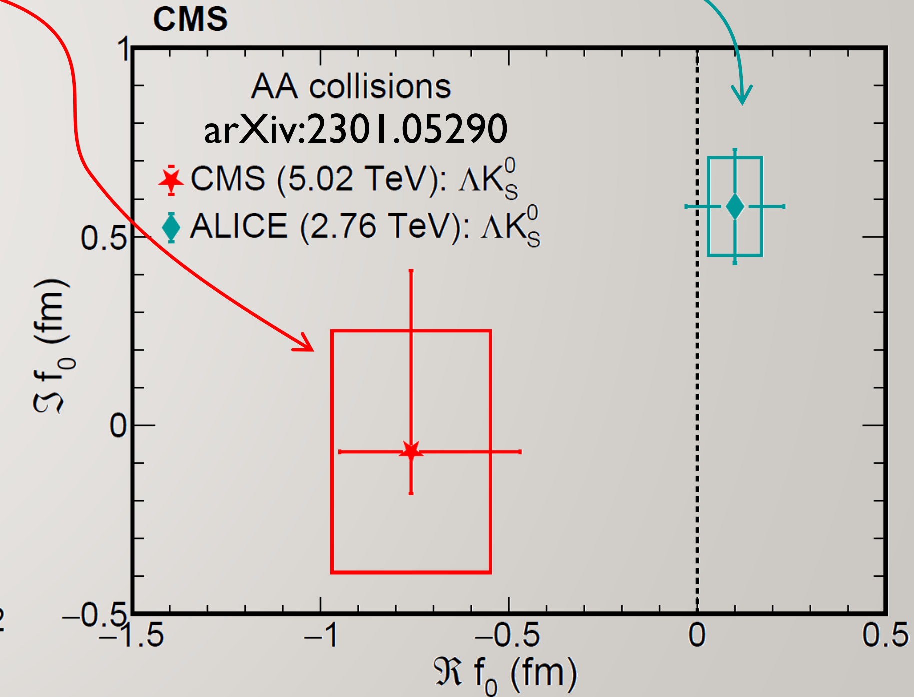


Parameter	$K^0_S K^0_S$	ΛK^0_S	$\Lambda\Lambda$
R_{inv} (fm)	$3.30 \pm 0.10 \pm 0.37$	$2.1^{+1.4}_{-0.5} \pm 0.7$	$1.3^{+0.4}_{-0.2} \pm 0.3$
λ	$0.38 \pm 0.02 \pm 0.08$	$0.34^{+0.41}_{-0.12} \pm 0.16$	$1.5^{+1.2}_{-1.1} \pm 1.4$
$\langle m_T \rangle$ (GeV)	1.53	2.09	2.60

STRONG SCATTERING PARAMETERS MEASURED

- Scattering amplitude $f(q) = \left[\frac{1}{f_0} + \frac{1}{8} d_0 q^2 + i\frac{q}{2}\right]^{-1}$: scattering length: $\Re f_0, \Im f_0$; effective range d_0
- $\Re f_0 < 0$ for ΛK: attractive strong interaction (potential), difference compared to ALICE
- $\Re f_0 > 0$ for $\Lambda\Lambda$: repulsive strong potential
- Bound state (H-dibaryion) disfavored
- $\Lambda\Lambda$ scattering helps constrain interaction models



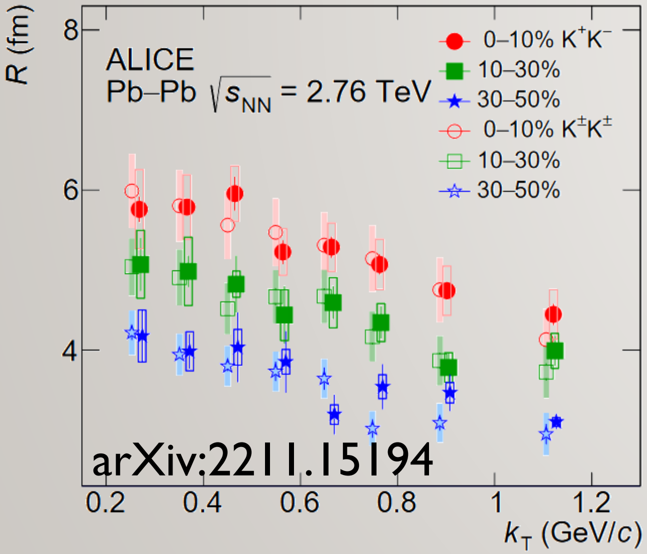


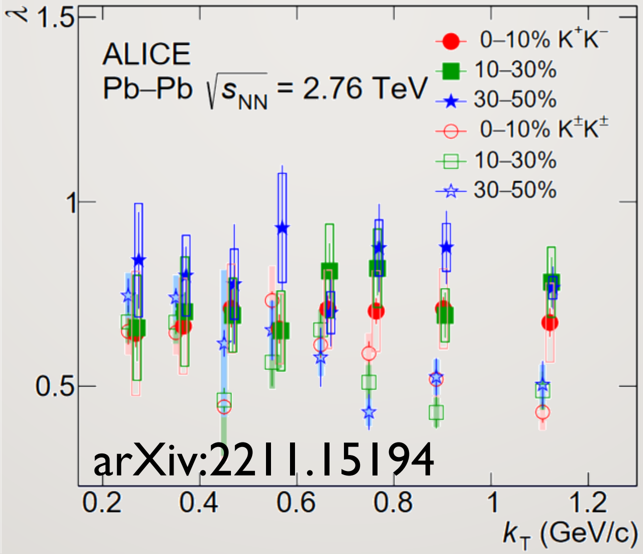
NON-IDENTICAL KAON FEMTOSCOPY WITH ALICE

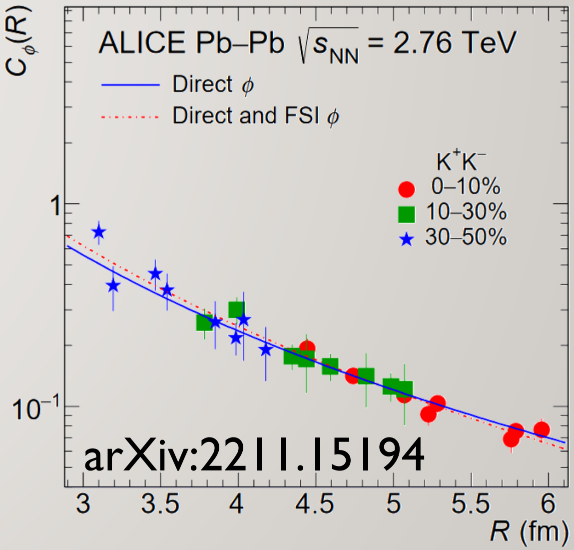
- Non-identical femtoscopy: test of strong interaction, resonances: $a_0(980)$, $f_0(980)$
  - Same and opposite sign source radii similar, by constraint in χ^2; correlation strength: $\lambda_{+-} \gtrsim \lambda_{\pm\pm}$ (non-Gauss?)
  - Mass and coupling parameters can be extracted
  - $\phi(1020)$ meson peak C_ϕ in opposite sign correlations, versus size R
    - Yield dominated by direct particles, from hadronization (not by FSI)

$$K_0(k^*) = \frac{\gamma_{f_0 \to K\bar{K}}}{m_{f_0}^2 - s - i\gamma_{f_0\to\pi\pi} k_{\pi\pi}}$$

$$K_1(k^*) = \frac{\gamma_{a_0 \to K\bar{K}}}{m_{a_0}^2 - s - i\gamma_{a_0\to\pi\eta} k_{\pi\eta}}$$

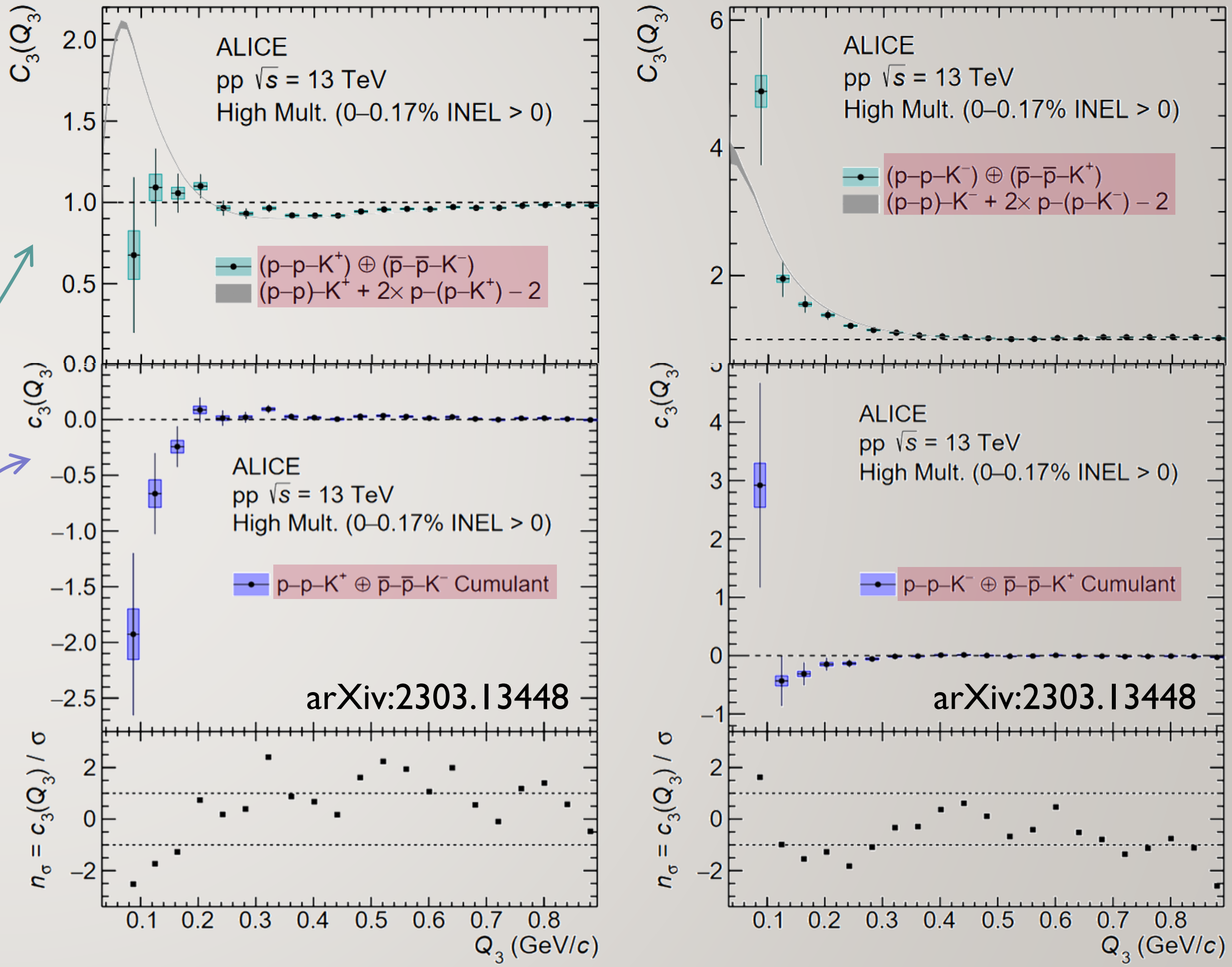






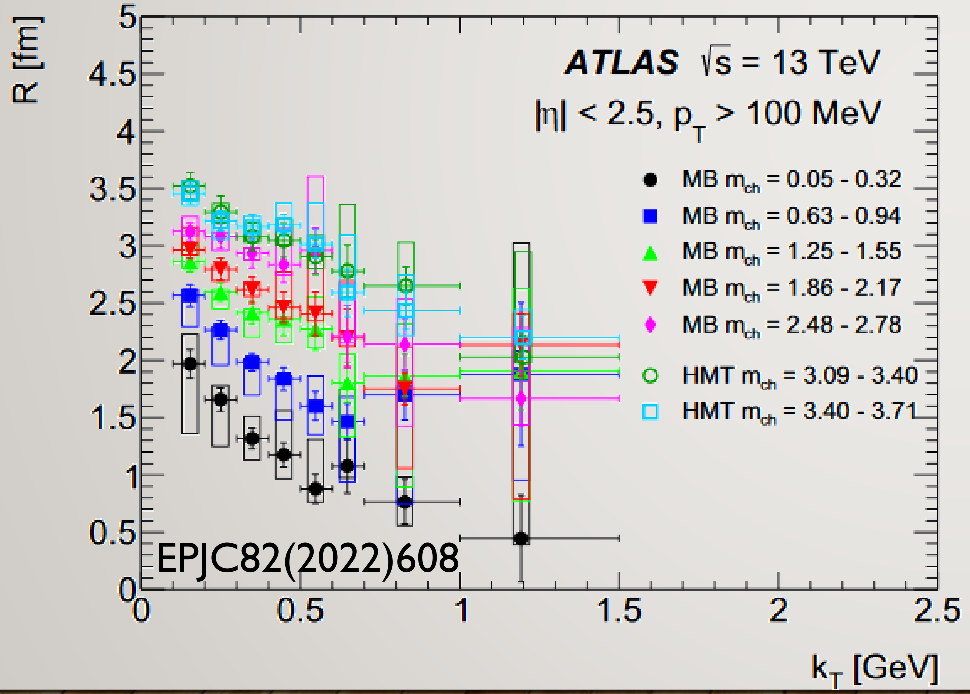
KAON-NUCLEON DYNAMICS IN PP WITH ALICE

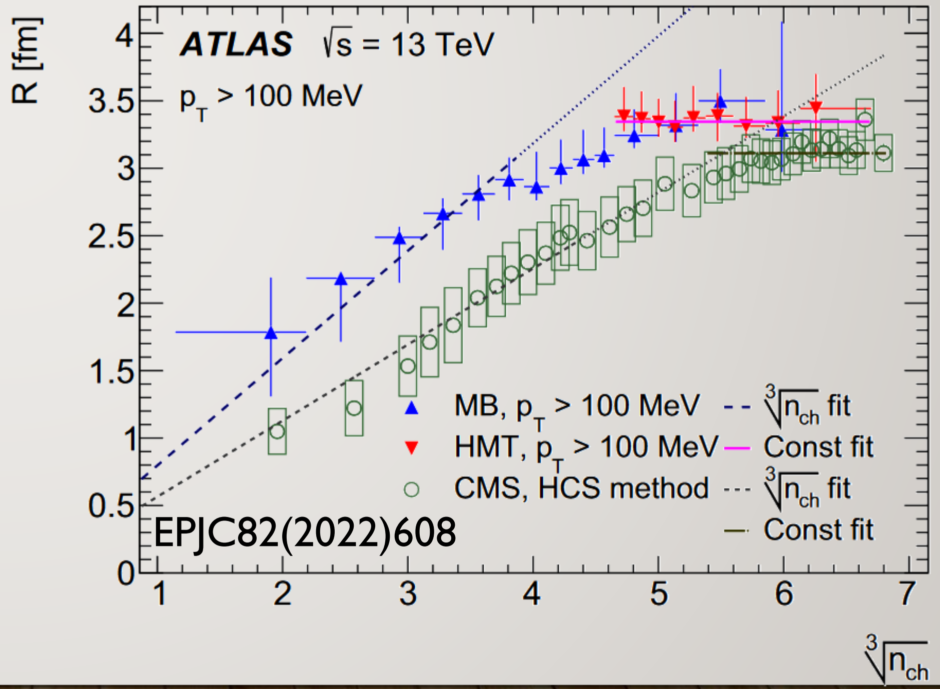
- Kaon/antikaon-nucleon interactions: studied usually via kaonic atoms
- Details of KNN dynamics not well understood
  - Problem: multi-nucleon interactions in nuclei
- Can be studied via final state interactions of triplets created in pp collisions
  - No effects due to bound nucleons
- ALICE studied ppK correlations & cumulants in high multiplicity pp collisions at 13 TeV
  - Two-body interactions appear to dominate
  - Three-body interactions not significant
  - Run 3 makes more detailed studies possible



TWO-PARTICLE FEMTOSCOPY WITH ATLAS IN 13 TEV PP

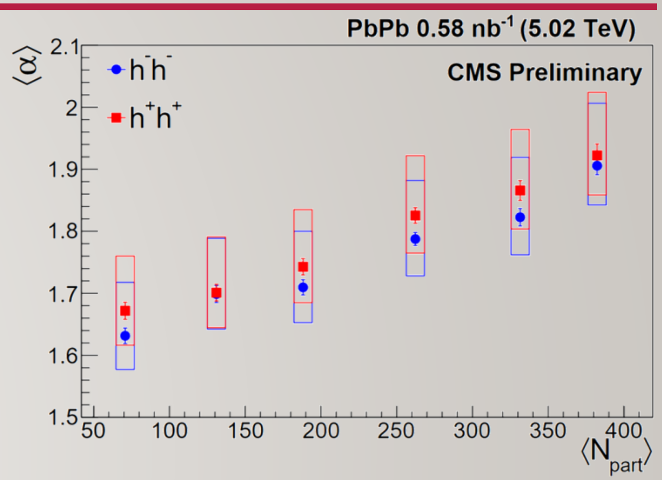
- Measurement of non-identified same-sign two-particle correlations in 13 TeV pp collisions
  - Relates to similar CMS analysis [JHEP 03 (2020) 014]
- Scaling with k_T and multiplicity (n_{ch}) compatible with radial flow and saturation at large multiplicities

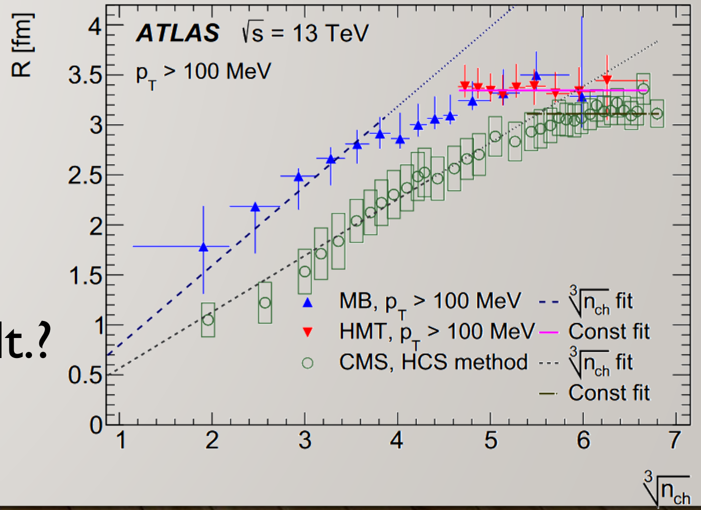


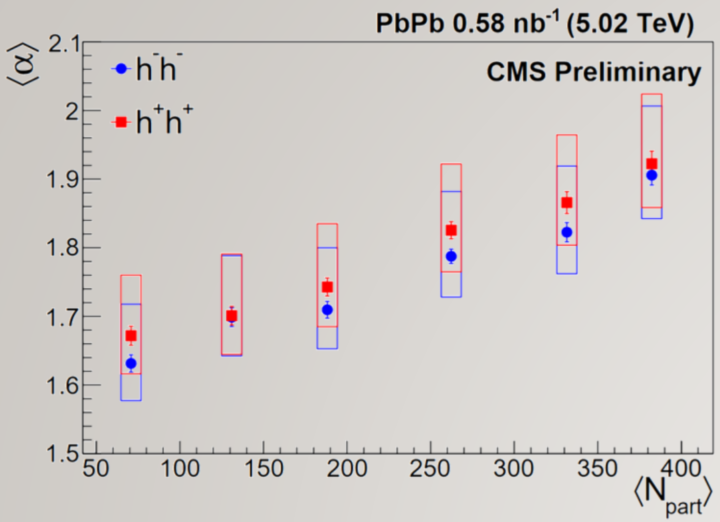


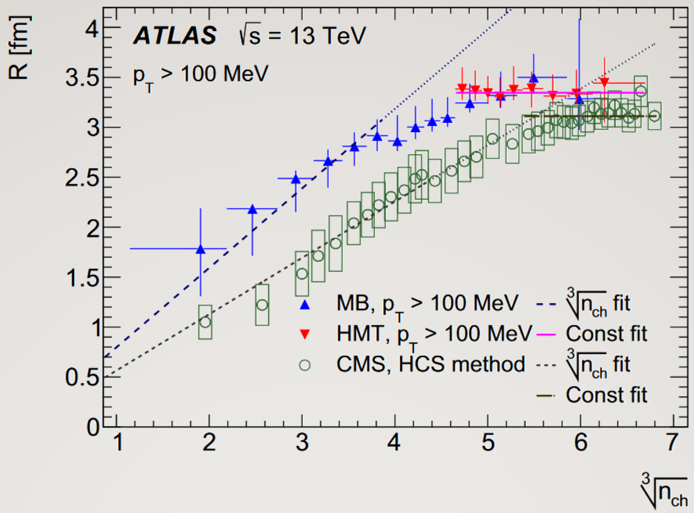
CONCLUSIONS AND OUTLOOK

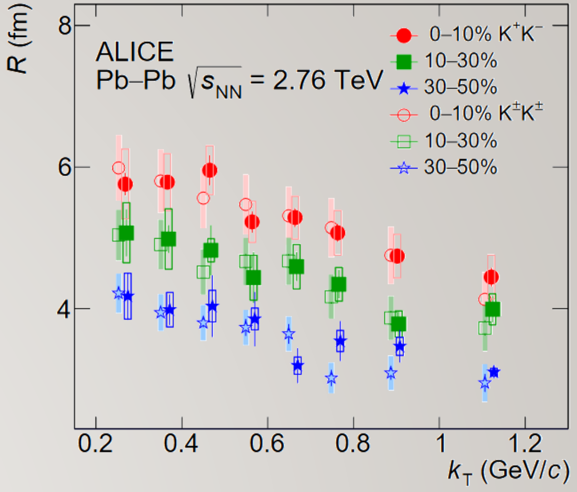
- Lévy sources appear in $\sqrt{s_{NN}} = 5.02$ PbPb collisions at LHC
  - **Lévy α**: between 1.6 and 2
  - **Lévy R**: hydro scaling versus m_T, despite not Gaussian source
  - Possible reason: Lévy flight → cross-checked with EPOS
- Many more topics in femtoscopy:
  - K^0 and Λ femtoscopy: **strong scattering parameters** by CMS
  - Non-identical kaon correlations: **resonance parameters** by ALICE
  - **Kaon-nucleon dynamics** explored by ALICE
  - Signs of radial flow in pp by ATLAS and CMS, **saturation** at high mult.?











THANK YOU FOR YOUR ATTENTION

… and if you are interested in these subjects:

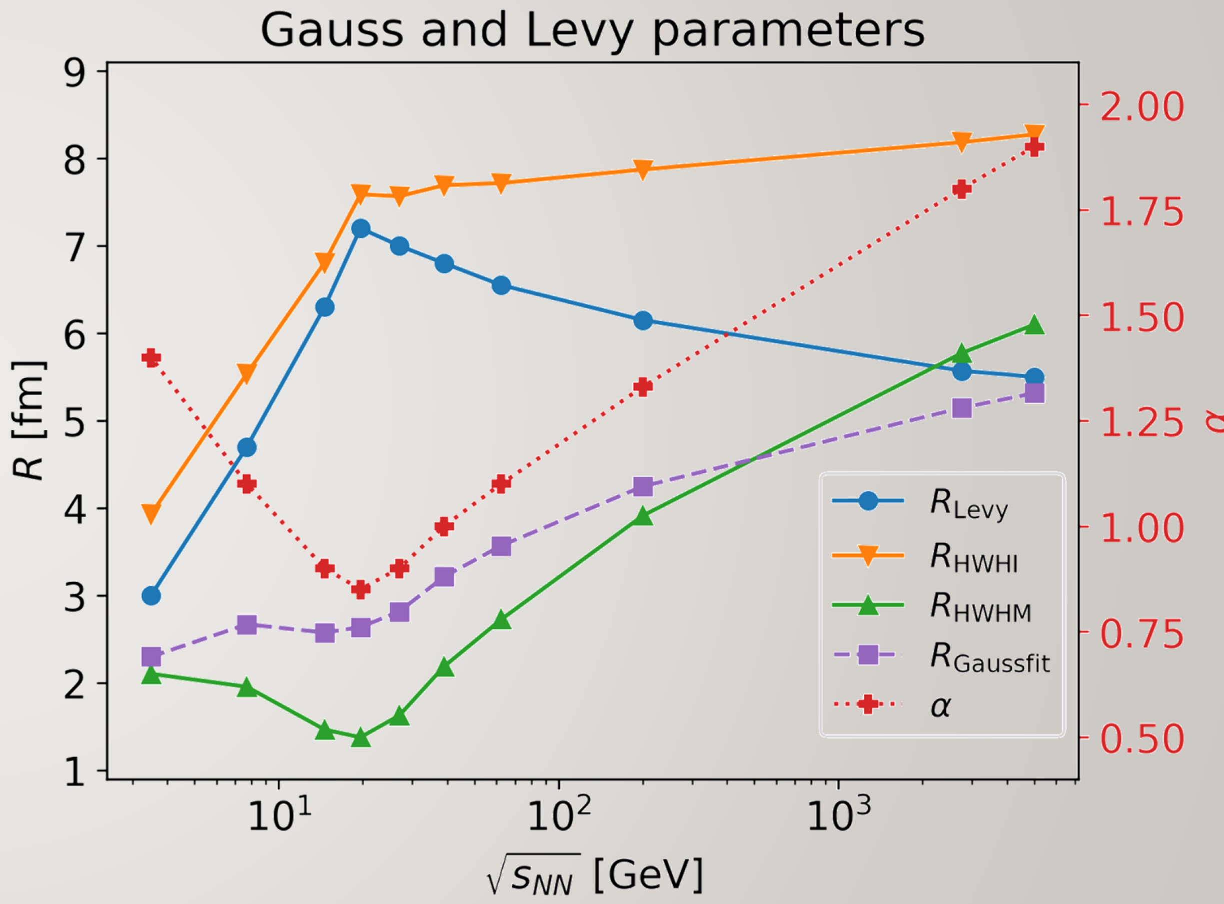
https://zimanyischool.kfki.hu/

WHY DOES LÉVY APPEAR, WHY IS IT IMPORTANT?

- A more comprehensive list of possible reasons:
  - Jet fragmentation (Csörgő, Hegyi, Novák, Zajc, Acta Phys.Polon. B36(2005)329-337)
  - Critical phenomena (Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828(2006)1,525-532)
  - Direction averaging and non-sphericality (Cimerman et al., Phys.Part.Nucl. 51(2020)282)
  - Event averaging (Cimerman et al., Phys.Part.Nucl. 51(2020)282)
  - Resonance decays (Csanád, Csörgő, Nagy, Braz.J.Phys. 37(2007)1002; Kincses, Stefaniak, Csanád, Entropy 24(2022)308)
  - Hadronic rescattering, Lévy flight (Braz.J.Phys. 37(2007)1002; Entropy 24(2022)308)
- Importance of utilizing Lévy sources:
  - Measuring α and R
    - Order of quark-hadron transition, critical point search
    - General understanding of source dynamics
  - Measuring λ also requires correct shape assumption
    - In-medium mass modification, coherent pion production

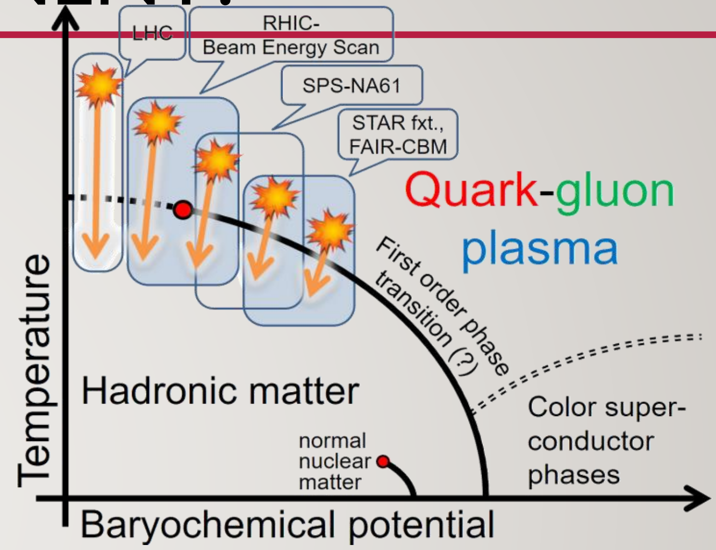
WHAT IS THE TRUE SIZE OF THE SOURCE?

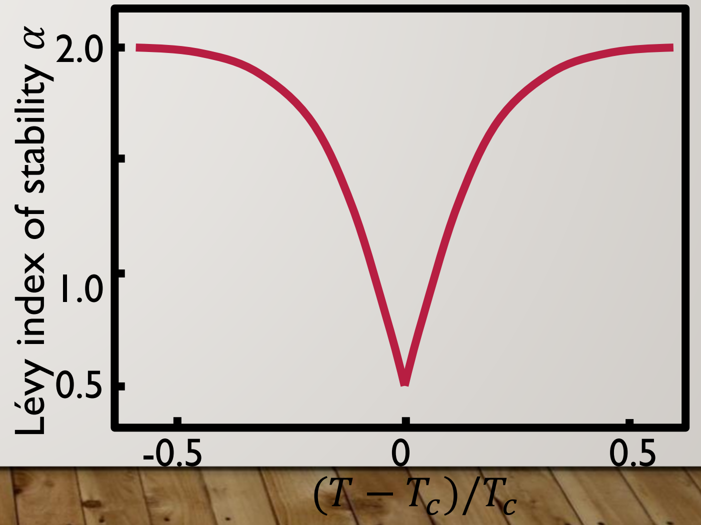
- Levy source parameters: R_{Levy}, α
- R_{Gaussfit}: $C(Q; R_{\text{Levy}}, \alpha)$ fitted with α = 2 fixed
- R_{HWHM}: half width at half max. forLevy source
- R_{HWHI}: half width of Levy source at half integral
- **Supposed scenario:**
  min. in α vs. s_{NN}, max. in R_{Levy} vs. s_{NN}
- **Observation:**
  - R_{Gaussfit} approx. Monotonic increase
  - min. in R_{HWHM}
  - Trend change in R_{HWHI}



LÉVY INDEX AS A CRITICAL EXPONENT?

- Critical spatial correlation: $\sim r^{-(d-2+\eta)}$;
  Lévy source: $\sim r^{-(1+\alpha)}$; $\alpha \Leftrightarrow \eta$?
  Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67,
- QCD universality class ↔ 3D Ising
  Halasz et al., Phys.Rev.D58 (1998) 096007
  Stephanov et al., Phys.Rev.Lett.81 (1998) 4816
- At the critical point:
  - Random field 3D Ising: η = 0.50±0.05
    Rieger, Phys.Rev.B52 (1995) 6659
  - 3D Ising: η = 0.03631(3)
    El-Showk et al., J.Stat.Phys.157 (4-5): 869
- Motivation for precise Lévy HBT!
- Change in α_{Levy} proximity of CEP?
- Finite size/time & non-equilibrium effects
  → what does power law mean?





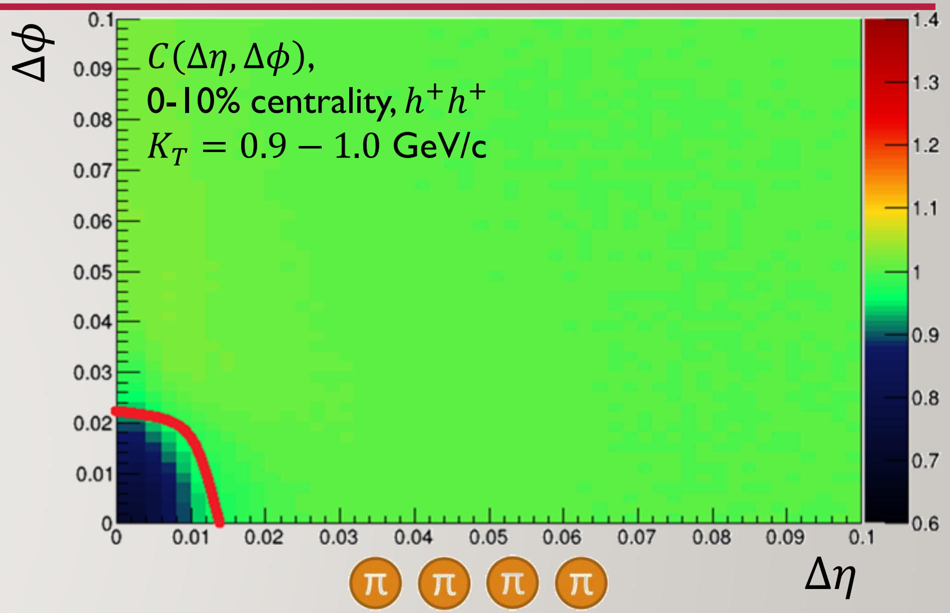
SYSTEMATIC UNCERTAINTY SOURCES

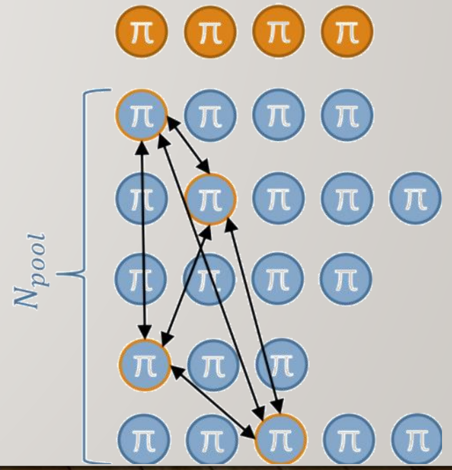
- Event and track cuts: 2-4%
- Pair cuts: 4-6%
- Fit range: 2-9%, largest effect
- Largest uncertainties:
  - Central collisions
  - λ parameter
  - Substantial asymmetry in some cases
- Separation
  - Point-to-point (fluctuating) part
  - Constant part (overall factor)

Systematic source	Default	Low	High
Zvertex cut	< 15 cm	< 12 cm	< 18 cm
p_T cut	> 0.5 GeV/c	> 0.55 GeV/c	> 0.5 GeV/c
δp_T cut	$< 10\%$	$< 5\%$	$< 15\%$
$\|\eta\|$ cut	< 0.95	< 0.9	< 1
$N_{\text{pixel hit}}$ cut	> 1	> 2	> 0
$\chi^2/N_{\text{dof}}/N_{\text{layer}}$ cut	< 0.18	< 0.15	< 0.18
$\|d_{xy}/\sigma(d_{xy})\|$ cut	< 3	< 2	< 5
$\|d_z/\sigma(d_z)\|$ cut	< 3	< 2	< 5
$\Delta\eta, \Delta\phi$ pair cut	$\Delta\eta_{\text{cut}}$=0.014 $\Delta\phi_{\text{cut}}$=0.022	η_{cut}=0.017 $\Delta\phi_{\text{cut}}$=0.028	η_{cut}=0.011 $\Delta\phi_{\text{cut}}$=0.016
q_{min} lower fit limit	$q^0_{\text{min}}(K_T,\text{cent})$	q^0_{min}-0.004	q^0_{min}+0.004
q_{max} upper fit limit	$q^0_{\text{max}}(K_T,\text{cent})$	$0.85 \cdot q^0_{\text{max}}$	$1.15 \cdot q^0_{\text{max}}$
Cent. edges	Default values	Lower values	Higher values

MEASURING CORRELATION FUNCTIONS

- 2018 data: $\sqrt{s_{NN}} = 5.02$ TeV PbPb, 3 billion MinBias events
  - z-vertex cut, centrality classes, standard single track cuts
- Pair cuts needed in Δη, Δϕ to reduce splitting and merging
- Measuring correlation functions via mixed event background:
  - $C(q) = A(q)/B(q)$
- $C(q)$ for large q may contain remaining effects
  - Energy and momentum conservation, resonances, minijets…
- $BG(q)$ long-range background fit to $C(q)$
- Double ratio calculated: $DR(q) = \frac{C(q)}{BG(q)}$
- Very small remaining linear background handled in fit





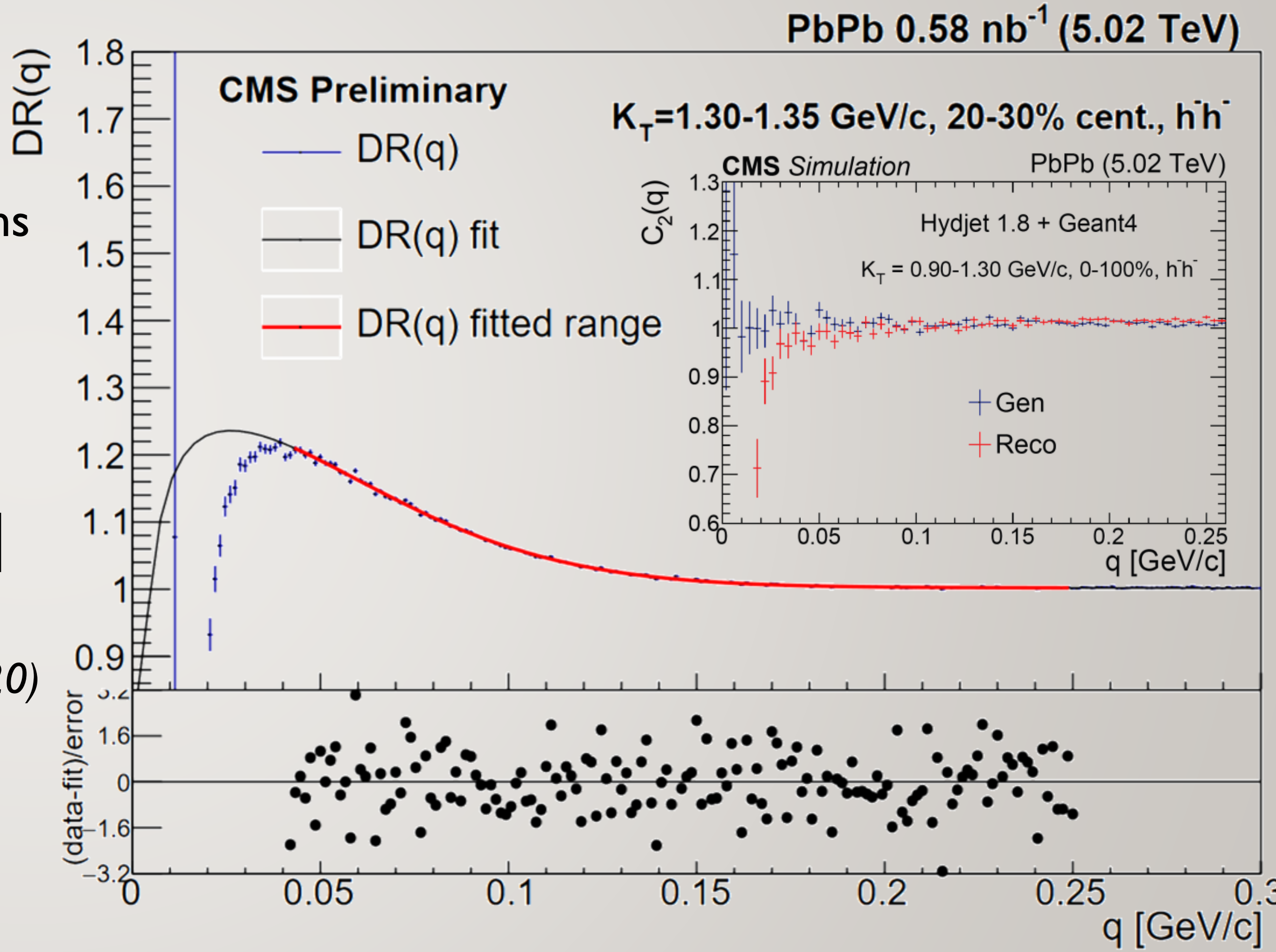
FITTING CORRELATION FUNCTIONS

- Background effects → $DR(q)$ double ratio
- Minuit2 χ^2 fit; statistical uncertainties: MINOS
- 6 centrality (0-60%) & 24 K_T (0.5-1.9 GeV/c) bins
- Fit range determined experimentally & via MC
- Small q values: measurement not reliable
- Bowler-Sinyukov fit function

  $$N(1+\varepsilon q)\left[1-\lambda+\lambda\left(1+e^{-|qR|^{\alpha}}\right)K_C(q;R,\alpha)\right]$$

  $K_C(q;R,\alpha)$: Coulomb correction with Lévy source

  Csanád,Lökös,Nagy: Phys. Part. Nuclei 51, 238-242 (2020)
- 5 fit parameters
  - R,α,λ: physical parameters of Lévy source
  - N: normalization; ε: linear mid-range background

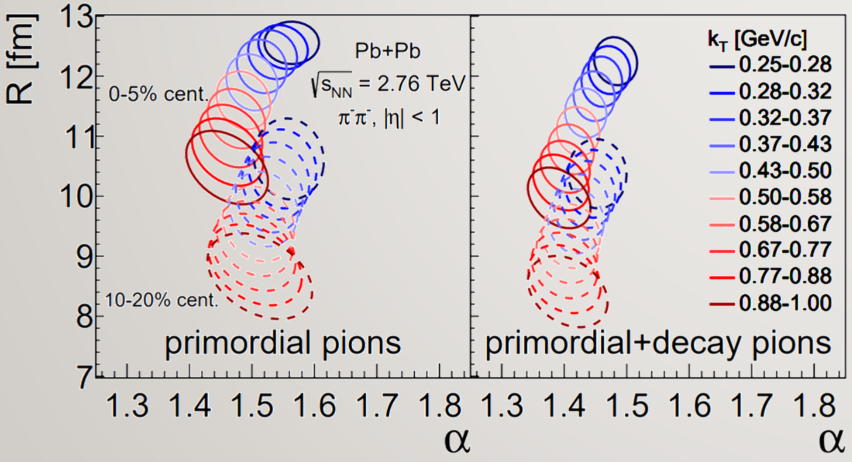


EVENT-BY-EVENT INVESTIGATION WITH EPOS

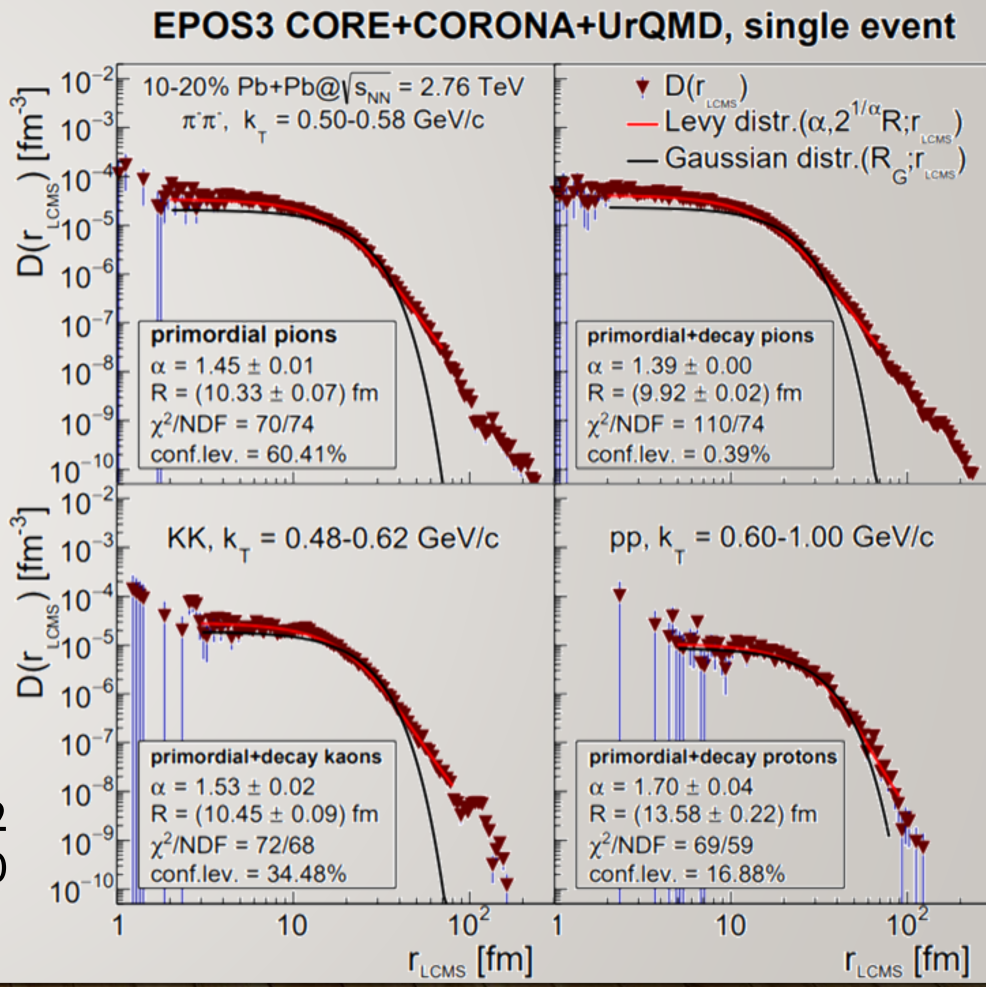
- Pion and kaon pair distributions calculated in individual EPOS events

$$D(r_{LCMS}) = \int d\Omega dt D(t, r_x, r_y, r_z)$$

- Lévy source parameters determined for each event separately
  - Fit limits: from 2-5 fm to 70-100 fm
  - Criterion for acceptance: confidence level > 0.1%
  - Strongly non-Gaussian shapes observed
- Separately for various centrality and k_T classes, primordial or decays

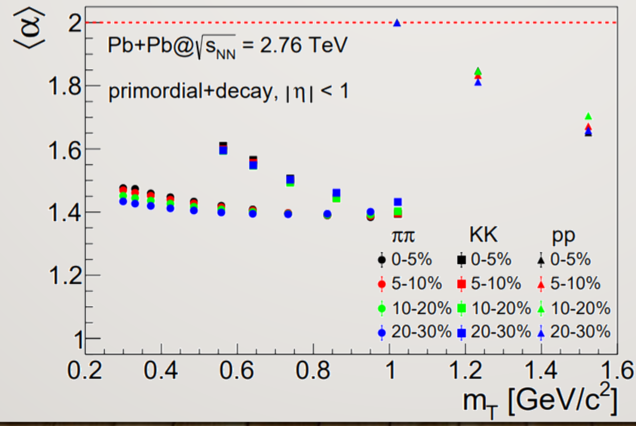


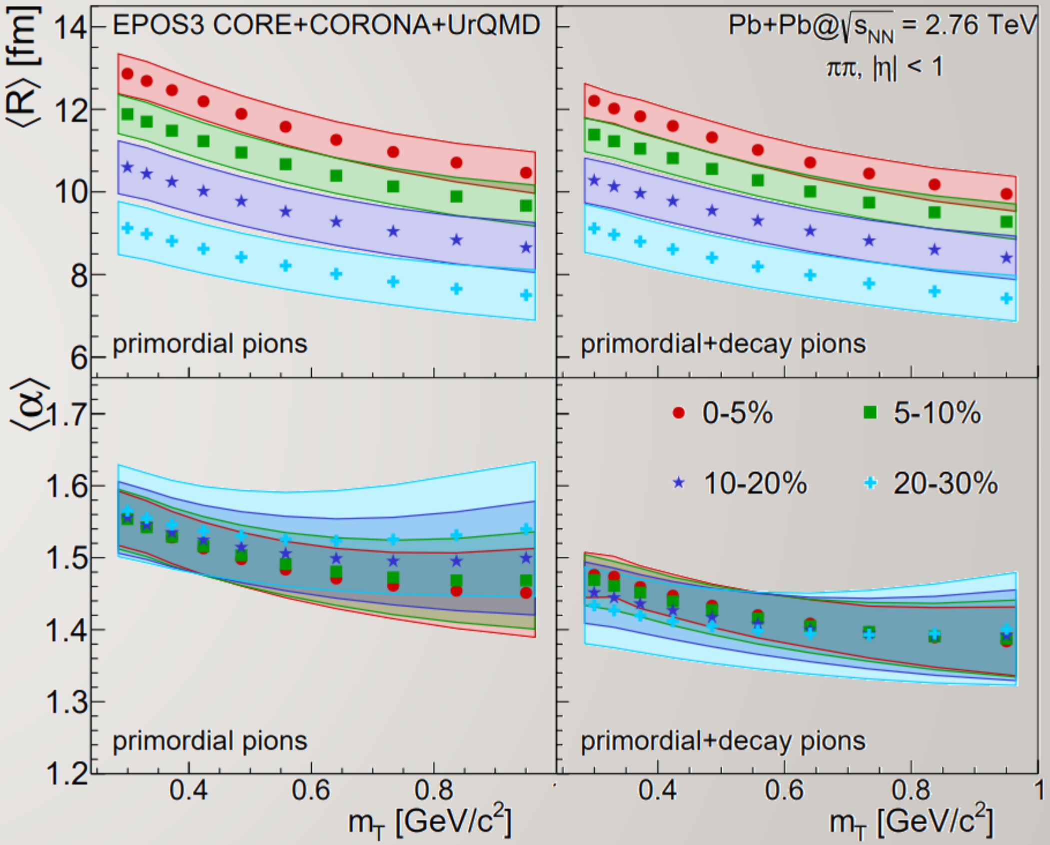
arXiv:2201.07962
arXiv:2212.02980



LÉVY SOURCE PARAMETERS IN EPOS

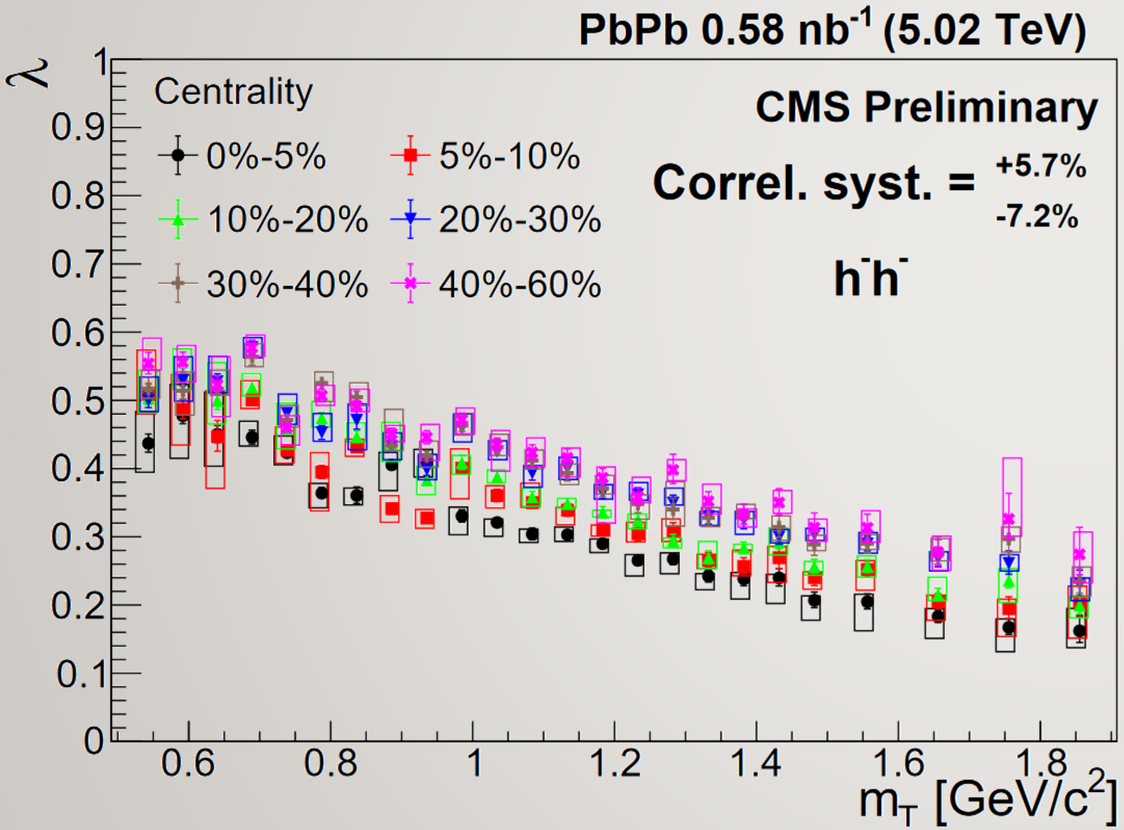
- Lévy scale parameter (R):
  - Larger in central events → connected to spatial size
  - Decreasing with m_T → hydro-like scaling
- Lévy stability index (α):
  - Small centrality dependence, decreasing trend with m_T
  - Larger effect of decays → larger α for primordial only
- Bands: variance of R, α in fits (not statistical uncertainty)
- Particle type dependence:
  - Anomalous diffusion suggests $\alpha_K < \alpha_\pi < \alpha_p$
  - Not exactly valid in EPOS
  - Effect of decays?

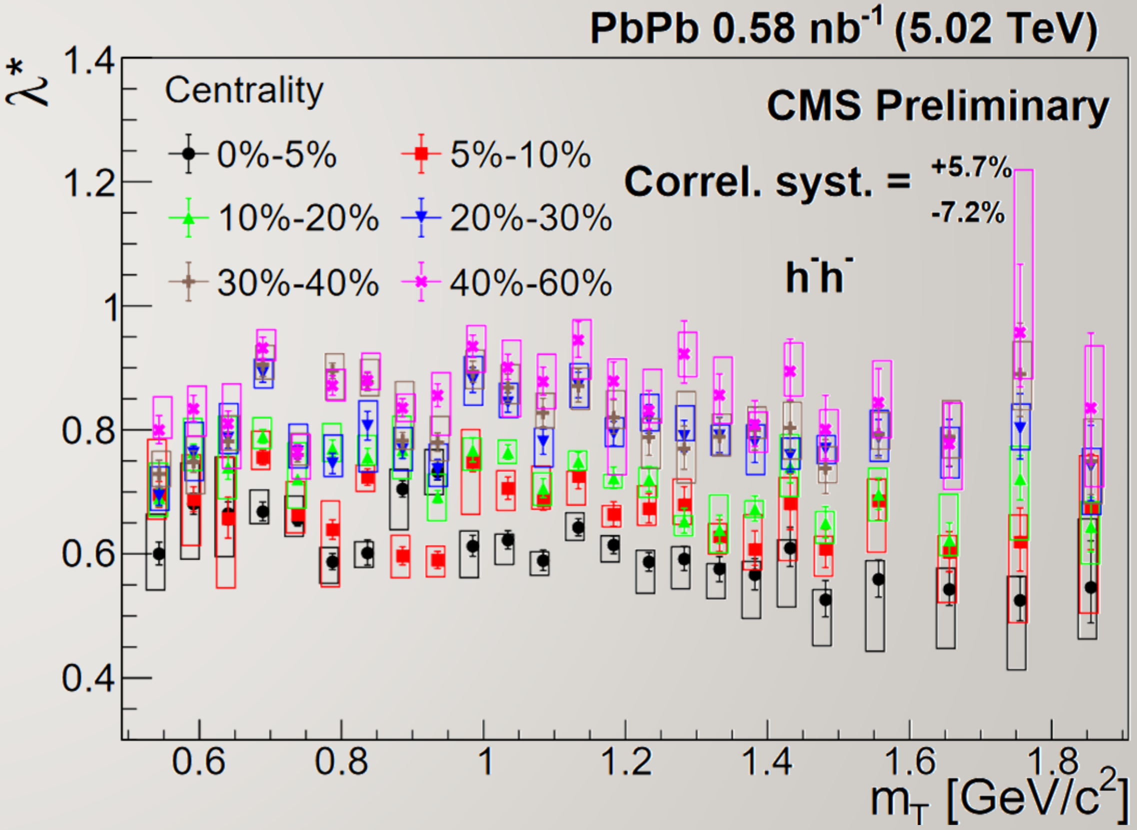




SAME PLOT FOR NEGATIVE PAIRS

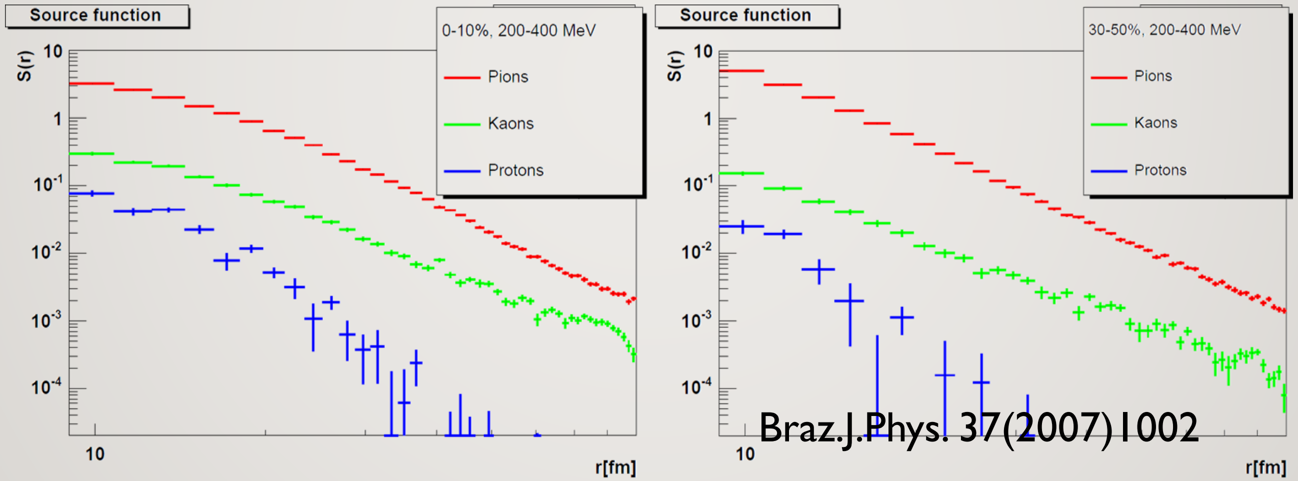
- Decrease with m_T appears mostly to be due to lack of PID





THE IMPORTANCE OF A KAON ANALYSIS

- Kaons: smaller cross-section, larger mean free path
- Heavier power-law tail?
- Prediction for π, K, p based on Humanic's Resonance Model (HRM): anomalous diffusion due to rescattering
  Humanic, Int.J.Mod.Phys. E15 (2006) 197 [nucl-th/0510049]
  Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002 [hep-ph/0702032]



Braz.J.Phys. 37(2007)1002

- Kaon HBT radii: m_T scaling or its violation for Lévy scale R?
- Prediction: $\alpha(p) > \alpha(\pi) > \alpha(K)$

EVENT BY EVENT SHAPE ANALYSIS WITH EPOS

- EPOS model: parton-based Gribov-Regge theory (PBGRT)
  - K. Werner et al., PRC82 (2010) 044904, PRC89 (2014) 064903, …
  - Core-Corona division, viscous hydro evolution (vHLLE), hadronic cascades (UrQMD)
- $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions generated by EPOS359
- Pair distribution calculated: $\boldsymbol{D(r_{LCMS}) = \int d\Omega dt D(t, r_x, r_y, r_z)}$
  angle-averaged radial source distribution of like-sign pion pairs

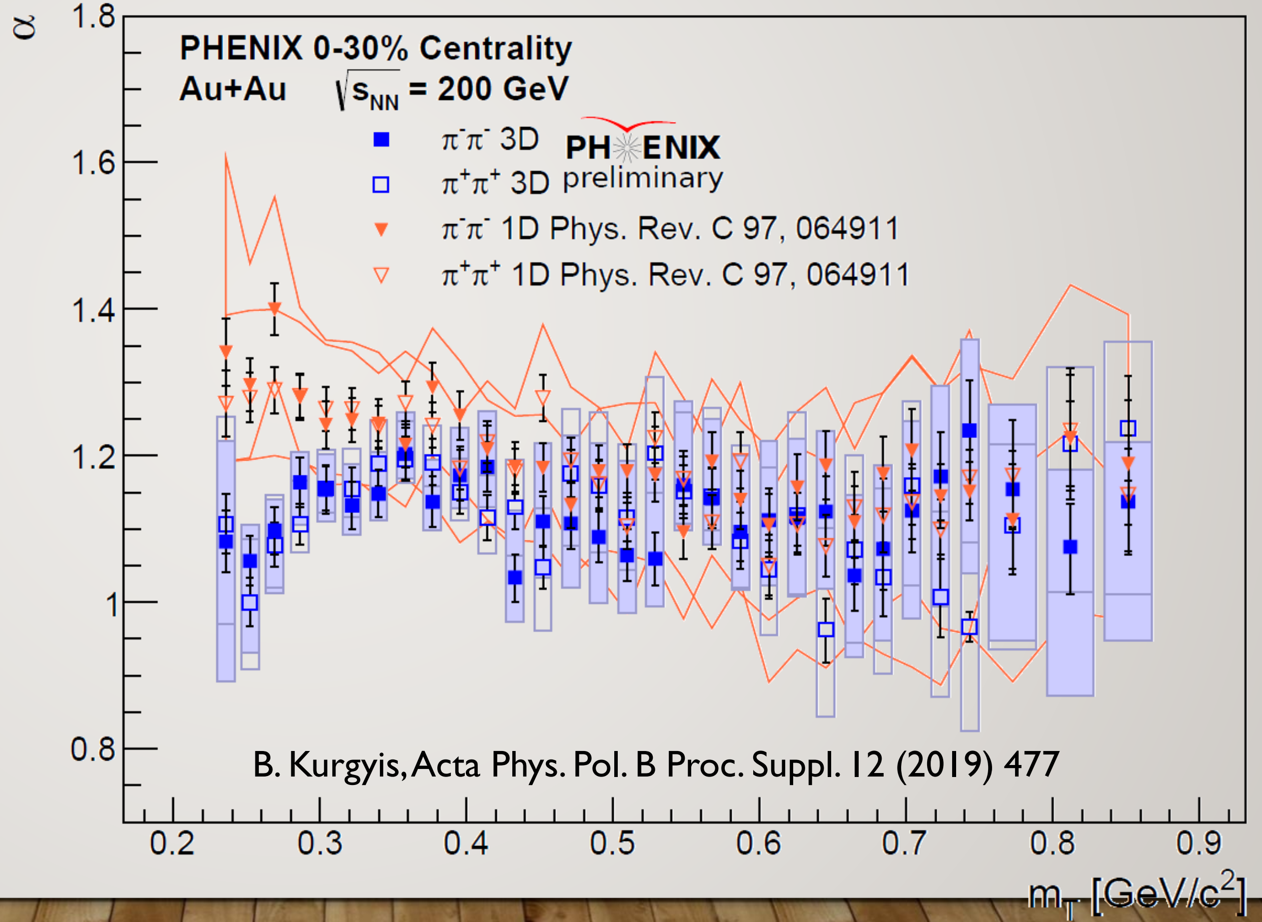
$$r_{LCMS} = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z_{LCMS})^2};\, \Delta z_{LCMS} = \Delta z - \frac{\beta(\Delta t)}{\sqrt{1-\beta^2}};\, \beta = \frac{p_{z,1} + p_{z,2}}{E_1 + E_2}$$

- Investigated cases:
  - CORE, primordial pions only
  - CORE, decay products included
  - CORE+CORONA+UrQMD, primordial pions only
  - CORE+CORONA+UrQMD, decay products included

Kincses, Stefaniak, Csanád, Entropy 24 (2022) 308 [arXiv:2201.07962]

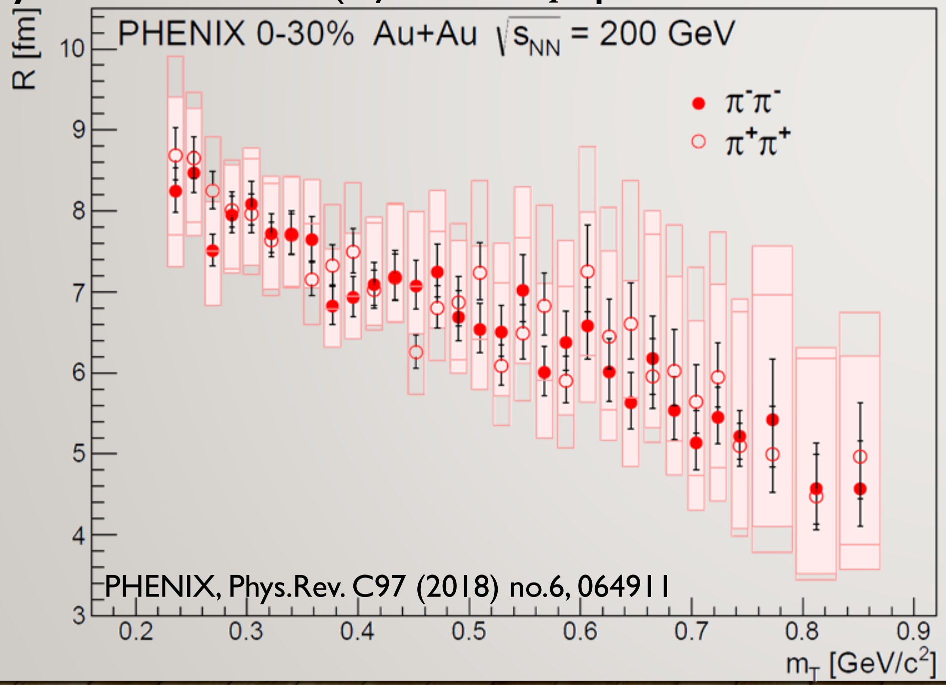
LÉVY EXPONENT IN 1D VS 3D

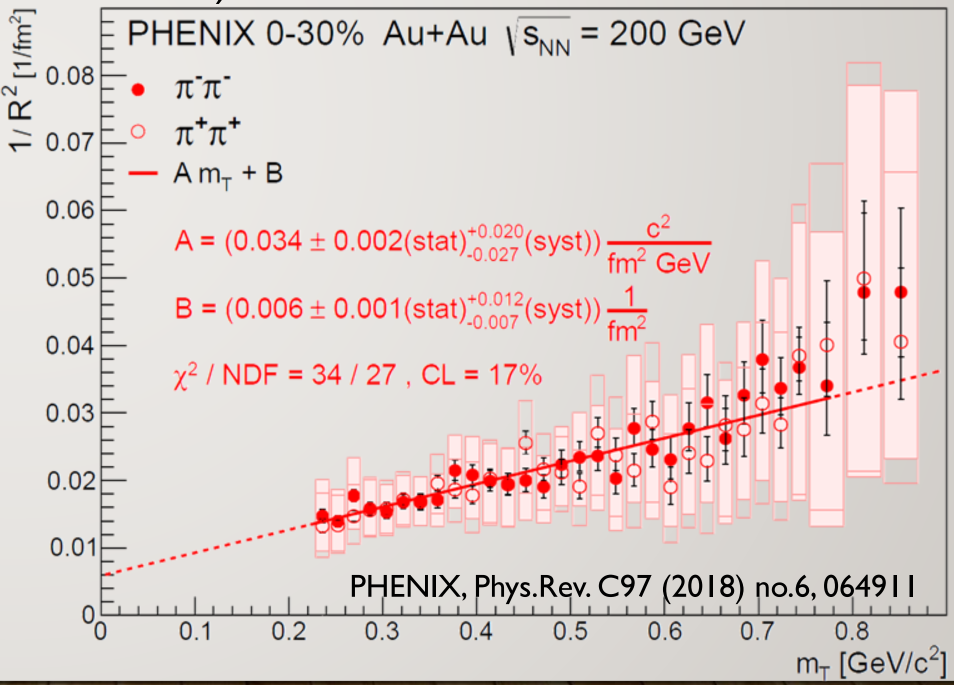
- Lévy exponent α in 3D analysis similar to 1D result
- On average still far from 2
- Observable differences at low m_T
- Maybe due to lack of spherical symmetry?
- Coulomb effect for non-spherical sources?
  - Approximation possible B. Kurgyis, arXiv:2007.10173



LÉVY SCALE PARAMETER R AT RHIC

- Similar decreasing trend as Gaussian HBT radii, but it is not an RMS!
  - RMS of a Lévy source: in principle infinity, obtained value depends on cutoff
- What do model calculations, simulations say about this?
- Hydro behavior ($1/R^2 \sim m_T$, predicted for Gaussian case) not invalid

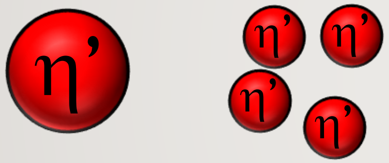




CORRELATION STRENGTH λ: IN-MEDIUM MASS?

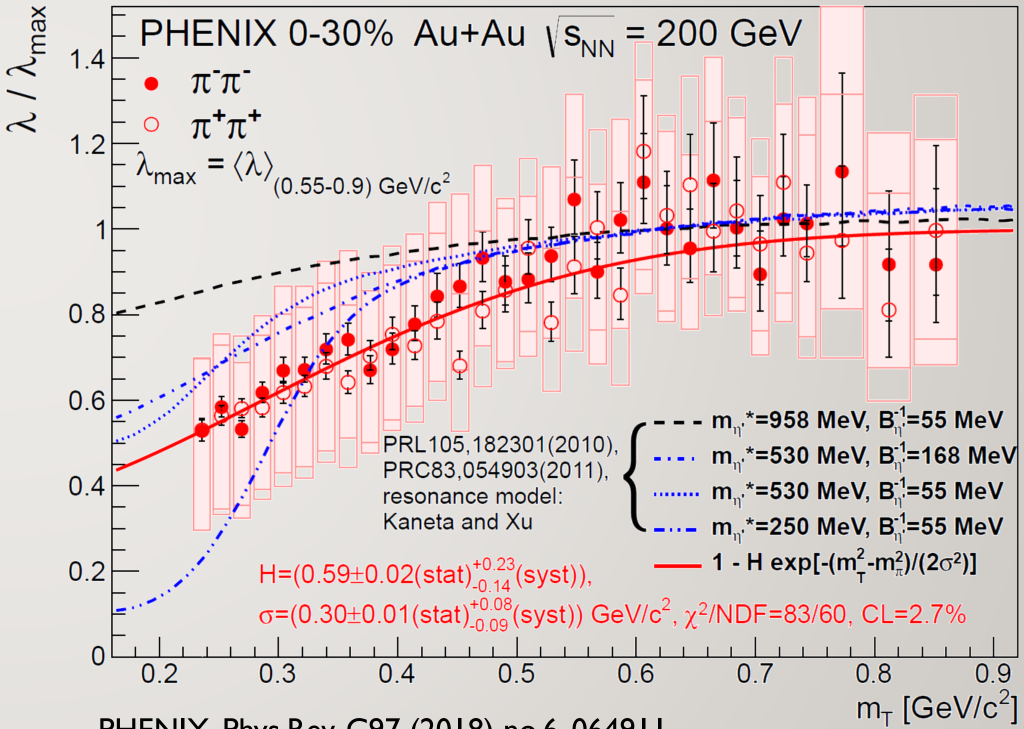
- Connection to chiral restoration
  - Decreased η’ mass → more η’ produced → more decay pions → λ decreases
  - Kinematics: η’ → ππππ with low m_T → decreased $\lambda(m_T)$ specifically at low m_T
  - Dependence on in-medium η’ mass?

Kapusta, Kharzeev, McLerran, PRD53 (1996) 5028
Vance, Csörgő, Kharzeev, PRL 81 (1998) 2205
Csörgő, Vértesi, Sziklai, PRL105 (2010) 182301



$$T < T^*$$
$$m_{\eta'} > m^*_{\eta'}$$
$$N_{\eta'} < N^*_{\eta'}$$
$$\lambda > \lambda^*$$

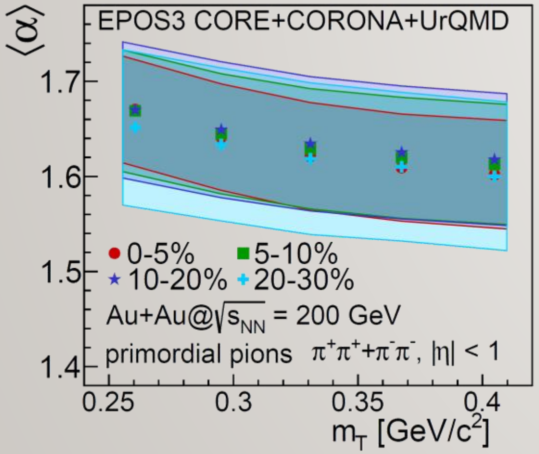
- Results not incompatible with this
- 3D results similar to 1D
- Need direct check with photons

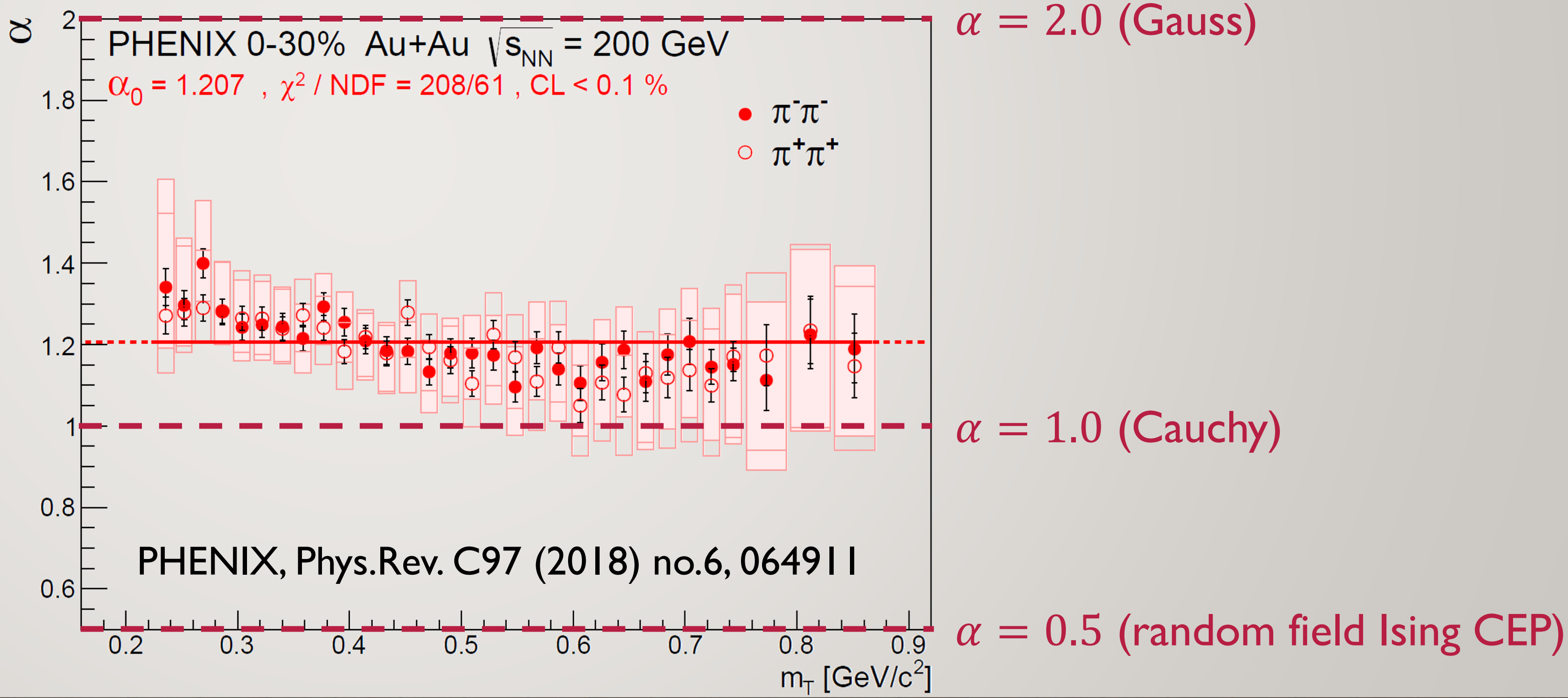


PHENIX, Phys.Rev. C97 (2018) no.6, 064911

LÉVY EXPONENT α IN 200 GEV AU+AU AT RHIC

- Measured value far from Gaussian (α = 2), inconsistent with expo. (α = 1)
- Far from random field 3D Ising value at CEP (α = 0.5)
- Approximately constant (at least within systematic uncertainties)
- What do models and calculations say?
- EPOS evt-by-evt analysis:

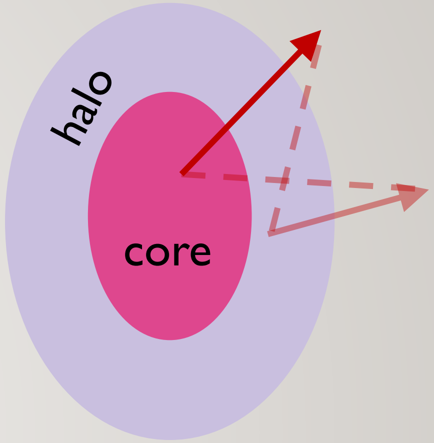


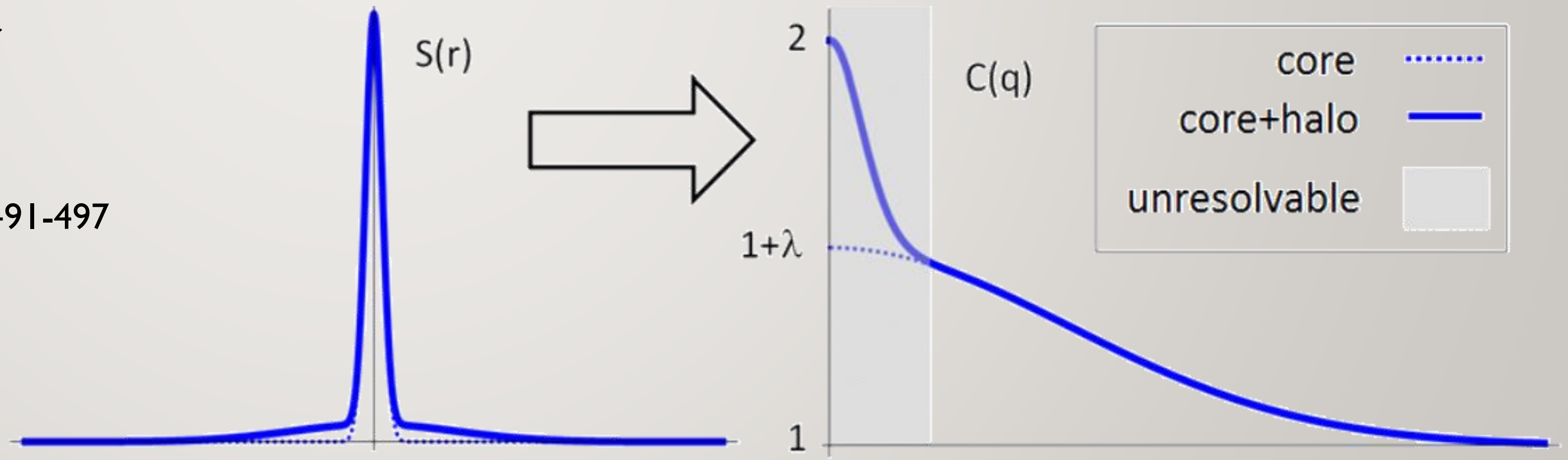


CORRELATION STRENGTH λ: CORE/HALO

- Two-component core+halo source
  - Core: hydrodynamically expanding, thermal medium
  - Halo: long lived resonances ($\gtrsim$ 10 fm/c, ω,η,η',K_0^S,...)
  - Unresolvable experimentally
  - Define $f_C = N_{\text{core}}/N_{\text{total}}$
- True $q \rightarrow 0$ limit: $C(0) = 2$
- Apparently $C(q \rightarrow 0) \rightarrow 1 + \lambda$
- $\lambda(m_\text{T}) = f_C^2(m_\text{T})$

Bolz et al, Phys.Rev. D47 (1993) 3860-3870;
Csörgő, Lörstad, Zimányi, Z.Phys. C71 (1996) 491-497





COHERENCE WITH THREE-PION LÉVY HBT

- Recall: two particle correlation strength $\lambda = f_C^2$ where $f_C = N_{\text{core}}/N_{\text{total}}$
- Generalization for higher order correlations: $\lambda_2 = f_C^2, \lambda_3 = 2f_C^3 + 3f_C^2$
- If there is partial coherence (p_C):

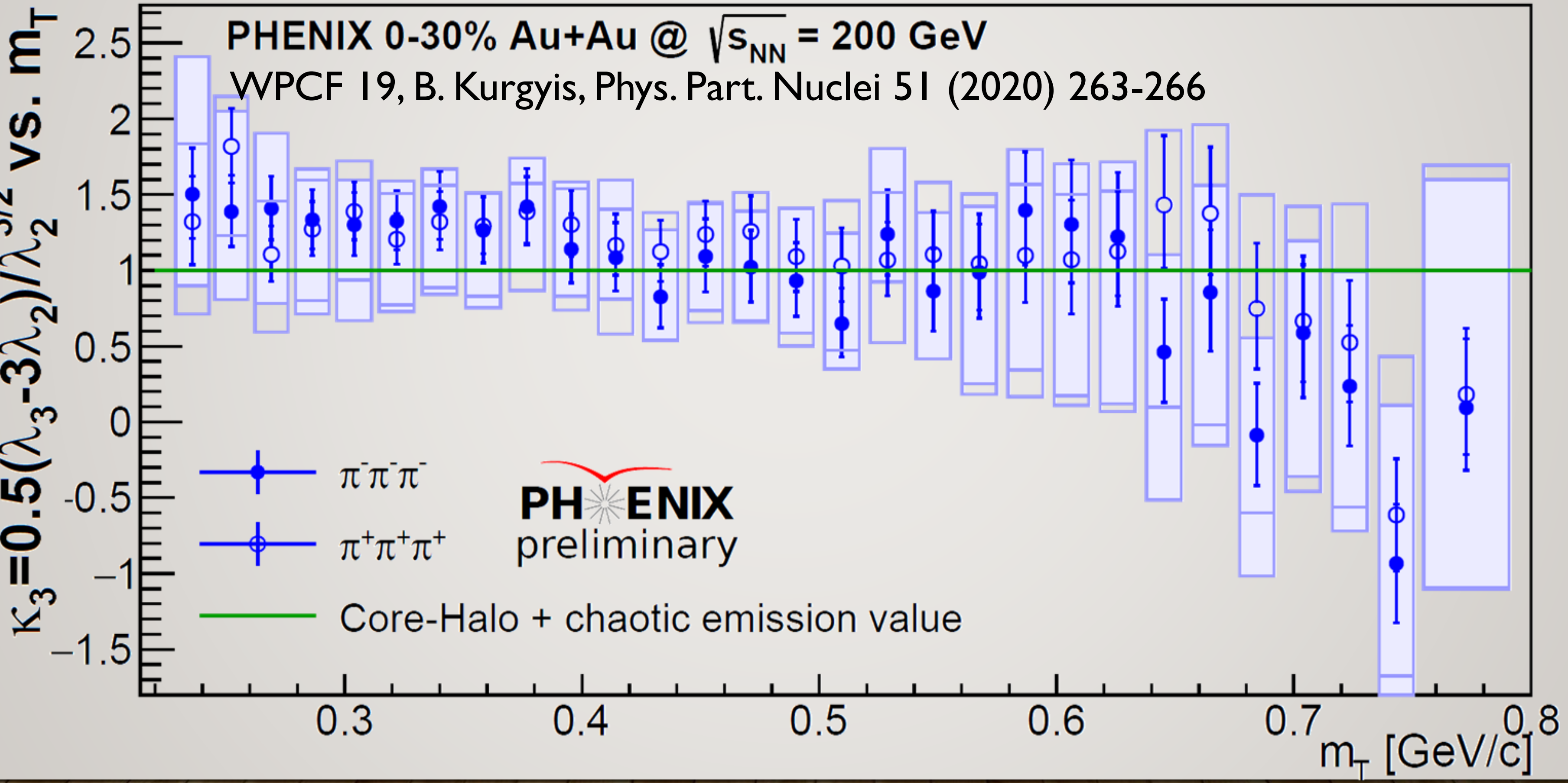
$$\lambda_2 = f_C^2[(1-p_C)^2 + 2p_C(1-p_C)]$$

$$\lambda_3 = 2f_C^3[(1-p_C)^3 + 3p_C(1-p_C)^2] + 3f_C^2[(1-p_C)^2 + 2p_C(1-p_C)]$$

- Introduce core-halo independent parameter $\kappa_3 = \frac{\lambda_3 - 3\lambda_2}{2\sqrt{\lambda_2}^3}$
  - does not depend on f_C
  - $\kappa_3 = 1$ if no coherence
- Finite meson sizes?
  - Gavrilik, SIGMA 2 (2006) 074 [hep-ph/0512357]
- Phase shift (a la Aharonov-Bohm) in hadron gas?
  - Random fields create random phase shift, on average distorts Bose-Einstein correlations Csanád et al., Gribov-90 (2021) 261-273 [arXiv:2007.07167]

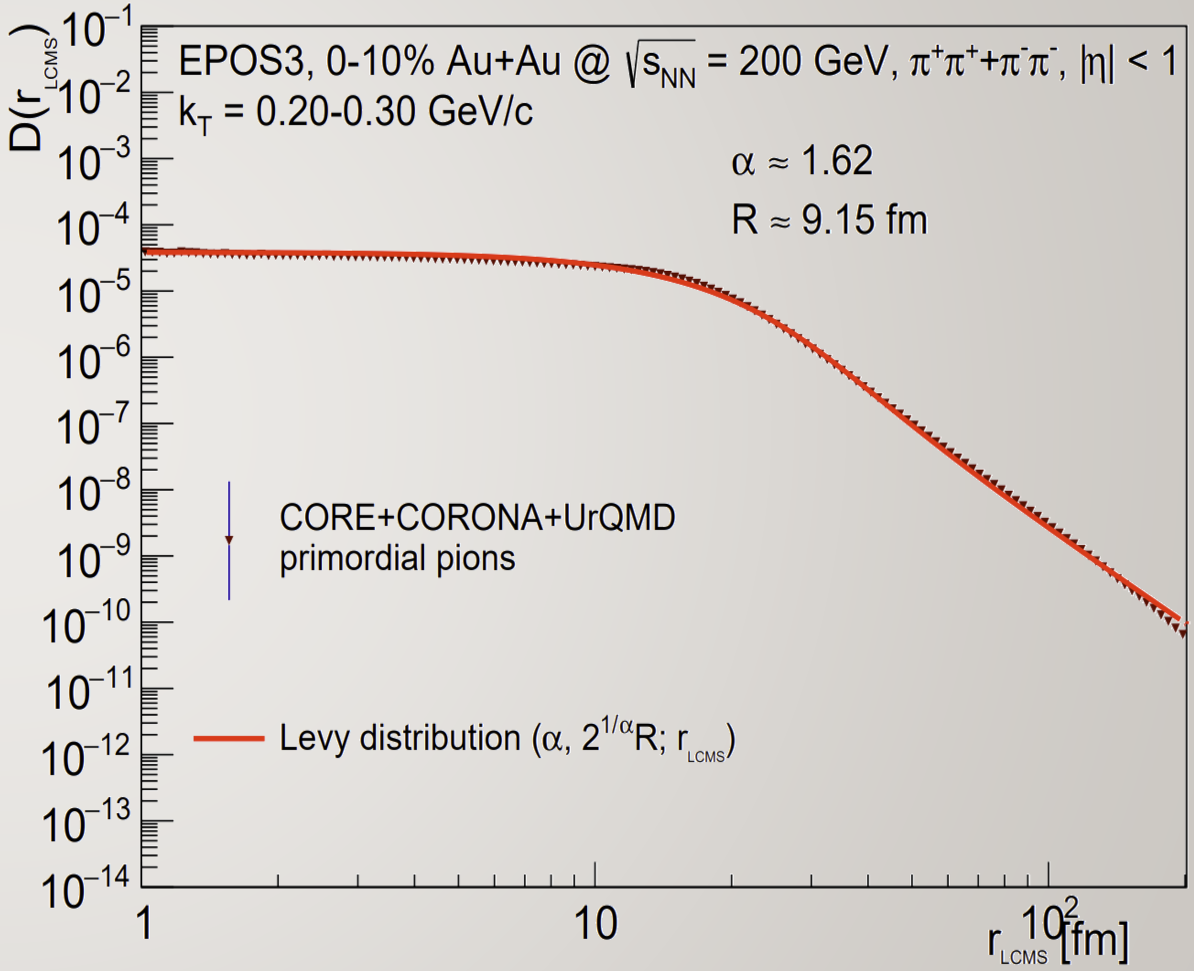
TEST OF CORE-HALO MODEL / COHERENCE

- Recall: $\kappa_3 = 1$ in pure core-halo model, $\kappa_3 \neq 1$ if coherence



ROLE OF EVENT AVERAGING?

- Event-averaged source also analyzed
- Not perfectly Lévy shape, very large χ^2
- Nevertheless: similar parameters achieved
  - Event averaged:
    $\alpha \approx 1.62, R \approx 9.15$ fm
  - Event-by-event:
    $\alpha \approx 1.66, R \approx 8.96$ fm
- More reasonable approach for kaons
  - No event-by-event analysis possible for kaons



SOURCE OR PAIR DISTRIBUTION?

- Under some circumstances (thermal emission, no interactions, …):

$$C_2(q,K) = \int S\left(r_1, K+\frac{q}{2}\right) S\left(r_2, K-\frac{q}{2}\right) |\Psi_2(r_1,r_2)|^2 dr_1 dr_2$$

$$\cong 1 + \left|\int S(r,K) e^{iqr} dr\right|^2$$

- Let us introduce the spatial pair distribution:

$$D(r,K) = \int S\left(\rho+\frac{r}{2},K\right) S\left(\rho-\frac{r}{2},K\right) d\rho$$

- Then the Bose-Einstein correlation function becomes:

$$C_2(q,K) \cong \int D(r,K)|\Psi_2(r)|^2 dr = 1 + \int D(r,K) e^{iqr} dr$$

- **Bose-Einstein correlations measure spatial pair distributions!**
- Coulomb and strong Final State Interactions? Under control for Lévy sources

Csanad, Lökös, Nagy, Phys. Part. Nuclei 51 (2020) 238 [arXiv:1910.02231]

Kincses, Nagy, Csanad Phys. Rev. C102, 064912 (2020) [arXiv:1912.01381]

INTERACTIONS: THE COULOMB-EFFECT

- Plane-wave result, based on $\left|\Psi_2^{(0)}(r)\right|^2 = 1 + e^{iqr}$:

$$C_2(q,K) \cong \int D(r,K)\left|\Psi_2^{(0)}(r)\right|^2 dr = 1 + \int D(r,K)e^{iqr}dr$$

- If there is interaction:

$$\Psi_2^{(0)}(r) \rightarrow \Psi_2^{(\mathrm{int})}(r_1, r_2)$$

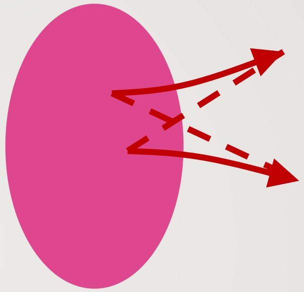
- For Coulomb:

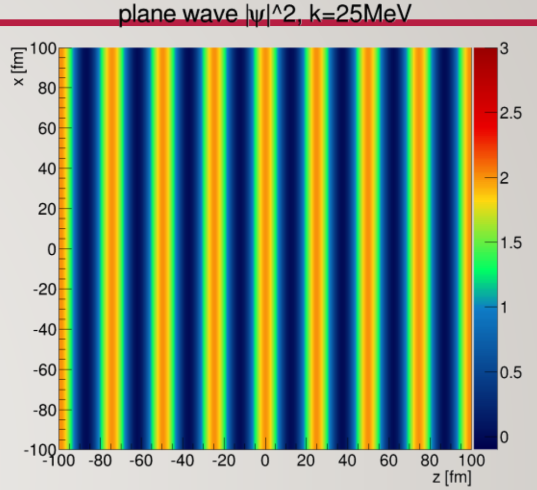
$$\left|\Psi_2^{(C)}(r)\right|^2 = \frac{\pi\eta}{e^{2\pi\eta}-1} \cdot (\text{complicated hypergeometric expression})$$

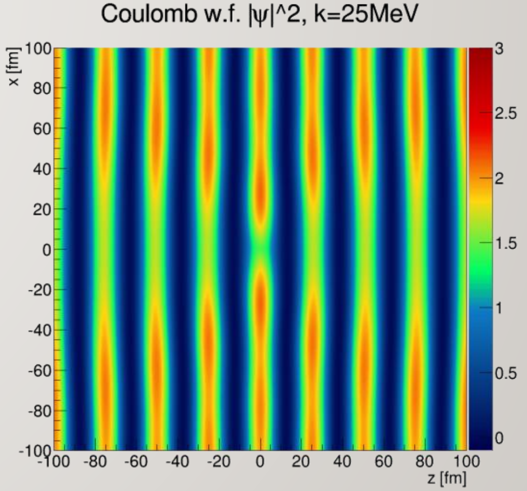
- Direct fit with this, or the usual iterative Coulomb-correction:

$$C_{\mathrm{Bose-Einstein}}(q)K(q), \text{ where } K(q) = \frac{\int D(r,K)\left|\Psi_2^{(C)}(r)\right|^2 dr}{\int D(r,K)\left|\Psi_2^{(0)}(r)\right|^2 dr}$$

- Complication: need for integrating power-law tails
- In this analyis: assuming spherical source
- Parametrization possible Csanád, Lökös, Nagy, Phys.Part.Nucl. 51 (2020) 238





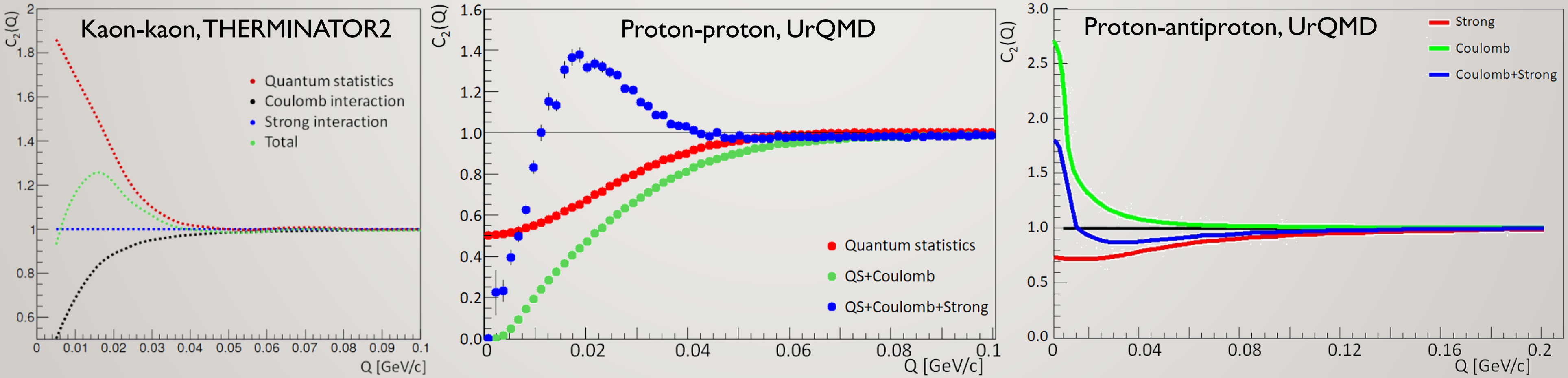


ROLE OF THE STRONG INTERACTION

- In case of other interactions or not identical bosons, the formula still works:

$$C_2(q,K) \cong \int D(r,K)|\Psi_2(r)|^2 dr$$

- Pair wave function determines $D \leftrightarrow C_2$ connection
- Mesons, baryons: strong interaction; fermions: anticorrelation
- Non-identical pairs: interaction modifies wave function



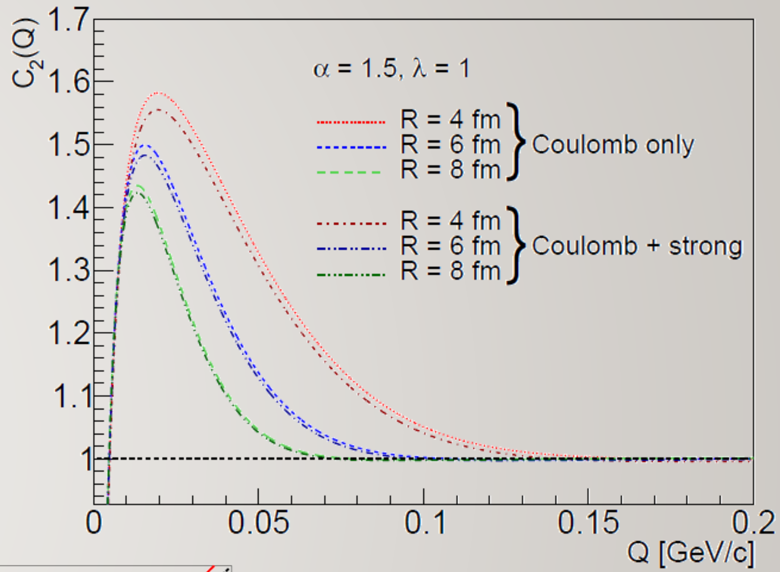
From e.g. H. Zbroszczyk's talk at Zimányi School 2019

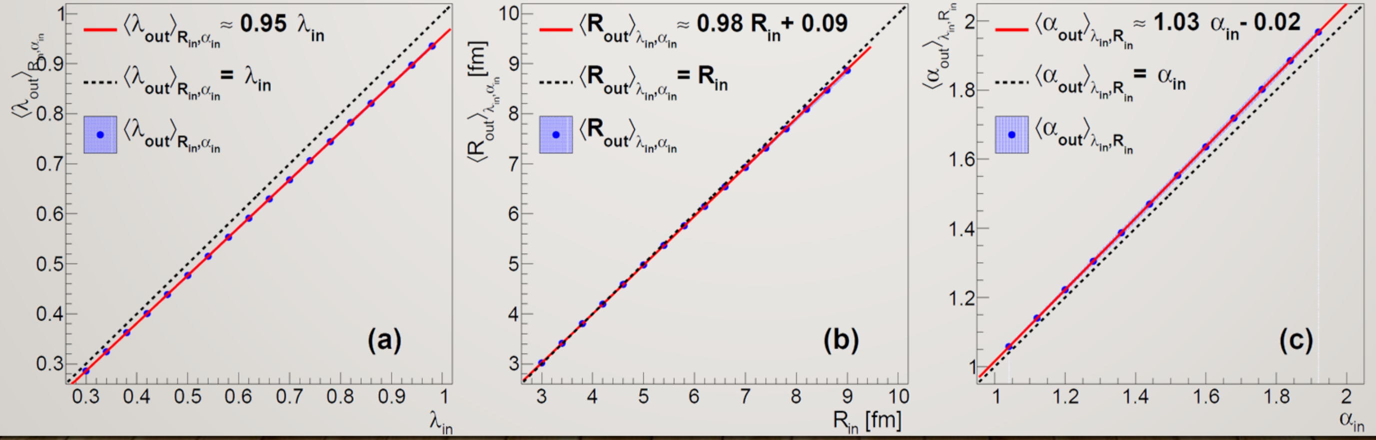
STRONG INTERACTION FOR PION PAIRS

- Additional potential appearing
- Possible handling: strong phase shift, Modify s-wave component in wave func.

  R. Lednicky, Phys. Part. Nucl.40, 307 (2009)
- Small difference in case of pions
- Few percent modification in λ, α

  Kincses, Nagy, Csanád, Phys.Rev.C 102 (2020) 064912



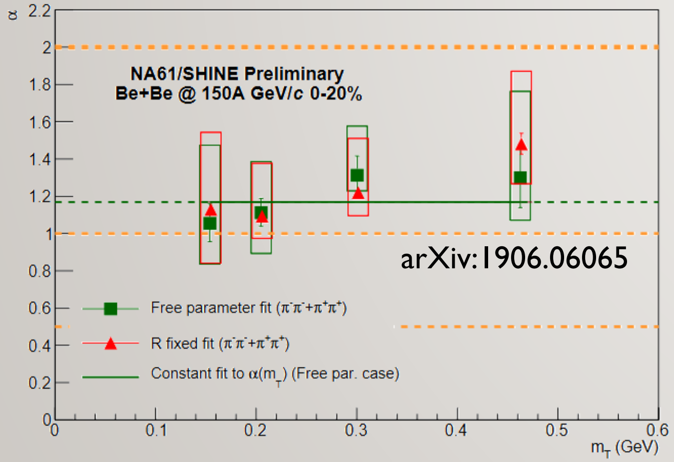


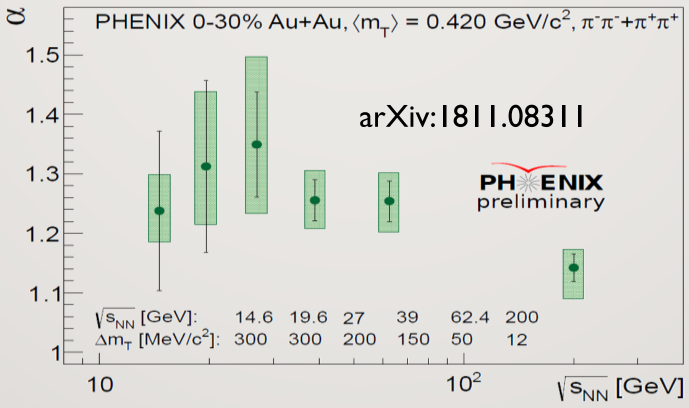
OPEN QUESTIONS

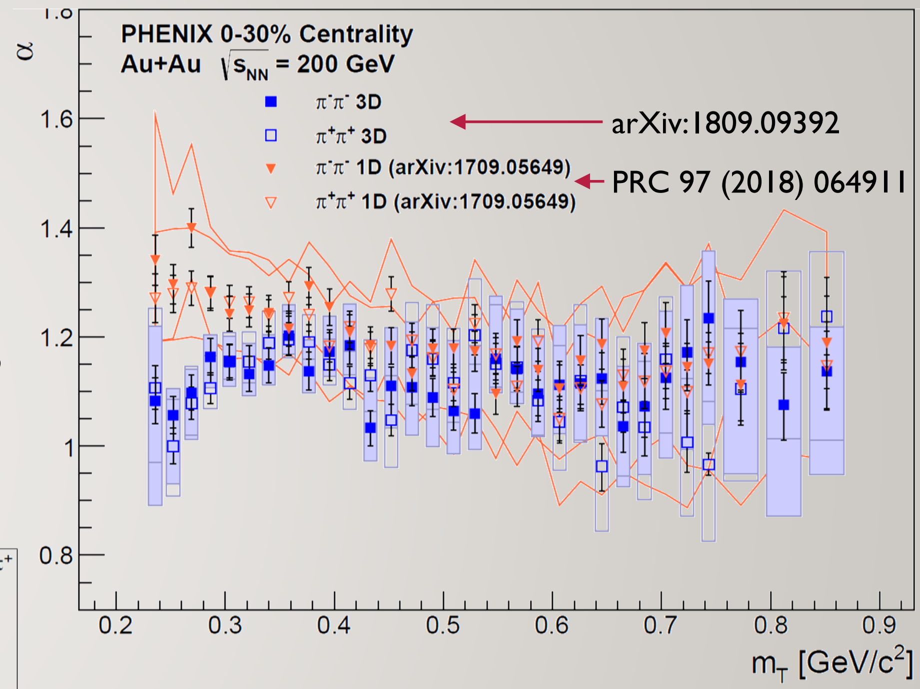
- Collision energy and centrality dependence of Lévy parameters?
  - Non-monotonicity in $\alpha\left(\sqrt{s_{NN}}\right)$ or $\alpha(\text{centrality})$?
  - Hole in $\lambda(m_T)$ at low $\sqrt{s_{NN}}$? Really due to η'?
- Reason for the appearance of Lévy distributions for pions?
  - What is the Lévy exponent for kaons?
  - Kaons have smaller total cross-section thus larger mean free path, heavier tail?
  - Does m_T scaling hold for Lévy scale R?
- Correlation strength versus core-halo picture: are there other effects?
  - Three-particle correlations may show if coherence or other effects play a role
  - Other effects may also play a role (finite meson sizes, random field phase shift, etc)

LÉVY HBT MEASUREMENTS

- Many experimental results
  - PHENIX Au+Au: $\alpha \approx 1 - 1.5$
  - STAR Au+Au: ongoing
  - NA61 Be+Be: $\alpha \approx 1 - 1.5$
  - CMS Pb+Pb: $\alpha = 1$ fixed
- Where does this Lévy shape come from? What does it mean?
  - Role of event class averaging?

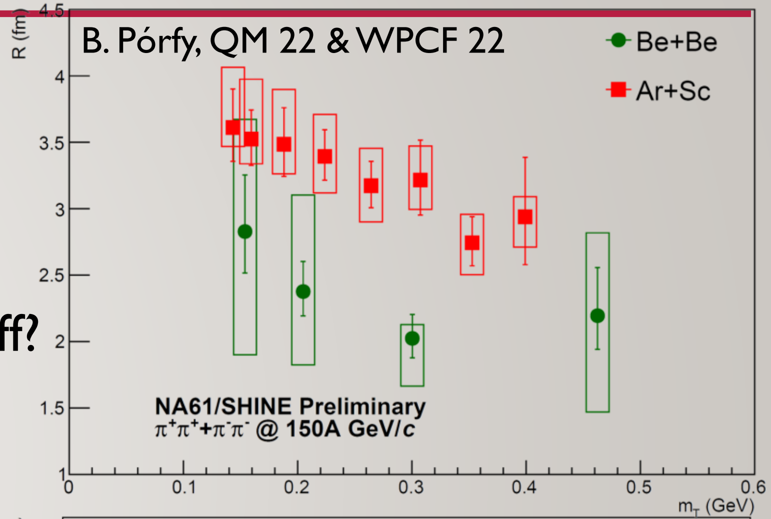


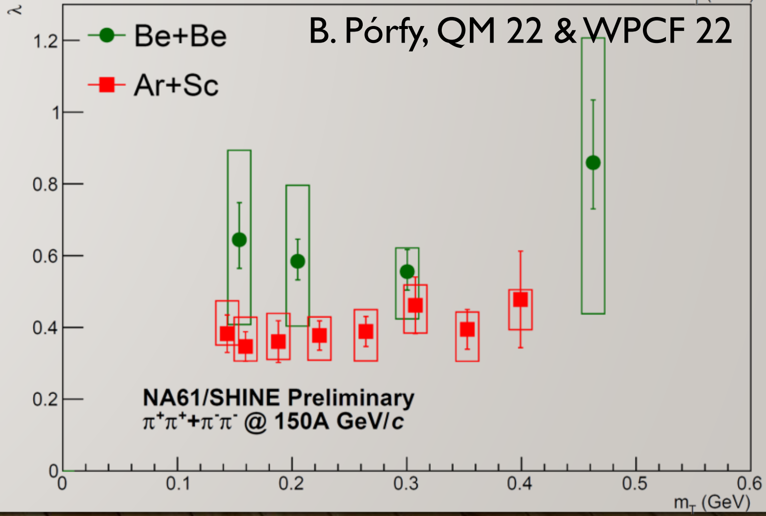


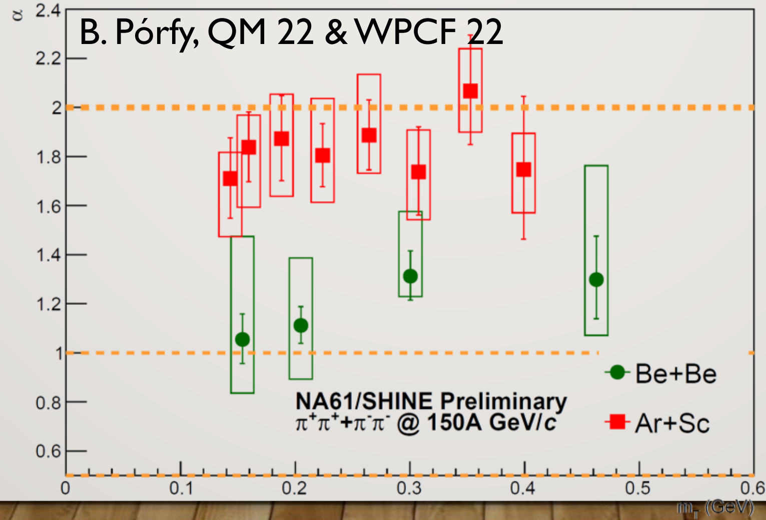


PION ANALYSIS AT SPS NA61/SHINE

- Lévy scale R of Ar+Sc and Be+Be:
  - Compatible with initial geometry factor 1.6
  - Decrease with m_T due to transverse flow?
- No m_T dependence in λ, in contrast to RHIC result – can be turned off?
- Lévy index α: significant difference







STABILITY PARAMETER α FROM SPS TO LHC

- Different values for small and medium systems at SPS
- Medium and large systems: increasing trend from SPS to RHIC to LHC
- Compare to „expectation”: