B-semileptonic form-factors on the lattice

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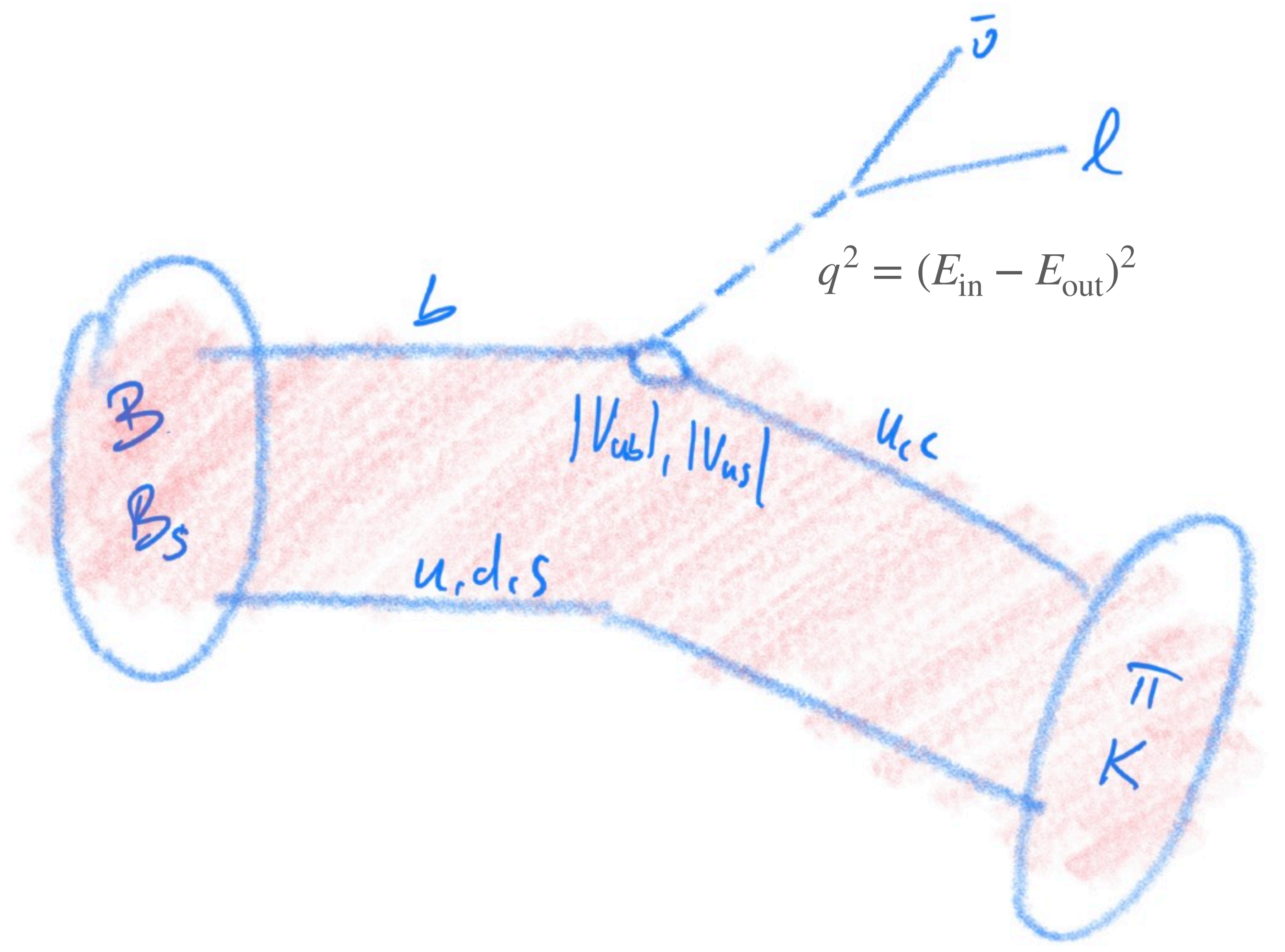
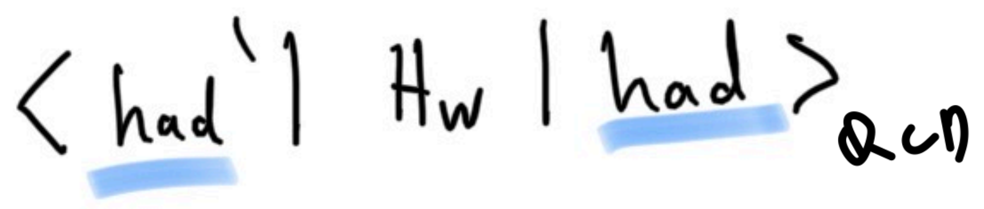
three points

- exclusive decay form-factor calculations $b \to u, b \to c$
- model- and truncation independent form-factor fitting
- inclusive decay rate calculations $b \to c$

Exclusive semileptonic meson decay



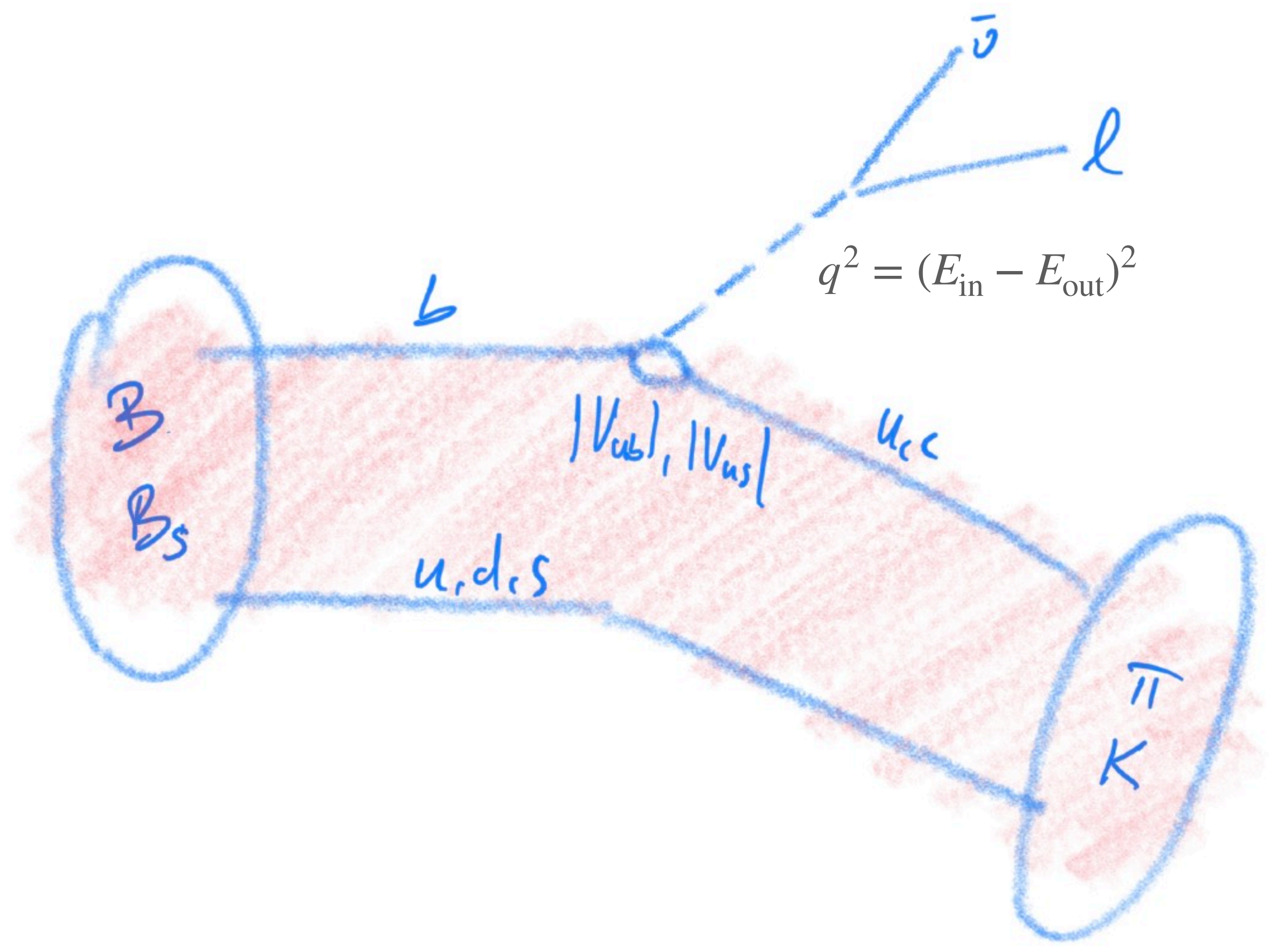
$$\frac{d\Gamma(B_s \to P\ell\nu)}{dq^2} = \eta_{EW}\frac{G_F^2|V_{xb}|^2}{24\pi^3}\frac{(q^2-m_\ell^2)^2|\vec{k}|}{(q^2)^2}\left[\left(1+\frac{m_\ell^2}{2q^2}\right)\vec{k}^2|f_+(q^2)|^2+\frac{3m_\ell^2}{8q^2}\frac{(M_{B_s}^2-M_P^2)^2}{M_{B_s}^2}|f_0(q^2)|^2\right]$$



- form factors computed on lattice QCD at high precision (see FLAG21)
- considered standard for tree-level decays
- heavily used for CKMology but also for lepton-flavour-universality tests
- only few collaborations competing

- estimating systematics in the last steps of the lattice-analysis can be rather challenging
- we (and others) have found instabilities in the extrapolation of lattice-form factors

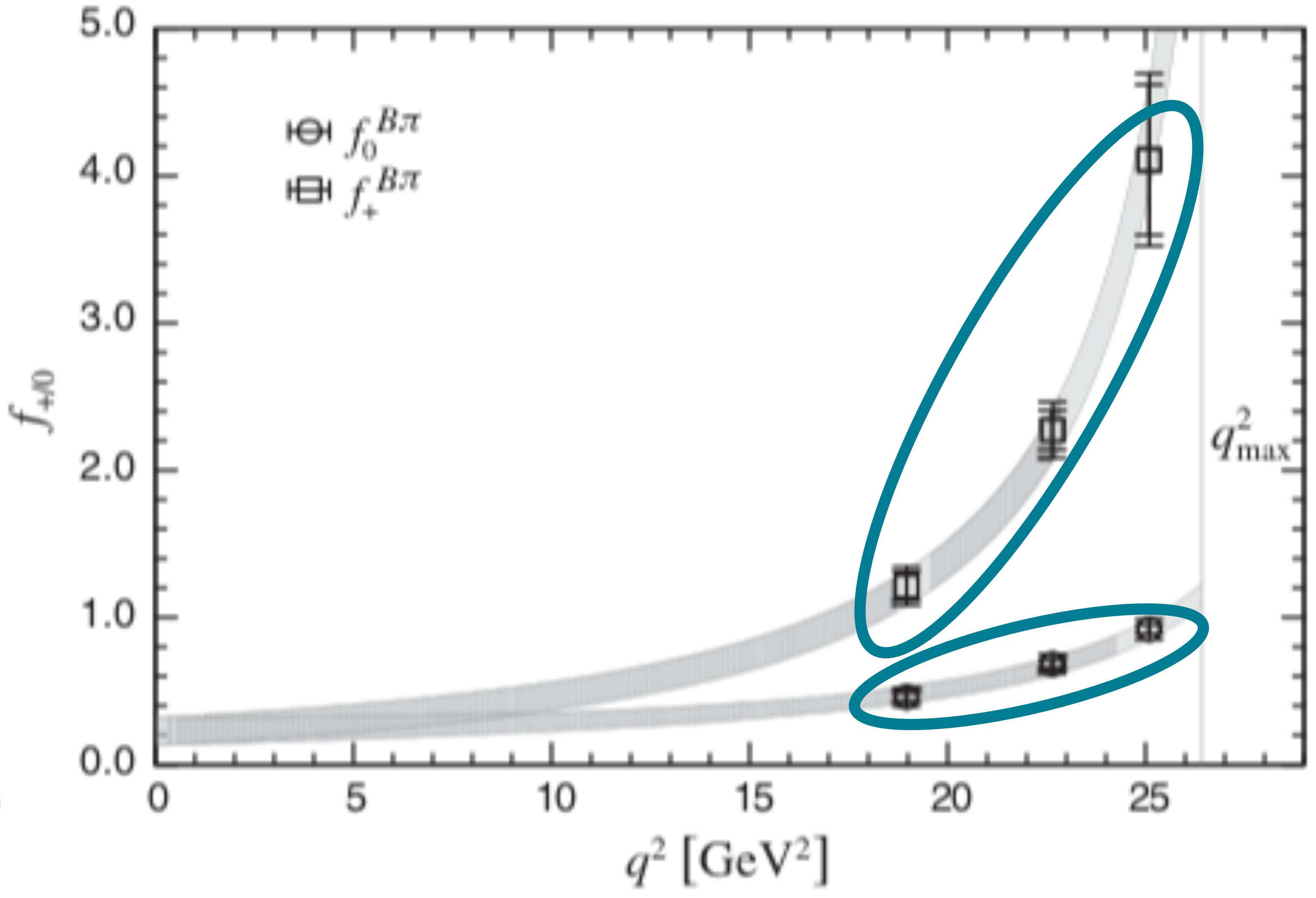
Exclusive semileptonic meson decay



Some novelties on the lattice:

- **heavy-quark mass vs. lattice spacing (UV cutoff)**
  - so far use of effective field theory for heavy quark
  - work on fully relativistic lattice b-quark going on
  - challenge: control systematics due to discretisation

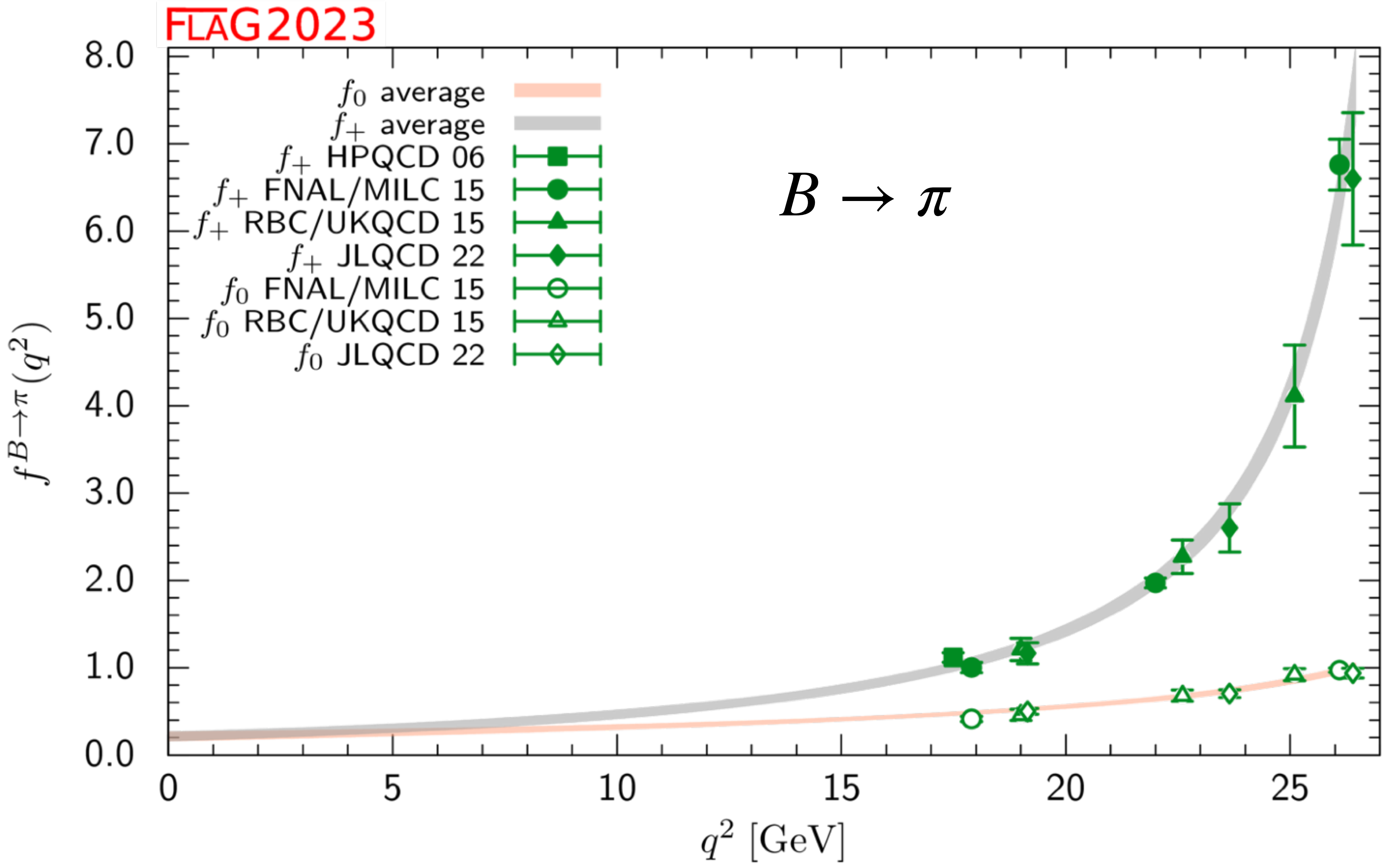
- **limited kinematic reach**
  - new finer lattice spacings allow to reach to lower q^2
  - novel ideas for model-independent parameterisations of form factors

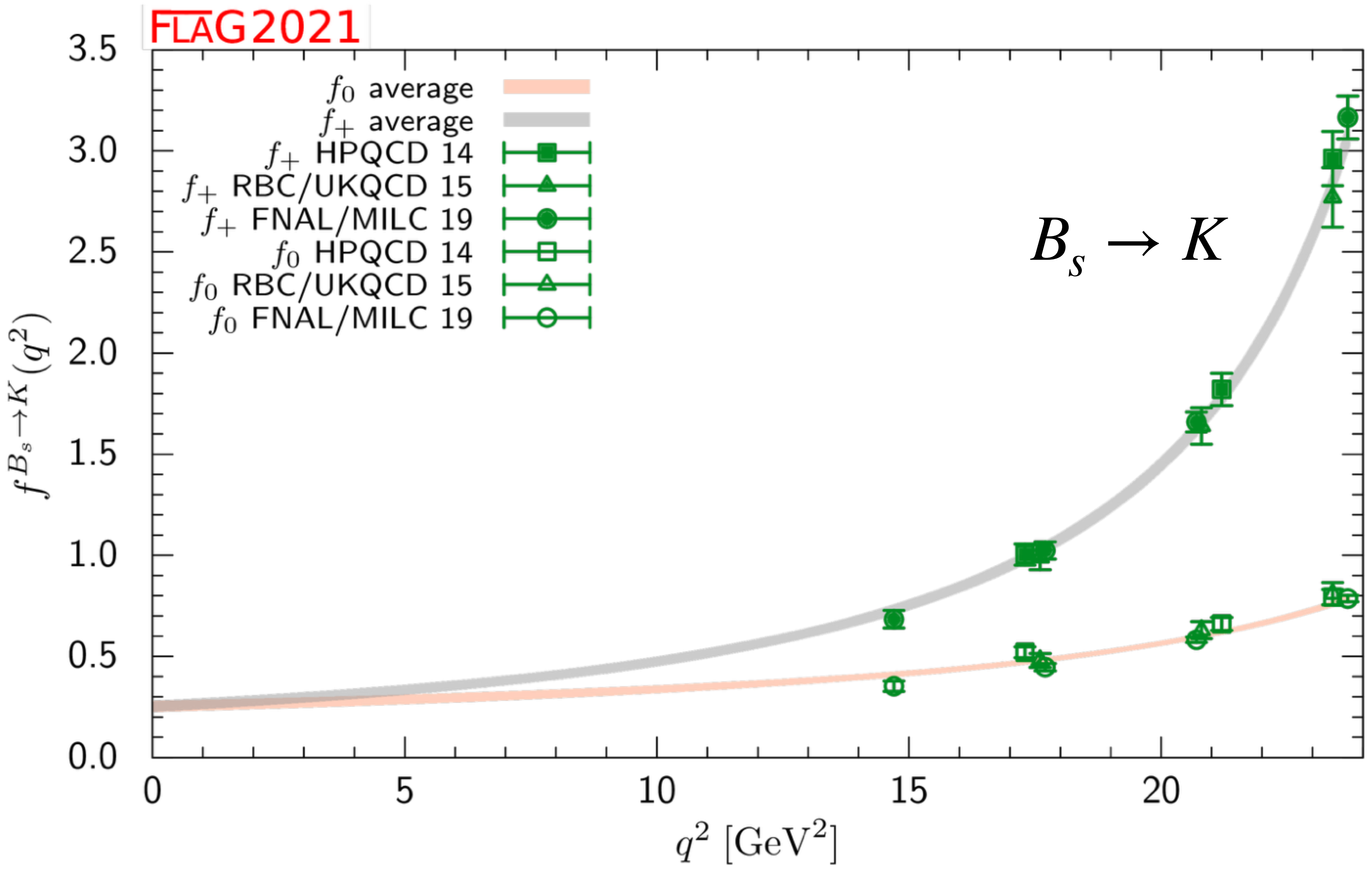


RBC/UKQCD PRD 91, 074510 (2015)2018

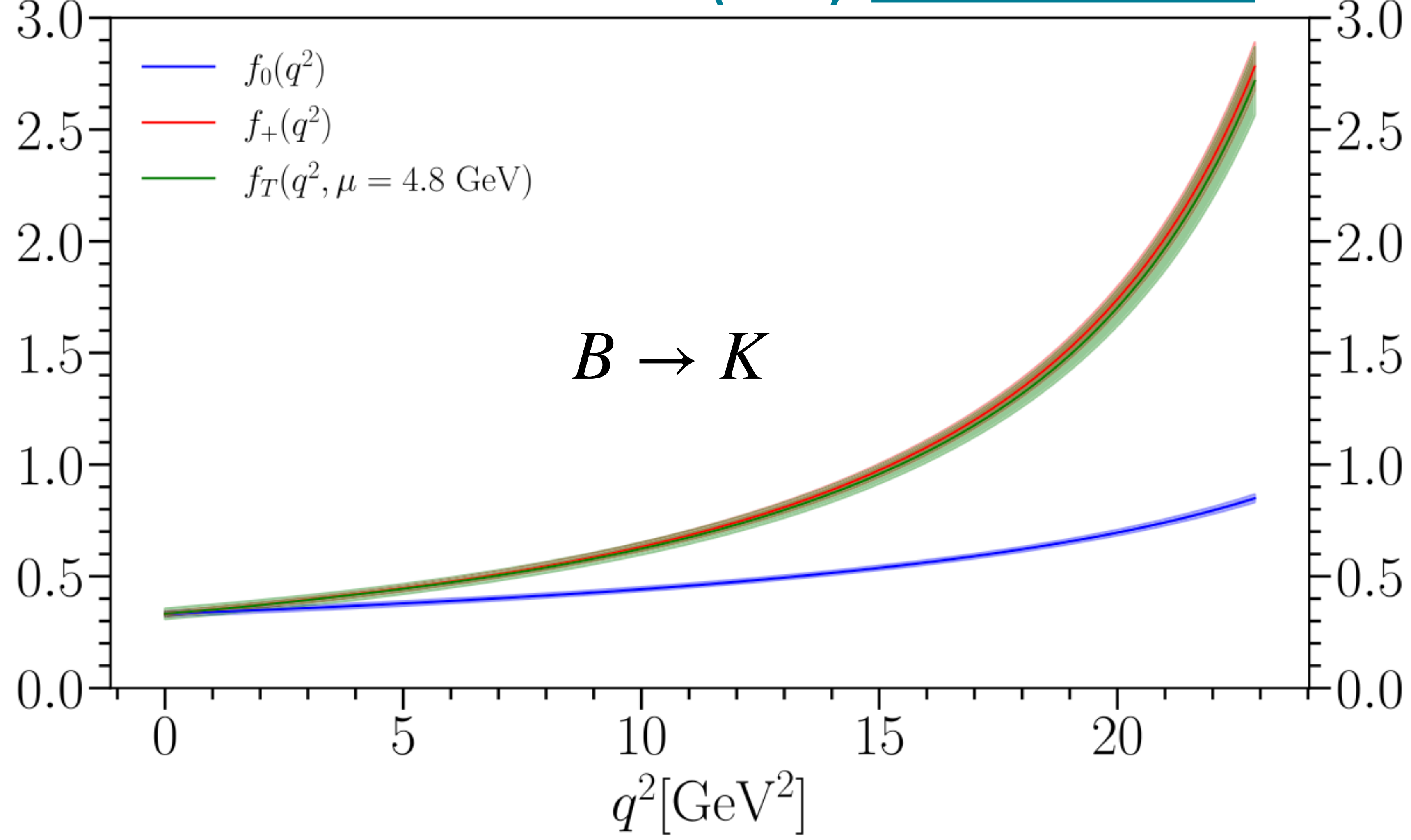
exclusive $b \to u$

$b \to u$ exclusive

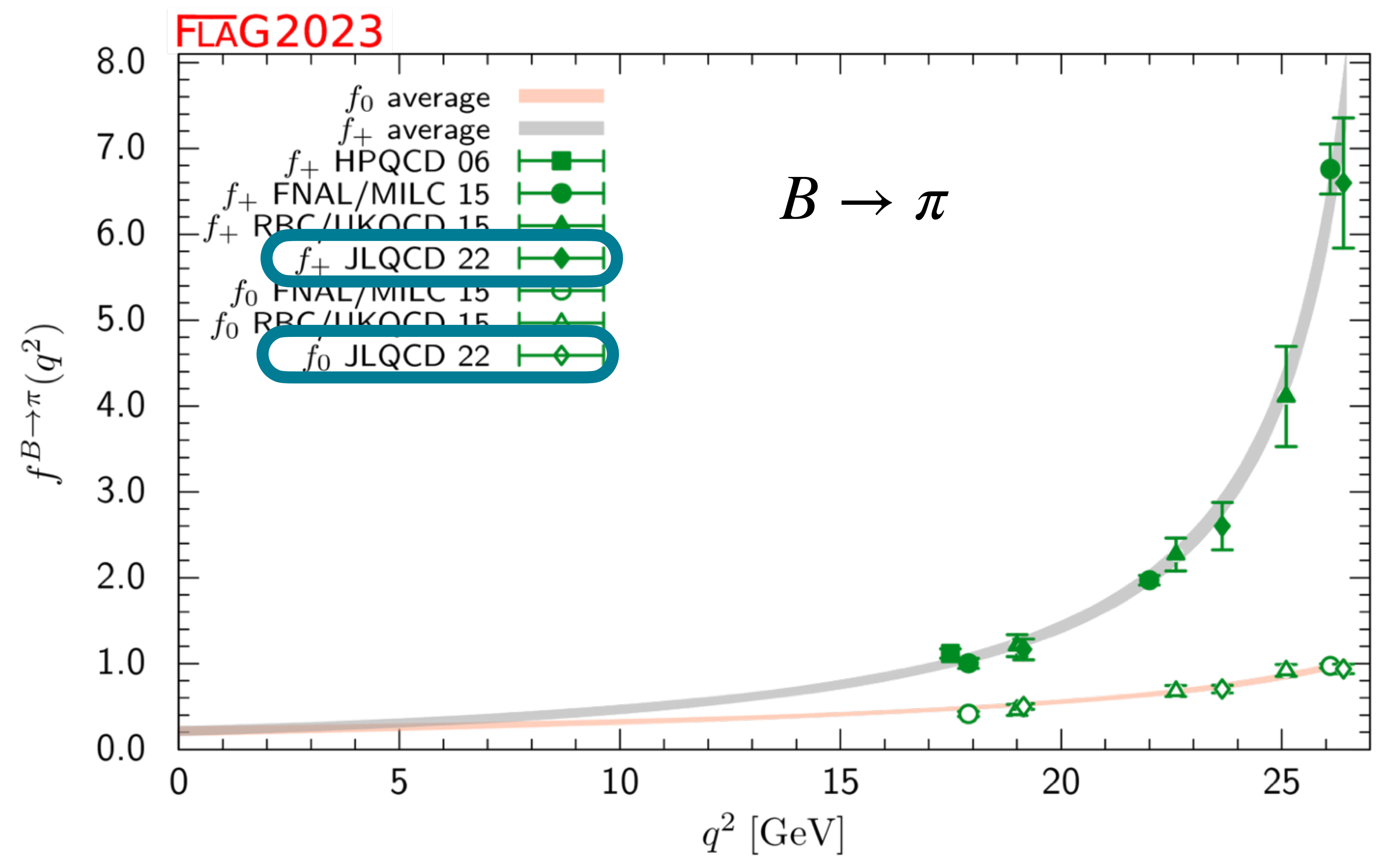


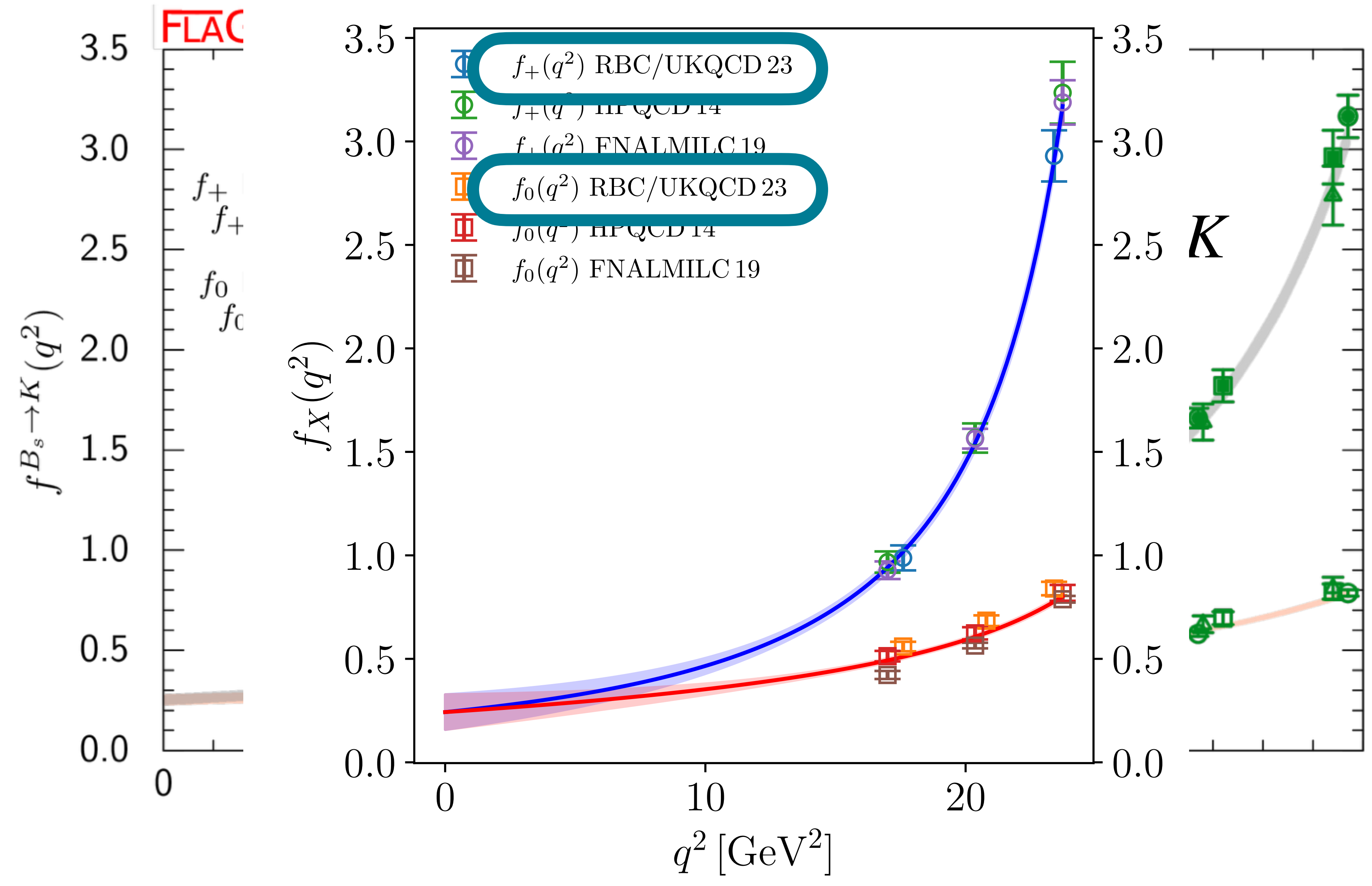


new HPQCD PRD 107 (2023): arXiv:2207.12468



$b \to u$ exclusive





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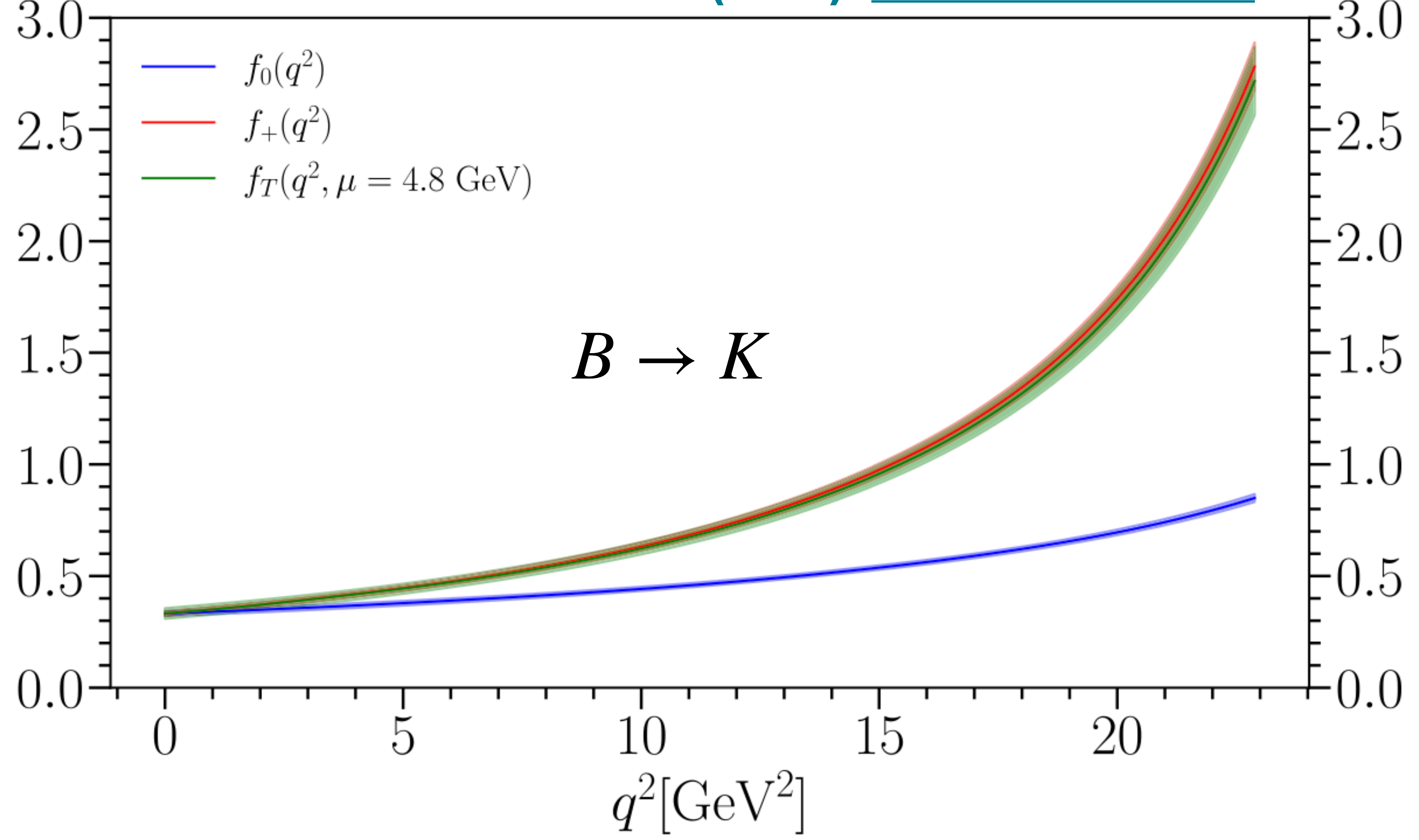


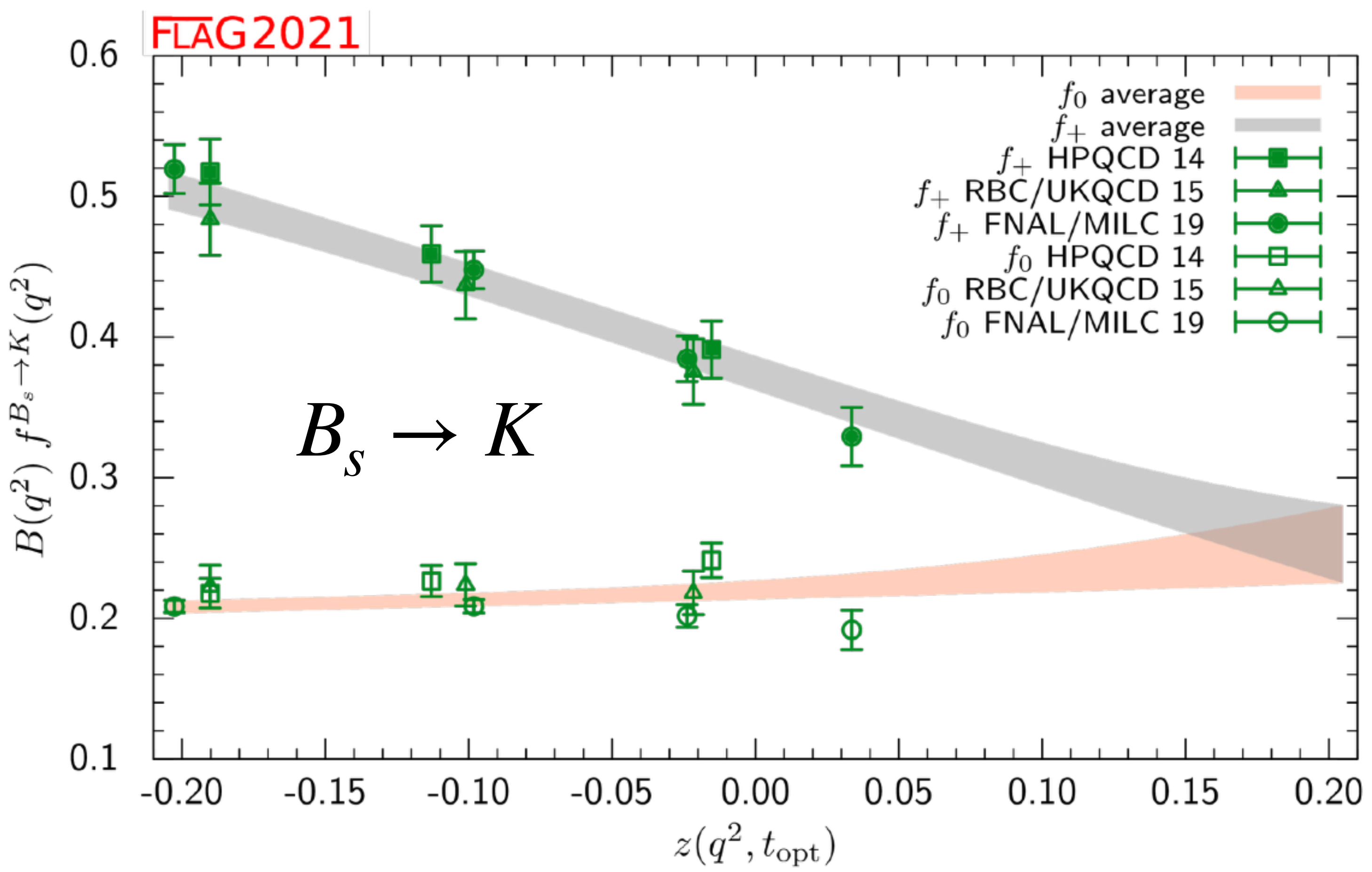
New:

$B \to \pi$ data by JLQCD 22 PRD arXiv:2203.04938

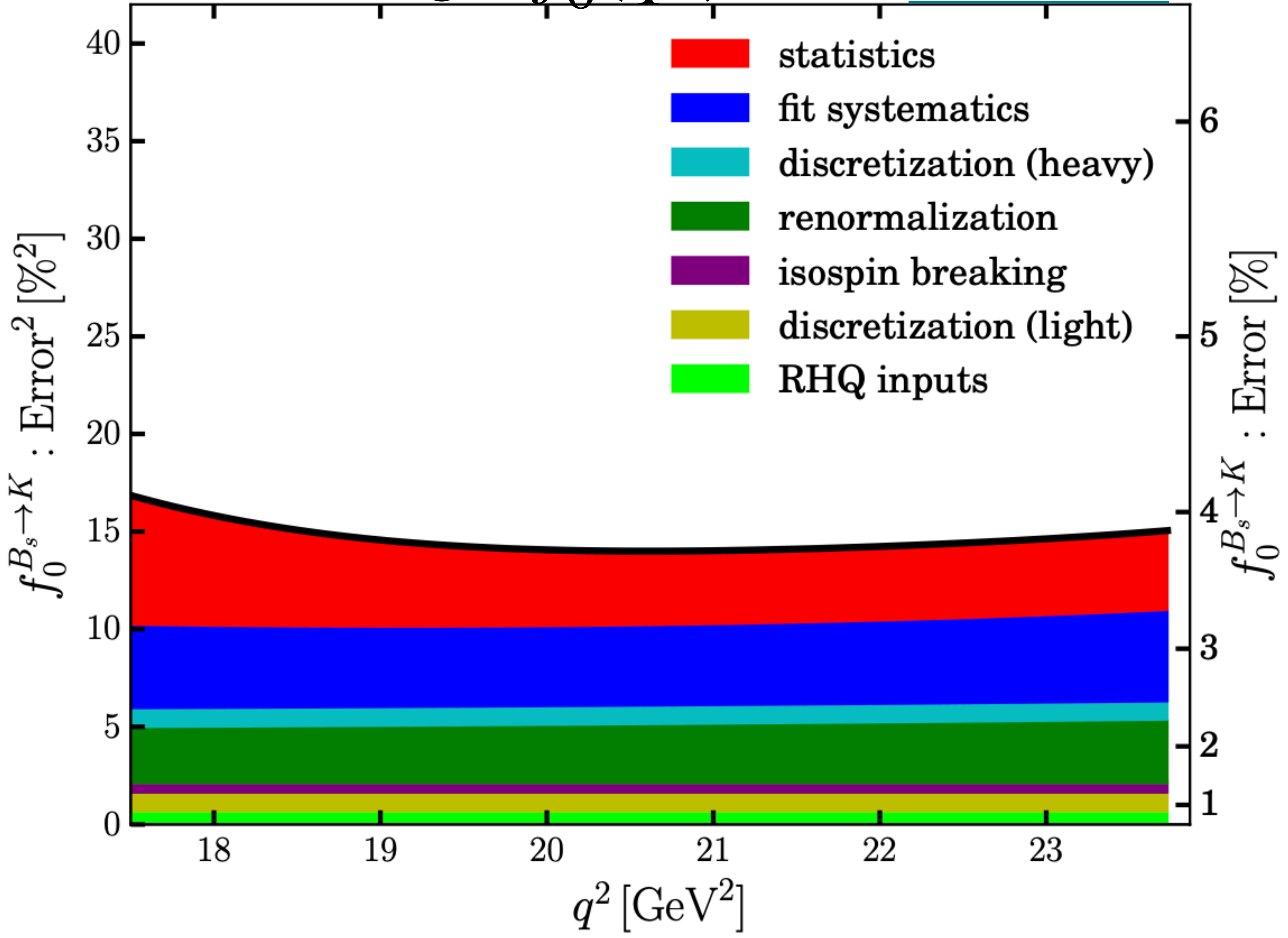
$B_s \to K$ data by RBC/UKQCD 23 arXiv:2303.11280

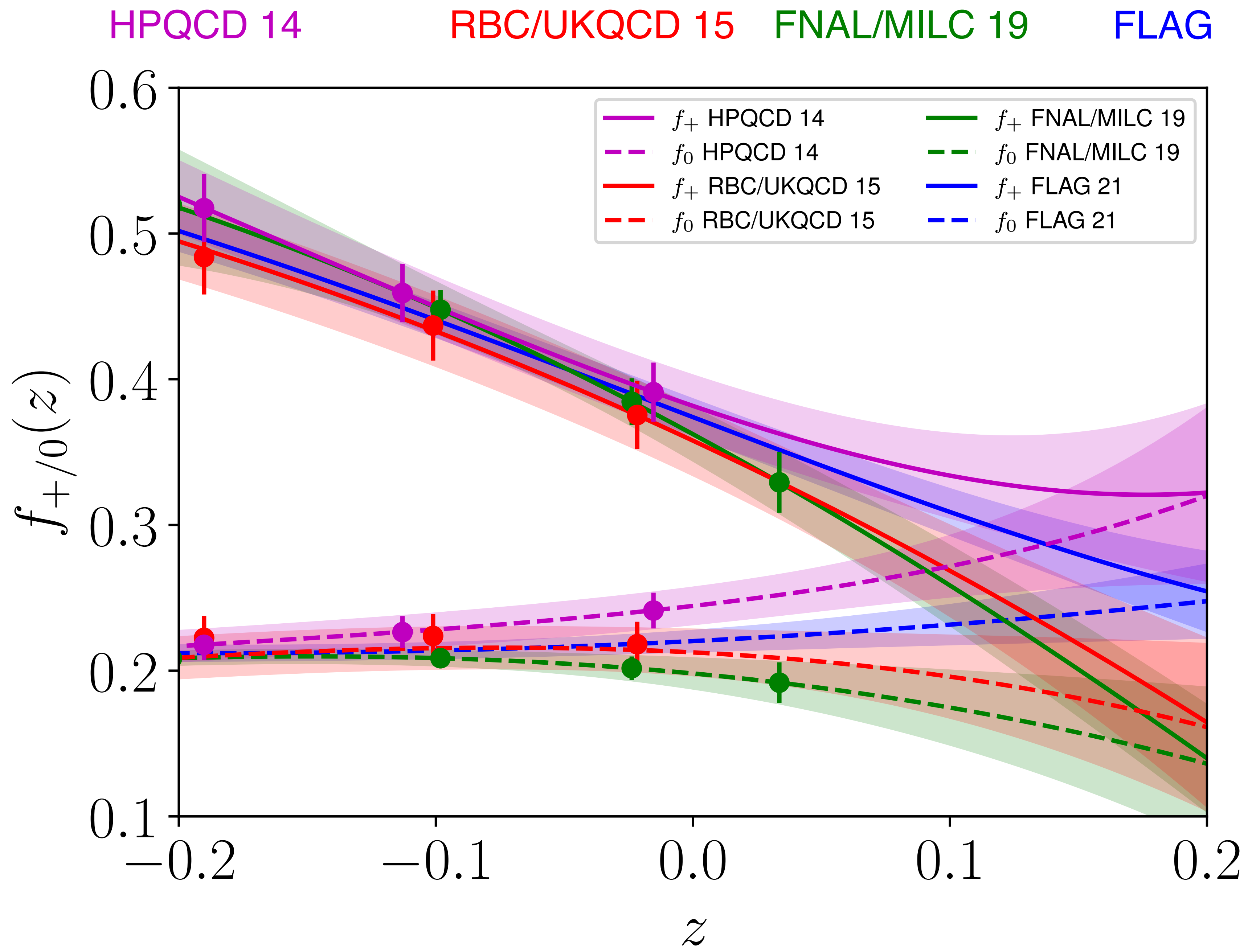
$B \to K$ data by HPQCD 23 PRD, arXiv:2207.12468

$b \to u$ exclusive: work to do



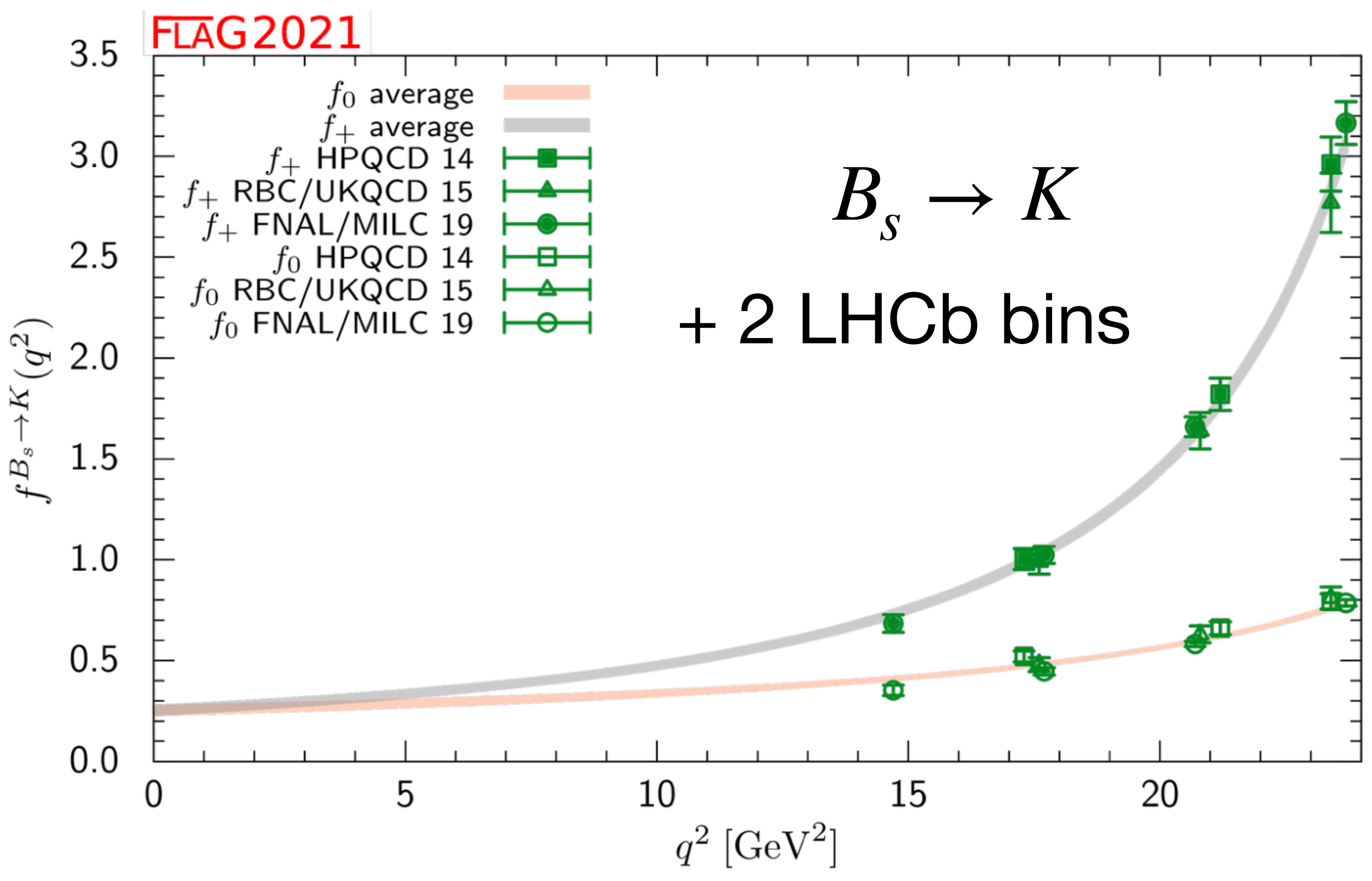
error budget $f_0(q^2)$ **RBC/UKQCD 23 arXiv:2303.11280**

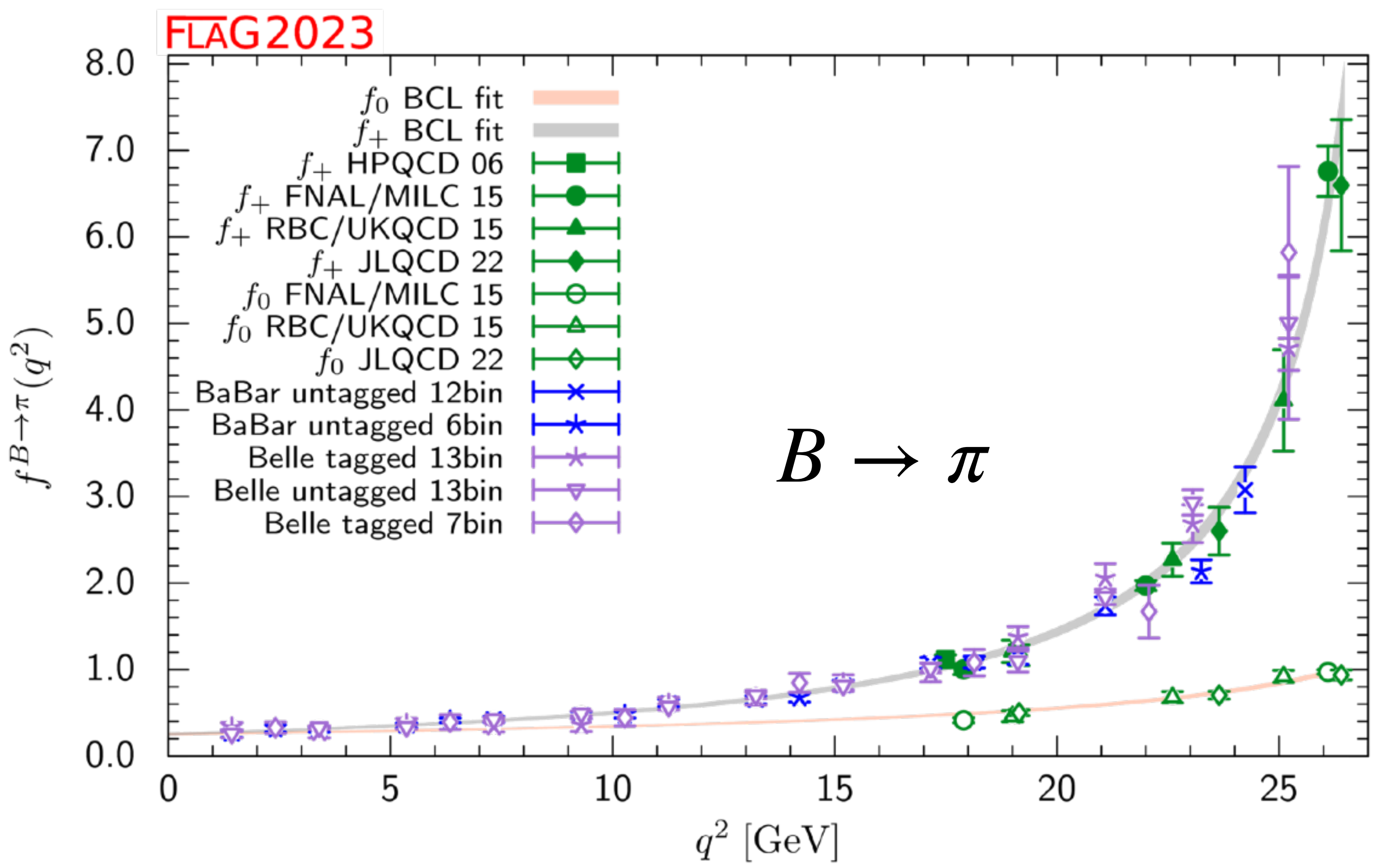


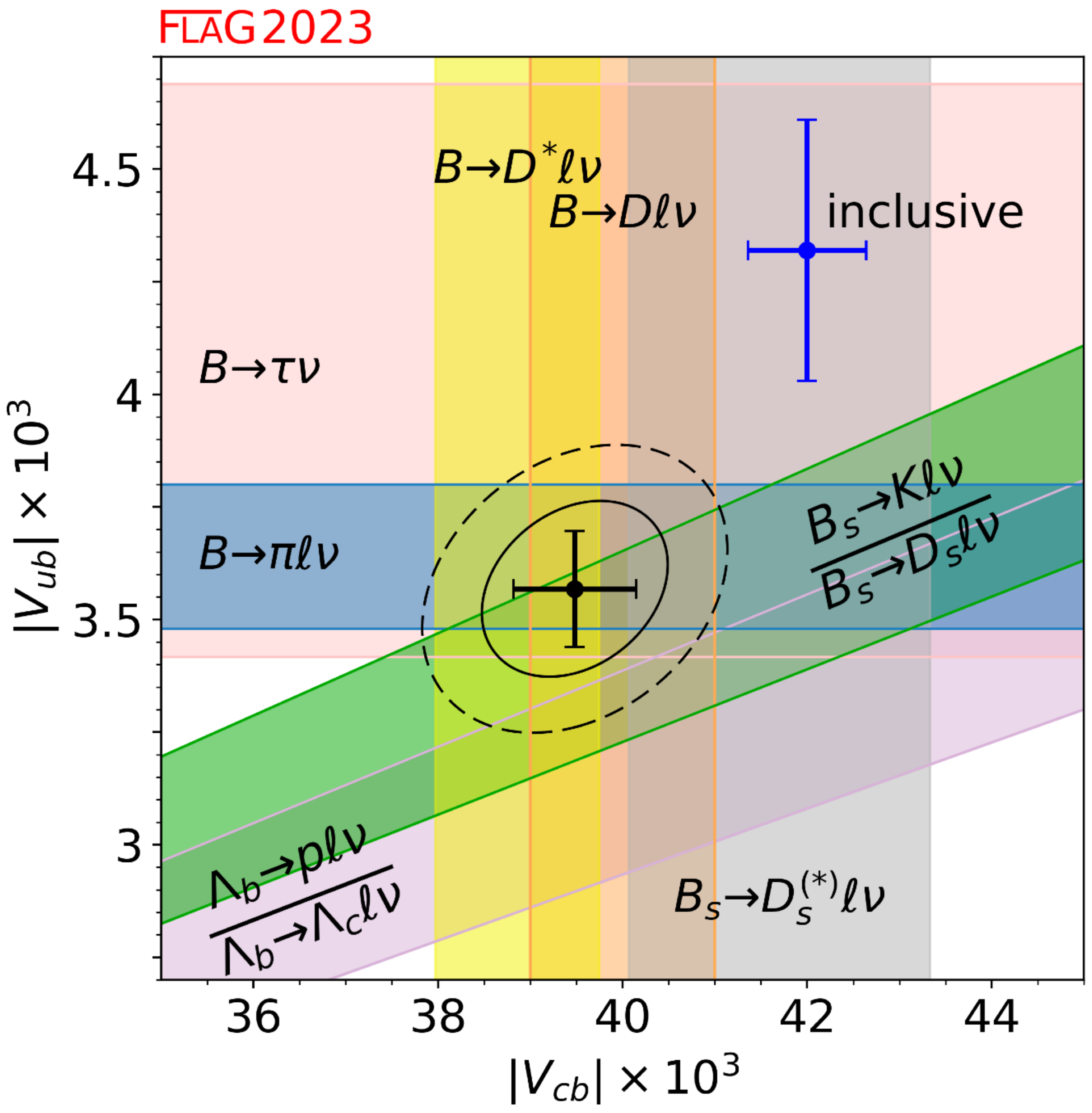


- Lattice data sets show tension $B \to \pi$, $B_s \to K$ (combination requires PDG-inflation *factor)*
- by definition they should agree
- reasons yet to be understood (excited states Bär et al. arXiv:2210.06863, 2210.06857, chiral/cont extrapolation RBC/UKQCD arXiv:2303.11280, ...)

$b \to u$ exclusive: $|V_{ub}|$







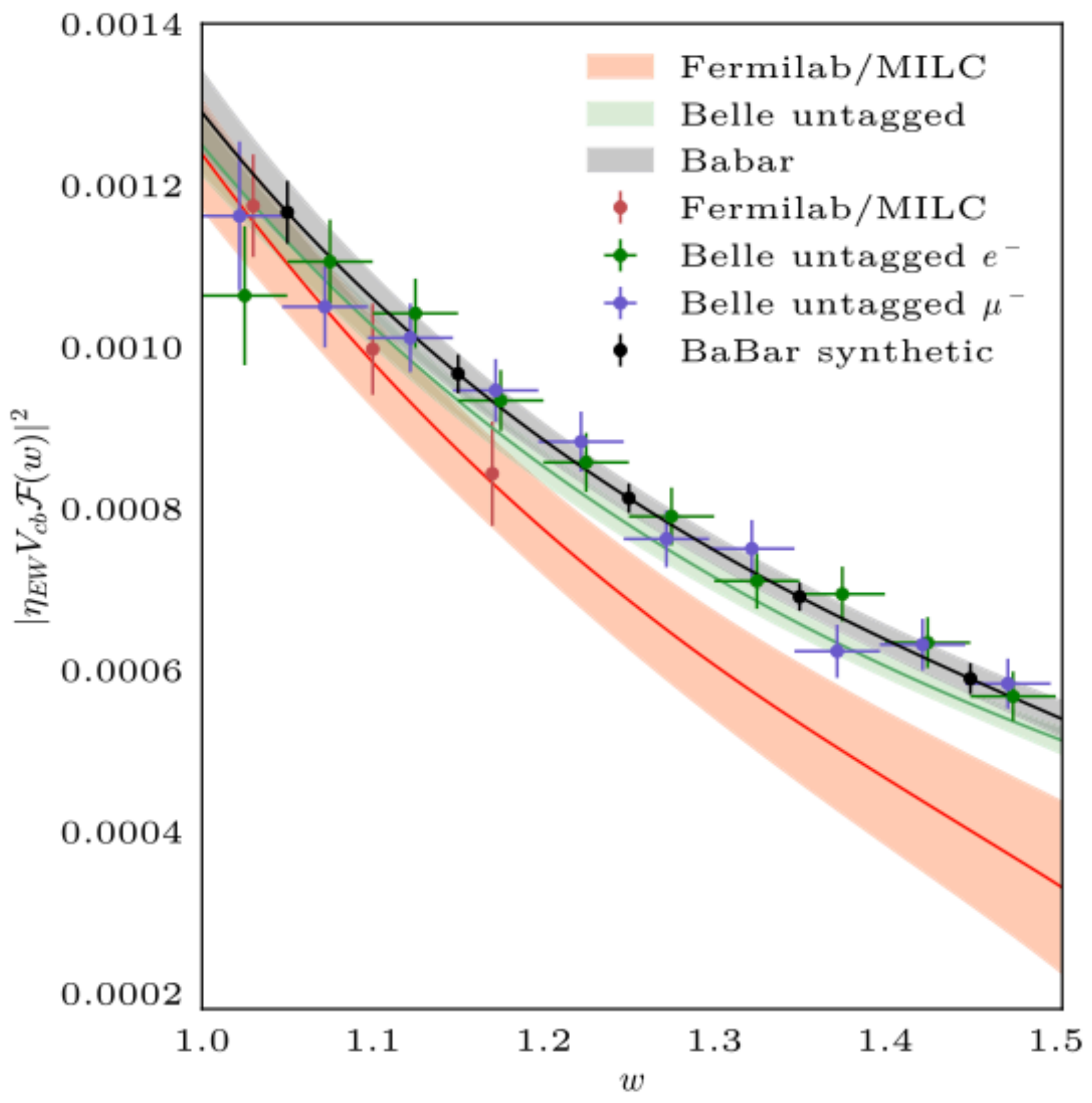
- $B \to \pi$ combined fit with experiment requires PDG inflation
- small tension inclusive vs exclusive

exclusive $b \to c$

A Puzzle in Flavour Physics?
SL meson decay $B \to D^* \ell \nu$

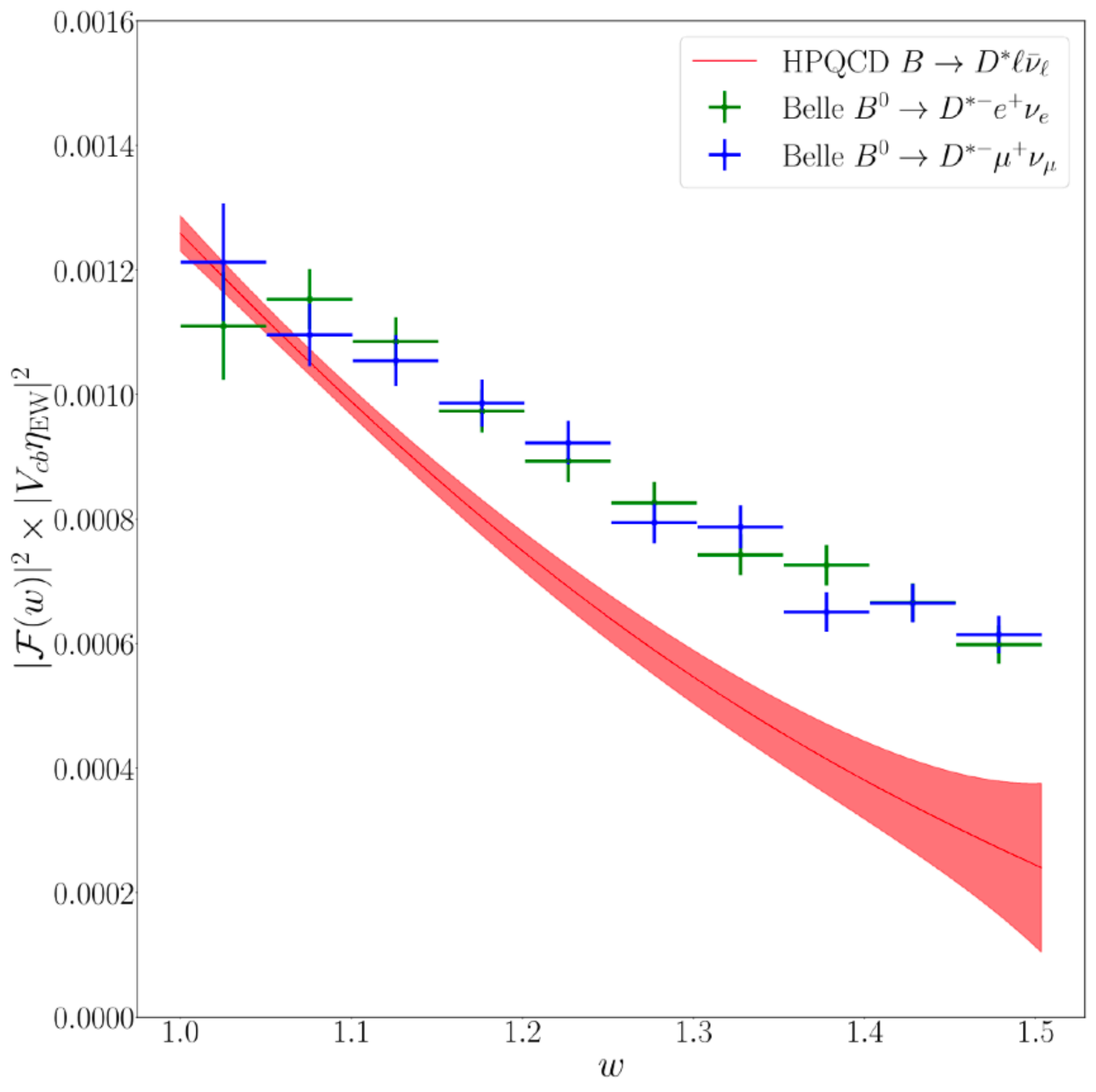
Novelties:
- form-factor shapes from the lattice FNAL/MILC, HPQCD
- JLQCD preliminary

FNAL/MILC EPJC (2022) arXiv:2105.14019



$$|V_{cb}| = (38.40 \pm 0.78) \times 10^{-3}$$
$$(\chi^2/N_{\rm dof}/p) = (\mathbf{128, 84, 0.001})$$

HPQCD arXiv:2304.03137



$$|V_{cb}| = 39.31 \pm (74)$$

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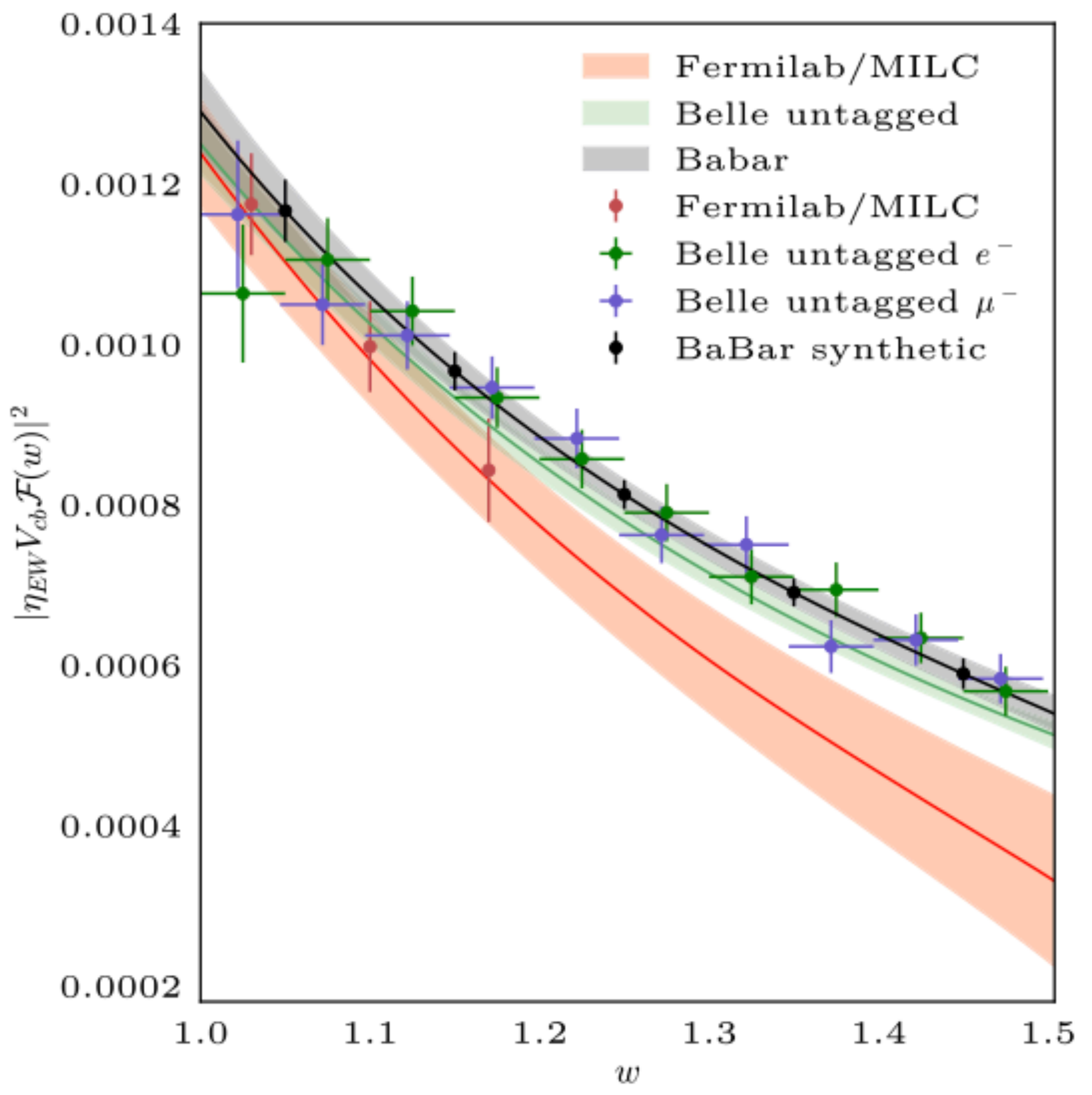

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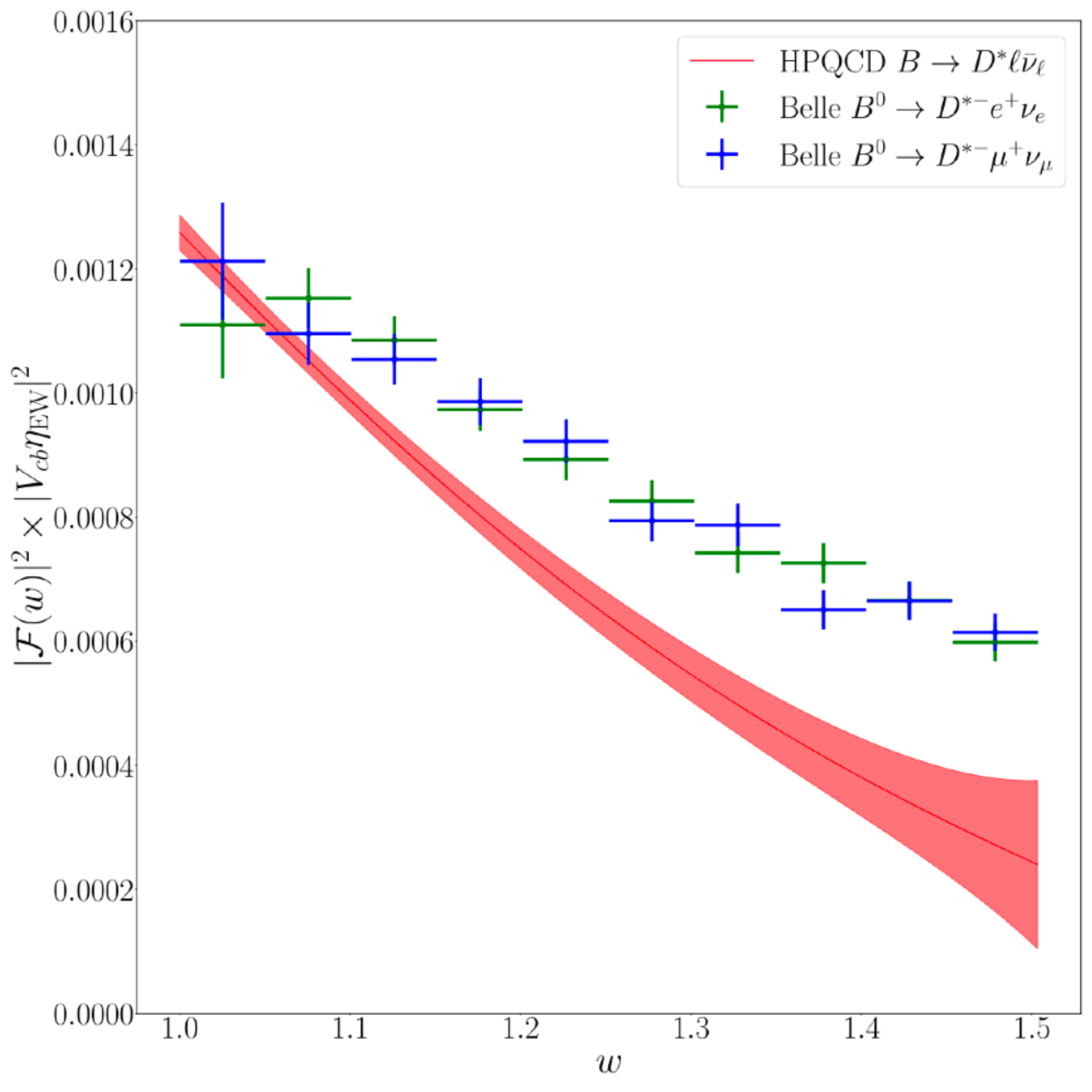

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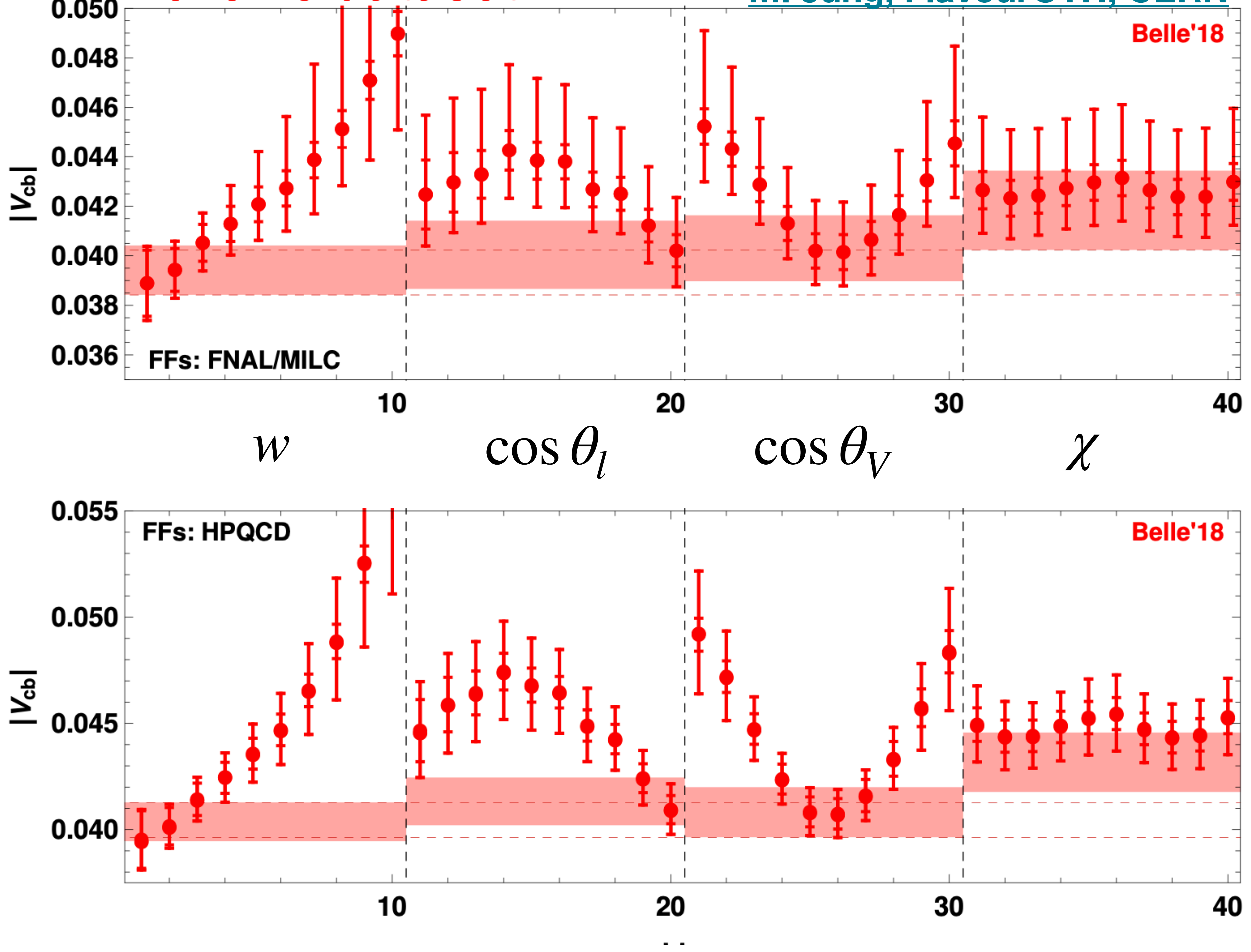
Belle 18 dataset

M. Jung, Flavour@TH, CERN



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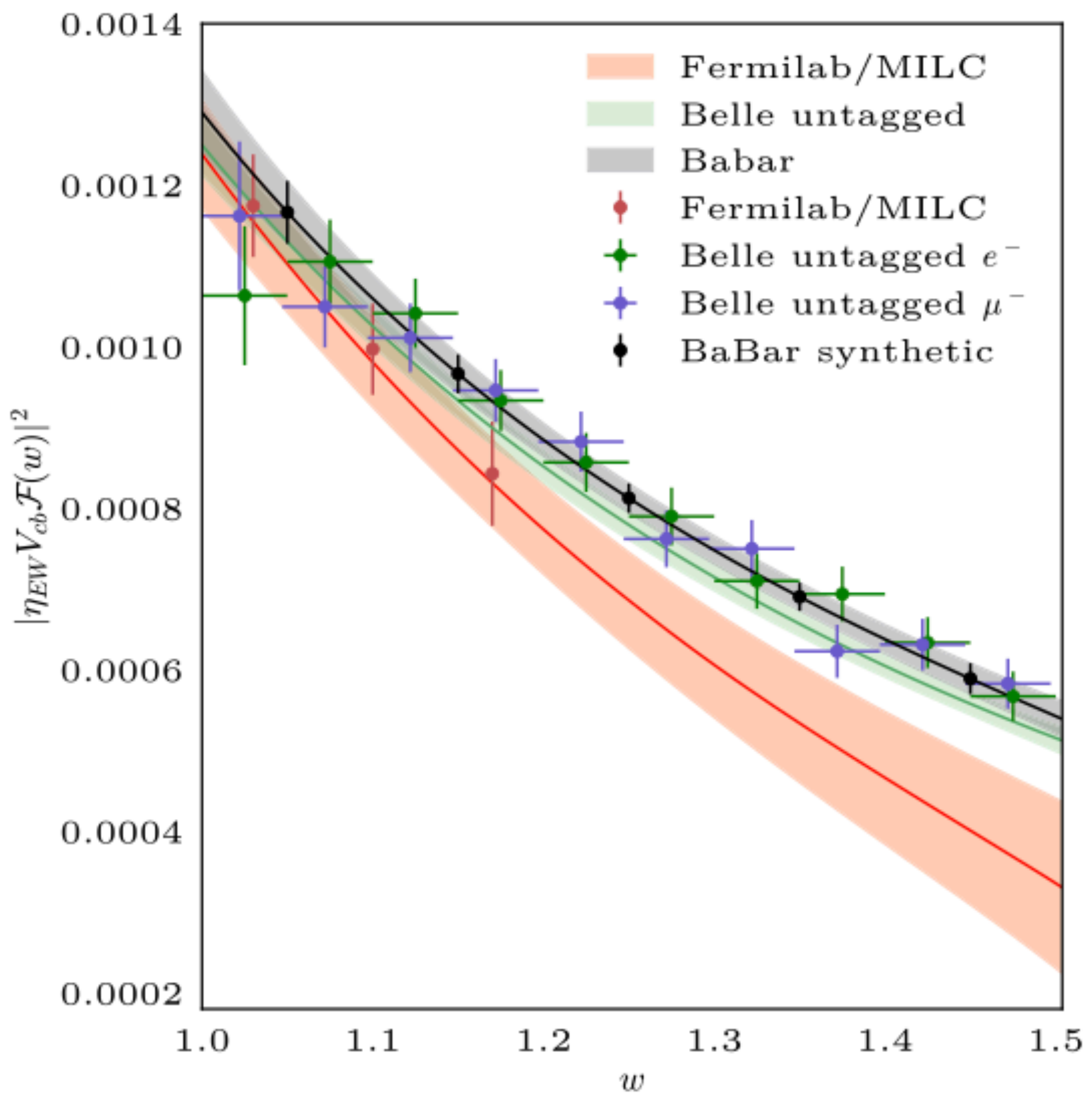

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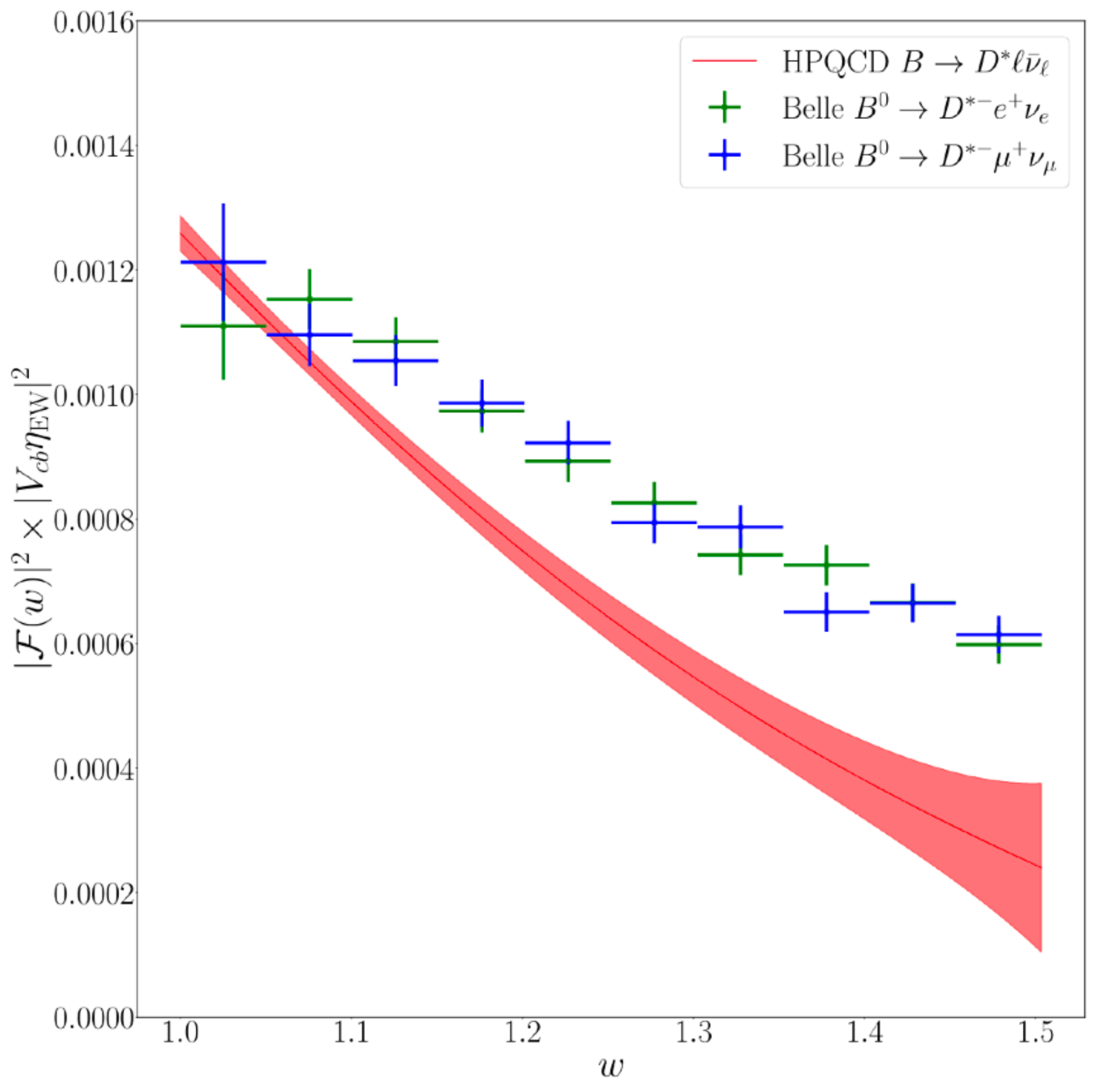

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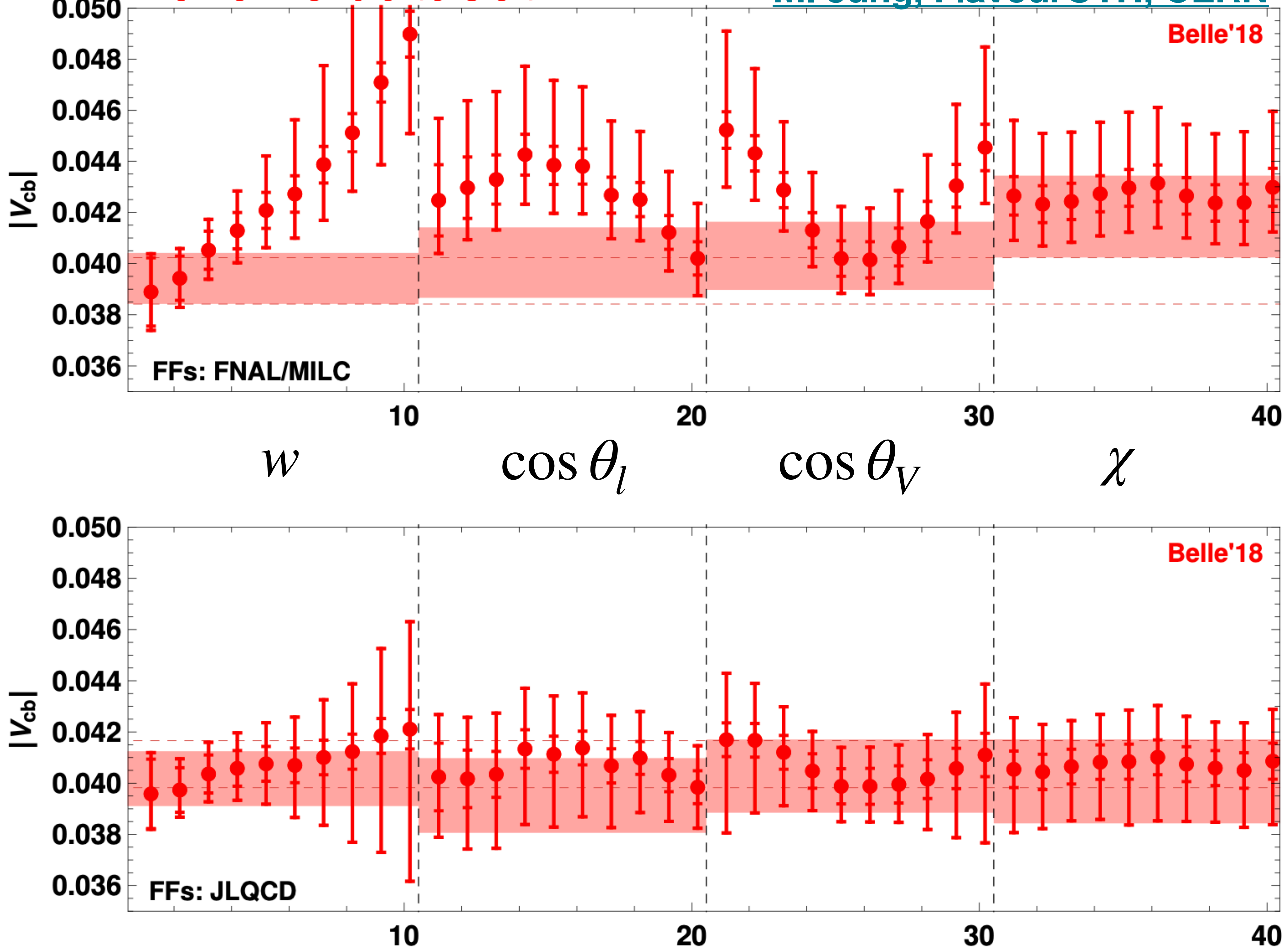


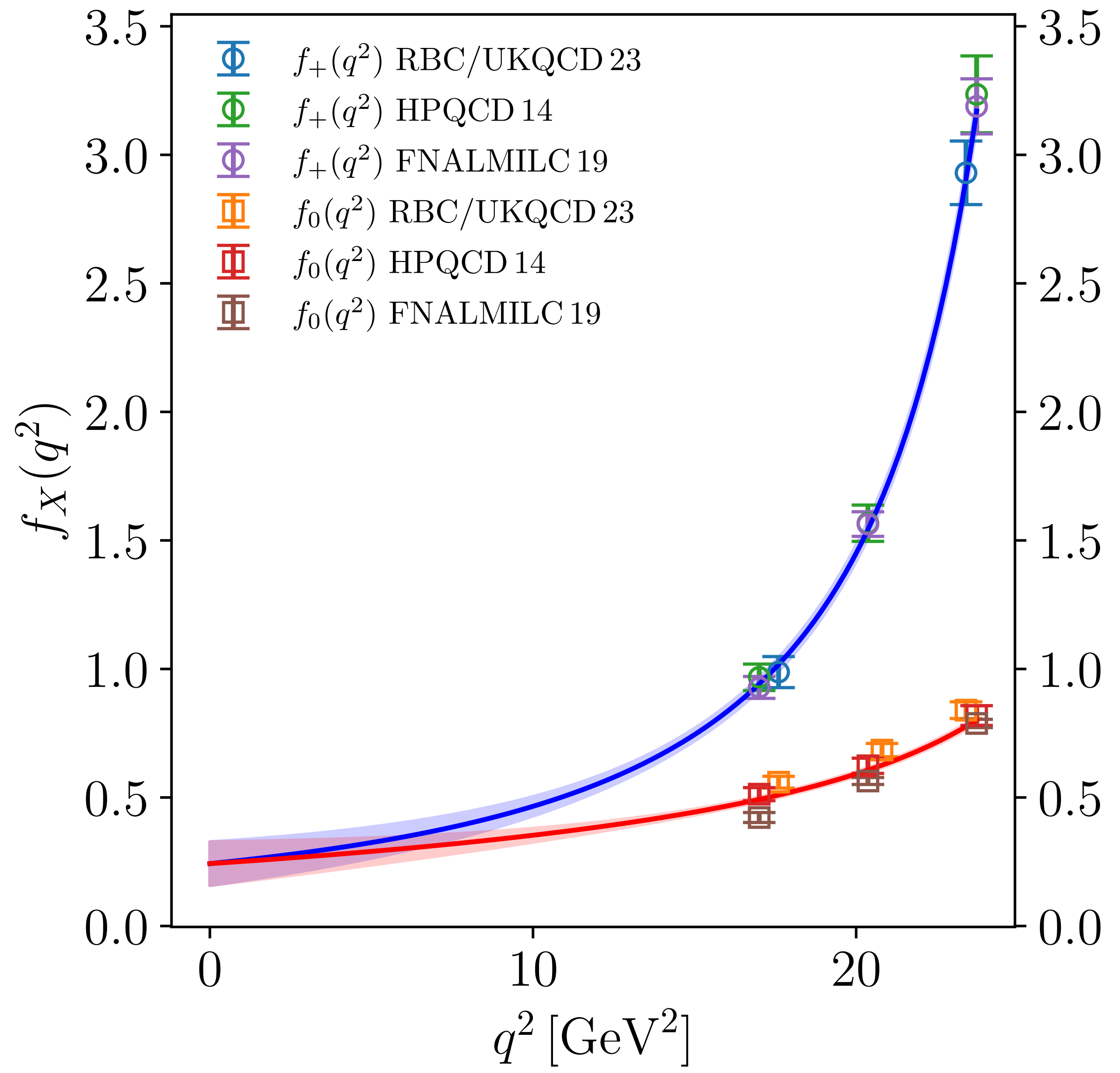
new Belle analysis: arXiv:2301.07529

data now publicly available:

https://www.hepdata.net/record/ins1512299

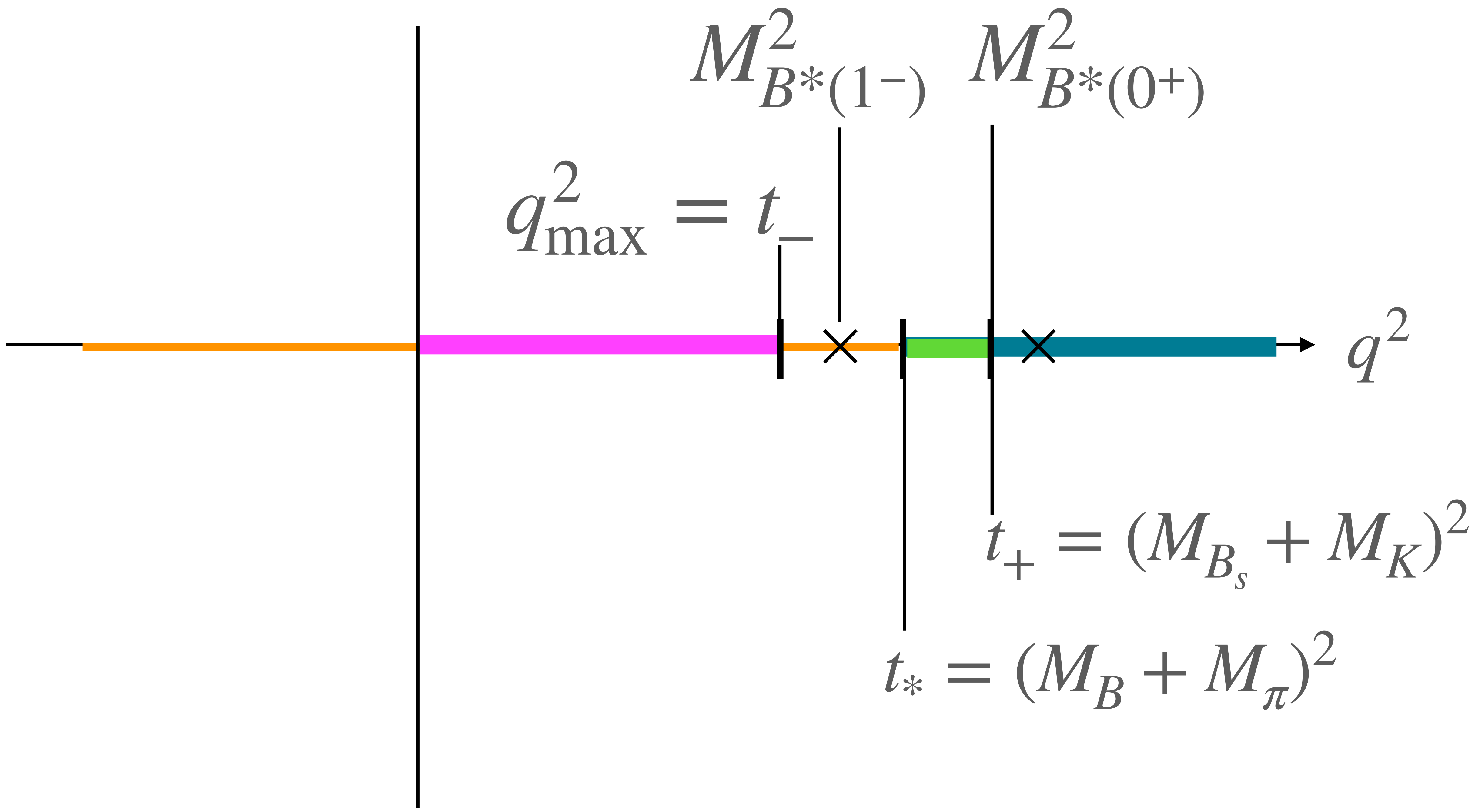
Today: Fedele arXiv:2305.15457

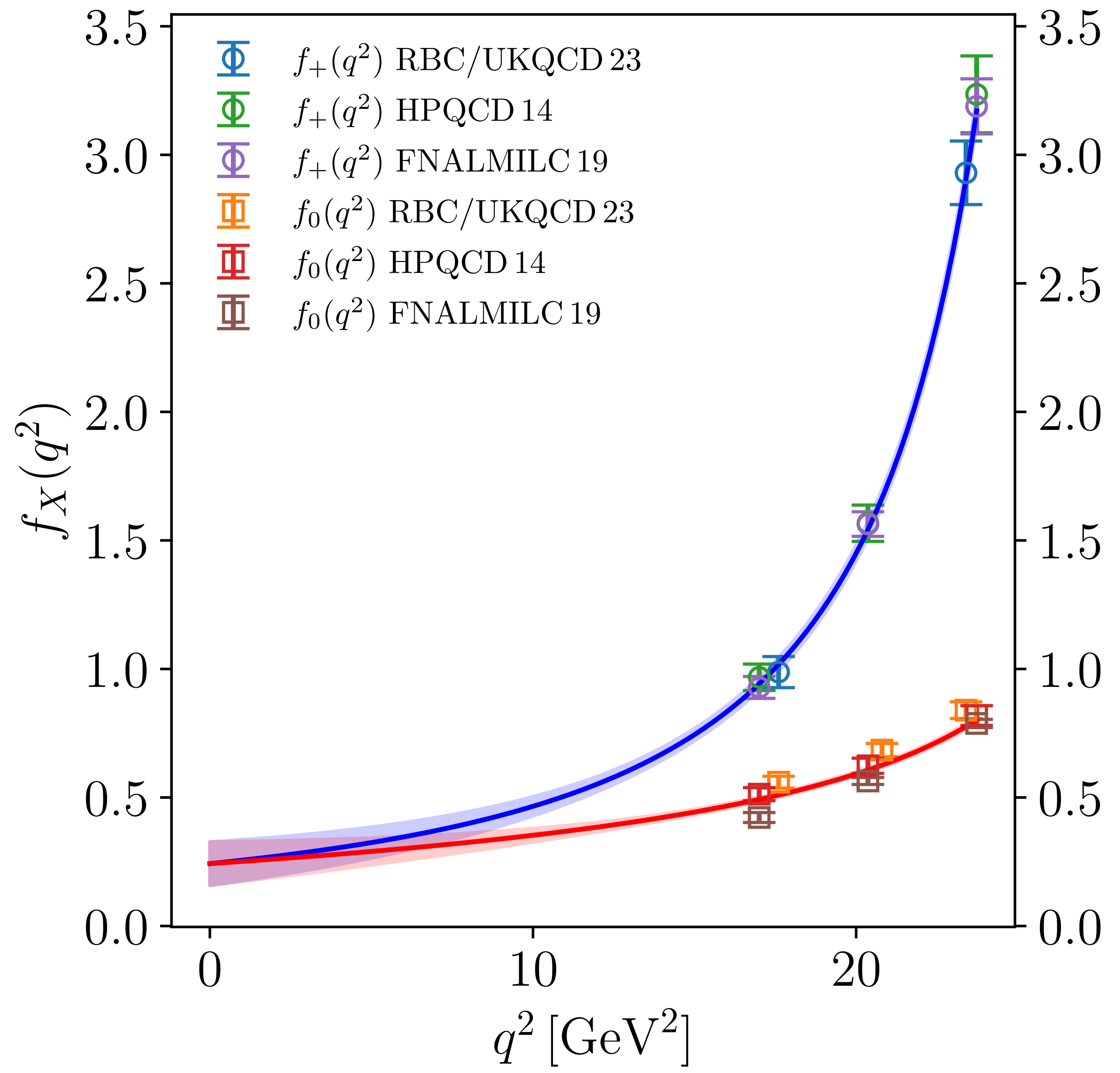
z-fit



Example: $B_s \to K l \nu$ kinematic extrapolation

Use unitarity/analyticity to devise model-independent and fast-converging polynomial expansion:





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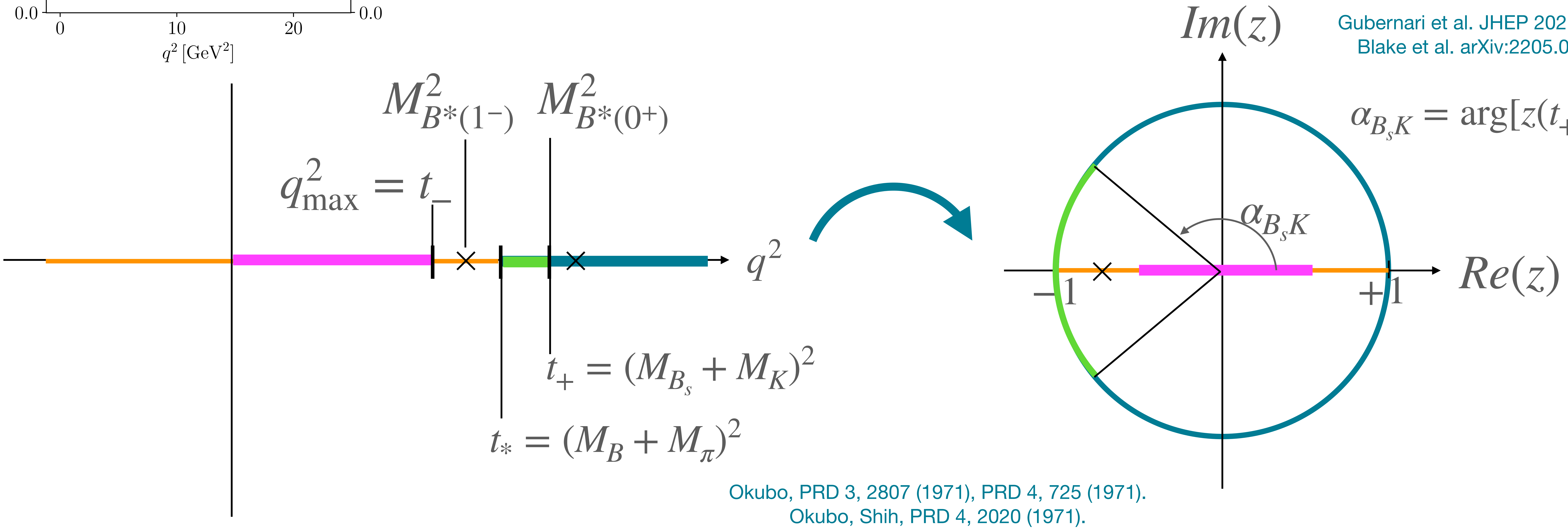
Gubernari et al. JHEP 2021,2022,
Blake et al. arXiv:2205.06041



Okubo, PRD 3, 2807 (1971), PRD 4, 725 (1971).
Okubo, Shih, PRD 4, 2020 (1971).
Boyd, Grinstein, Lebed, PLB 353, 306 (1995). NPB461, 493 (1996). PRD 56, 6895 (1997).

BGL expansion

$$f_X(q_i^2) = \frac{1}{B_X(q_i^2)\phi_X(q_i^2, t_0)} \sum_{n=0}^{K_X-1} a_{X,n} z(q_i^2)^n \qquad X = +,0$$

$$= Z_{XX,in} a_{X,n}$$

Boyd, Grinstein, Lebed, PRL 74 (1995)

Two constraints: kinematic $f_+(0) = f_0(0)$ (eliminates one parameter in combined fit)

unitarity constraint $\frac{1}{2\pi i}\oint_C \frac{dz}{z}\theta_{B_sK}|B_X(q^2)\phi_X(q^2, t_0) f_X(q^2)|^2 \leq 1$

$$a_{X,i}\langle z^i | z^j\rangle a_{X,j} \leq 1$$

Flynn, AJ, Tsang, arXiv:2303.11280

BGL fitting strategies

Frequentist fit:
- $N_{\text{dof}} = N_{\text{data}} - N_{\text{params}} \geq 1 \rightarrow$ in practice truncation of z expansion@low order
- induced systematic difficult to estimate

Bayesian fit:
- **IDEA:** fit full z expansion (i.e. no truncation)
- need regulator to control higher-order coefficients

Model-independent fit

Flynn, AJ, Tsang, arXiv:2303.11280

Compute BGL parameters as expectation values $\langle g(\mathbf{a}) \rangle = \mathcal{N} \int d\mathbf{a}\, g(\mathbf{a})\, \pi(\mathbf{a} \,|\, \mathbf{f}, C_{\mathbf{f}})\, \pi_{\mathbf{a}}$

where *probability for parameters given model and data*

$$\pi(\mathbf{a} \,|\, \mathbf{f}, C_{\mathbf{f}}) \propto \exp\left(-\frac{1}{2}\chi^2(\mathbf{a}, \mathbf{f})\right) \quad \text{where} \quad \chi^2(\mathbf{a}, \mathbf{f}) = (\mathbf{f} - Z\mathbf{a})^T C_{\mathbf{f}}^{-1} (\mathbf{f} - Z\mathbf{a})$$

where *prior knowledge just QFT:*

$$\pi_{\mathbf{a}} \propto \theta\left(1 - |\mathbf{a}_+|^2_{\alpha_{B_sK}}\right) \theta\left(1 - |\mathbf{a}_0|^2_{\alpha_{B_sK}}\right)$$

In practice MC integration: draw samples for **a** from multivariate normal distribution and drop samples not compatible with unitarity

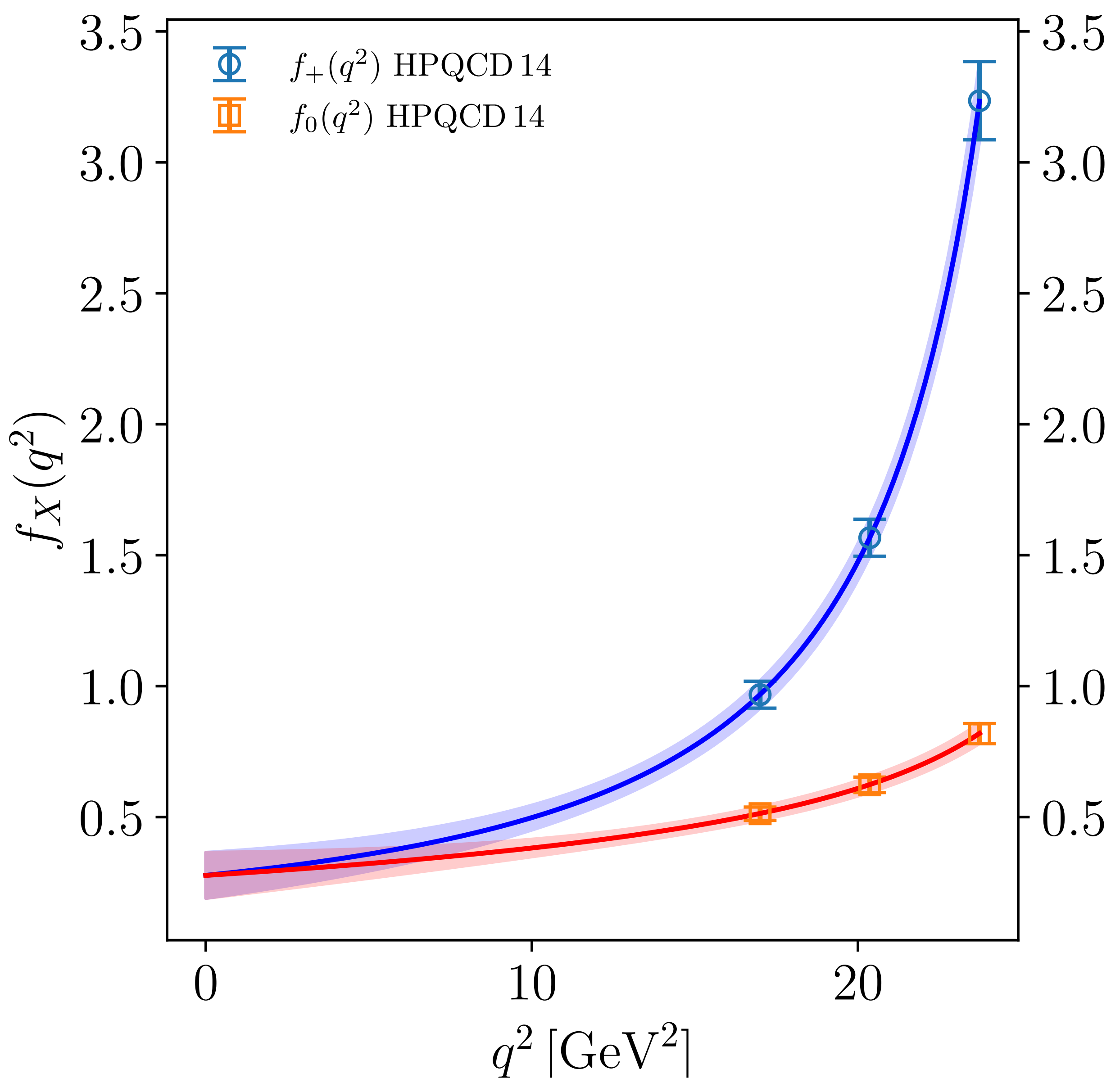
Example: $B_s \to K\ell\nu$

HPQCD 14
PRD 90 (2014) 054506

HPQCD 14 – $\mathbf{a}_+$

K_+	K_0	$a_{+,0}$	$a_{+,1}$	$a_{+,2}$	p	χ^2/N_{dof}	N_{dof}
2	2	0.0270(13)	-0.0792(50)	-	0.03	2.93	3
2	3	0.0273(13)	-0.0760(63)	-	0.02	4.06	2
3	2	0.0257(14)	-0.0805(50)	0.068(31)	0.15	1.89	2
3	3	0.0262(14)	-0.0727(64)	0.096(34)	0.97	0.00	1

Frequentist



HPQCD 14 – $\mathbf{a}_0$

K_+	K_0	$a_{0,0}$	$a_{0,1}$	$a_{0,2}$	p	χ^2/N_{dof}	N_{dof}
2	2	0.0883(44)	-0.250(17)	-	0.03	2.93	3
2	3	0.0880(44)	-0.242(19)	0.053(65)	0.02	4.06	2
3	2	0.0906(45)	-0.240(17)	-	0.15	1.89	2
3	3	0.0908(46)	-0.215(22)	0.138(71)	0.97	0.00	1

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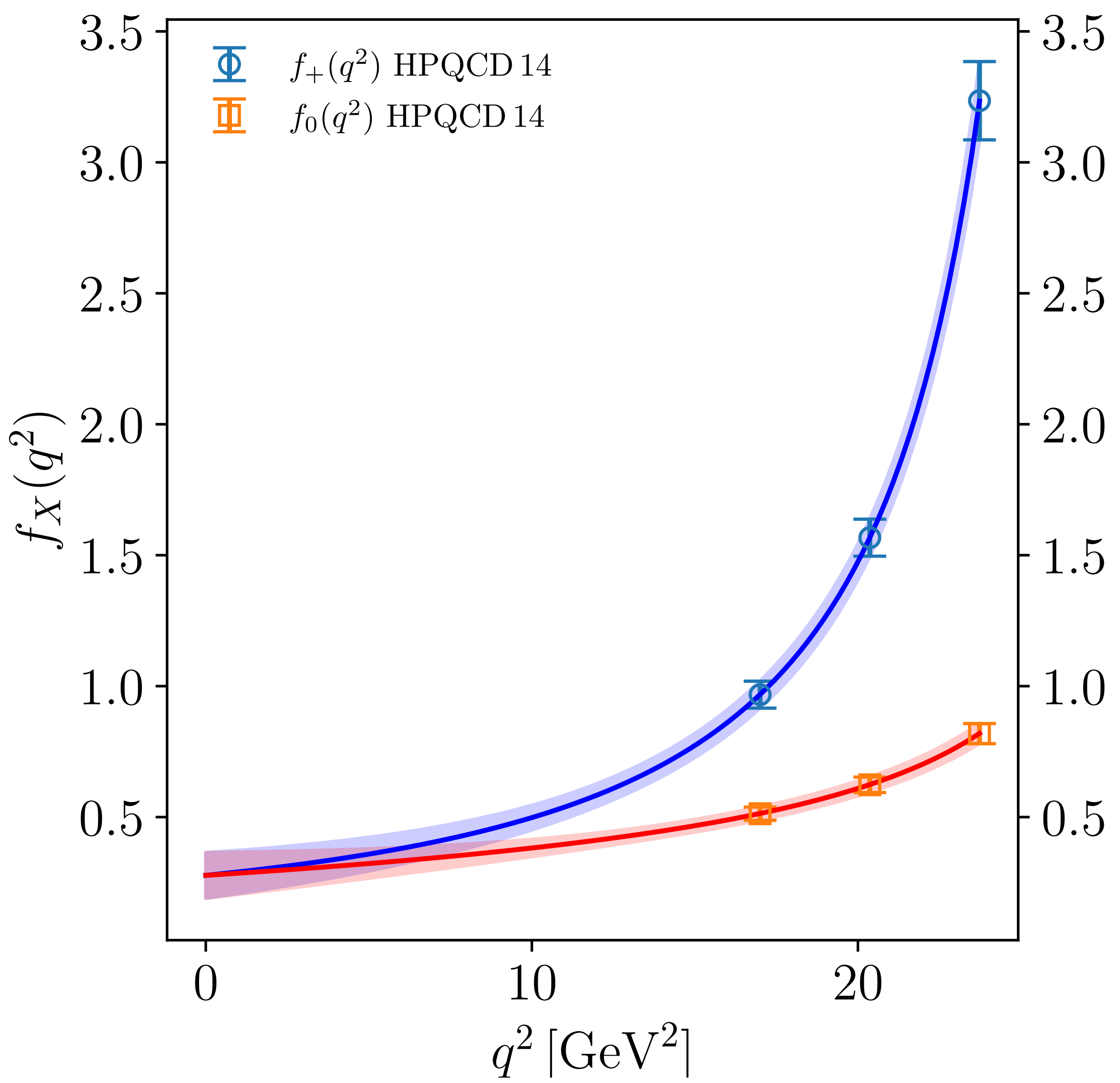

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Flynn, AJ, Tsang, arXiv:2303.11280

HPQCD 14 – $\mathbf{a}_+$

Bayesian

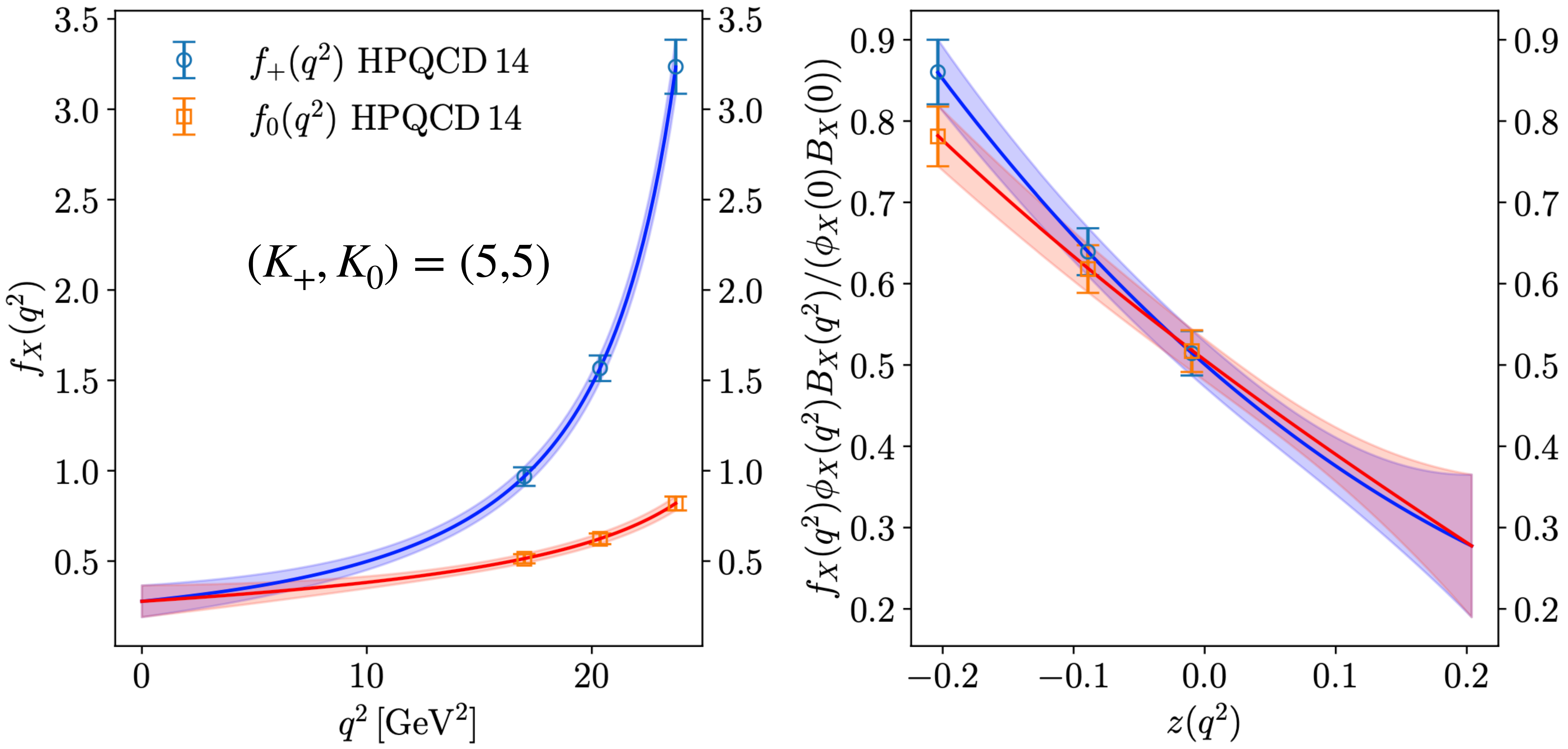
K_+	K_0	$a_{+,0}$	$a_{+,1}$	$a_{+,2}$	$a_{+,3}$	$a_{+,4}$	$a_{+,5}$	$a_{+,6}$	$a_{+,7}$	$a_{+,8}$	$a_{+,9}$
2	2	0.0270(12)	-0.0792(49)	-	-	-	-	-	-	-	-
2	3	0.0273(13)	-0.0761(63)	-	-	-	-	-	-	-	-
3	2	0.0257(14)	-0.0805(49)	0.069(30)	-	-	-	-	-	-	-
3	3	0.0261(14)	-0.0728(64)	0.096(34)	-	-	-	-	-	-	-
3	4	0.0261(14)	-0.0728(76)	0.096(39)	-	-	-	-	-	-	-
4	3	0.0261(14)	-0.0729(68)	0.096(35)	0.008(90)	-	-	-	-	-	-
4	4	0.0261(14)	-0.0730(77)	0.091(62)	-0.02(20)	-	-	-	-	-	-
5	5	0.0262(15)	-0.0735(79)	0.084(67)	-0.03(19)	0.03(68)	-	-	-	-	-
6	6	0.0261(14)	-0.0735(79)	0.086(69)	-0.03(19)	-0.00(64)	0.01(65)	-	-	-	-
7	7	0.0262(14)	-0.0732(84)	0.088(69)	-0.02(18)	0.01(65)	0.02(73)	-0.03(70)	-	-	-
8	8	0.0261(14)	-0.0732(80)	0.089(72)	-0.02(18)	-0.00(66)	0.03(86)	-0.04(90)	0.03(73)	-	-
9	9	0.0261(14)	-0.0729(84)	0.095(75)	-0.02(19)	-0.04(68)	0.1(1.0)	-0.1(1.2)	0.1(1.1)	-0.06(79)	-
10	10	0.0261(14)	-0.0726(89)	0.101(79)	-0.01(20)	-0.09(73)	0.2(1.3)	-0.3(1.7)	0.2(1.8)	-0.2(1.4)	0.08(87)

HPQCD 14 – $\mathbf{a}_0$

Bayesian

K_+	K_0	$a_{0,0}$	$a_{0,1}$	$a_{0,2}$	$a_{0,3}$	$a_{0,4}$	$a_{0,5}$	$a_{0,6}$	$a_{0,7}$	$a_{0,8}$	$a_{0,9}$
2	2	0.0883(44)	-0.250(17)	-	-	-	-	-	-	-	-
2	3	0.0880(44)	-0.243(19)	0.052(65)	-	-	-	-	-	-	-
3	2	0.0907(46)	-0.240(17)	-	-	-	-	-	-	-	-
3	3	0.0906(44)	-0.215(22)	0.137(73)	-	-	-	-	-	-	-
3	4	0.0907(47)	-0.215(22)	0.14(11)	-0.01(31)	-	-	-	-	-	-
4	3	0.0907(45)	-0.214(22)	0.139(72)	-	-	-	-	-	-	-
4	4	0.0907(46)	-0.215(25)	0.12(19)	-0.08(60)	-	-	-	-	-	-
5	5	0.0909(46)	-0.218(25)	0.10(19)	-0.12(55)	0.04(63)	-	-	-	-	-
6	6	0.0907(45)	-0.217(25)	0.10(19)	-0.11(53)	0.06(66)	-0.02(66)	-	-	-	-
7	7	0.0907(46)	-0.217(26)	0.11(20)	-0.08(51)	0.03(73)	0.03(81)	-0.04(70)	-	-	-
8	8	0.0908(46)	-0.217(25)	0.11(20)	-0.08(50)	-0.01(84)	0.1(1.0)	-0.09(96)	0.08(74)	-	-
9	9	0.0907(46)	-0.215(25)	0.13(22)	-0.05(50)	-0.06(95)	0.2(1.4)	-0.2(1.5)	0.1(1.2)	-0.05(82)	-
10	10	0.0907(46)	-0.214(27)	0.15(24)	-0.03(49)	-0.2(1.1)	0.4(1.8)	-0.5(2.2)	0.4(2.1)	-0.3(1.6)	0.13(90)

Summary z-fits



- new, clean, truncation and bias-free form-factor parameterisation
- works for any hadronic form factor
- use complementarity of frequentist and Bayesian analysis

Flynn, AJ, Tsang, arXiv:2303.11280

Also have a look:

- Dispersive-matrix method, Di Carlo et al. PRD 2021, arXiv:2105.02497
- Self-consistency checks of z expansinon, Simons, Gustafson, Meurice arXiv:2304.13045

Code: BFF

Flynn, AJ, Tsang, arXiv:2303.11280

Python3 available via github/Zenodo

https://github.com/andreasjuettner/BFF **https://zenodo.org/record/7799543#.ZEezTy8Ro80**

```
        }
##########################################################################################
# specify input for BGL fit
##########################################################################################
input_dict = {
         'decay':        'Btopi',
         'Mi':           pc.mBphys,      # initial-state mass
         'Mo':           pc.mpiphys,     # final-state mass
         'sigma':        .5,             # sigma for prior in algorithm
         'Kp':           4,              # target Kp (BGL truncation) - can be changed later
         'K0':           4,              # target K0 (BGL truncation) - can be changed later
         'tstar':        '29.349570696829012', # value of t*
         't0':           'self.tstar - np.sqrt(self.tstar*(self.tstar-self.tm))', # definition of t0
         'chip':         pc.chip_Btopi,  # susceptibility fp
         'chi0':         pc.chi0_Btopi,  # susceptibility f0
         'mpolep':       [pc.mBstar],    # fplus pole
         'mpole0':       [],             # fzero pole (no pole for BstoK)
         'N'    :        N,              # number of desired samples
         'outer_p':      [3./2,'48*np.pi',3,2], # specs for outer function fp
         'outer_0':      [3./2,'16*np.pi/(self.tp*self.tm)',1,1], # specs for outer function f0
         'seed':         123,            # RNG seed
```

```
input_data = {
          'RBCUKQCD 23 lat':
          {
           'data type':    'ff',
           'label'        :'RBC/UKQCD 23',
           'Np':           2,
           'N0':           3,
           'qsqp':         np.array([17.60,23.40]),
           'qsq0':         np.array([17.60,20.80,23.40]),
           'fp':           fparray,
           'f0':           f0array,
           'Cff':          cov_array
          }
}
```

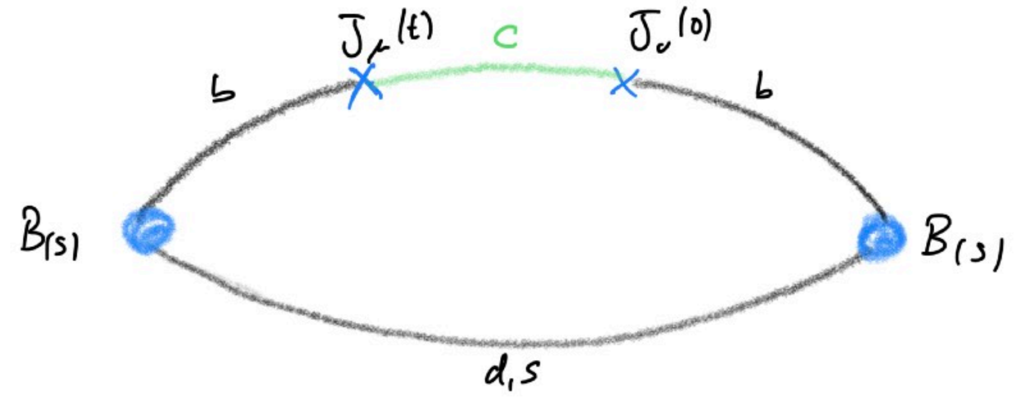
inclusive $b \to c$

$b \to c$ inclusive on the lattice

long-standing tension inclusive vs. exclusive CKM determination → find new ways to look at it

$$\Gamma(B_s \to X_c l\nu) = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \int_0^{\mathbf{q}^2_{\max}} d\mathbf{q}^2 \sqrt{\mathbf{q}^2}\, \bar{X}(\mathbf{q})$$

$$\bar{X}(\mathbf{q}) = \sum_k c_{\mu\nu,k}(\mathbf{q})\, C_{\mu\nu}(ak, \mathbf{q})$$



Gambino, Hashimoto, PRL (2020) arXiv:2005.13730
Hansen, Meyer, Robaina, PRD (2017), arXiv:1704.08993

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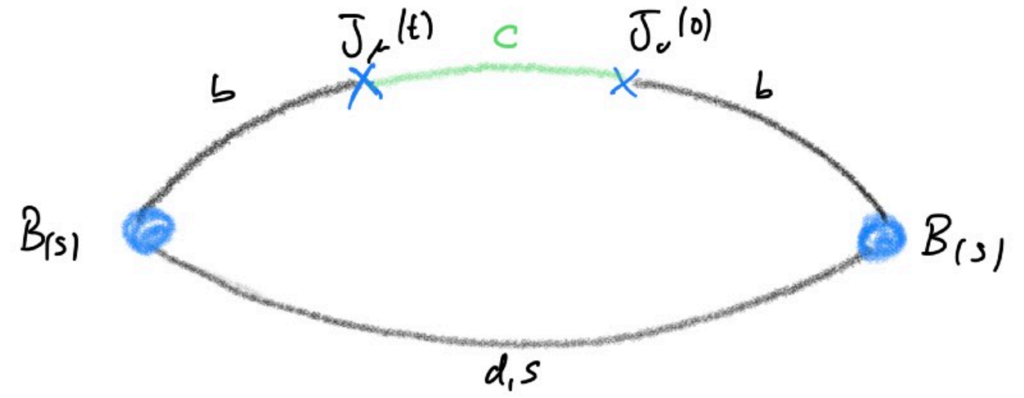
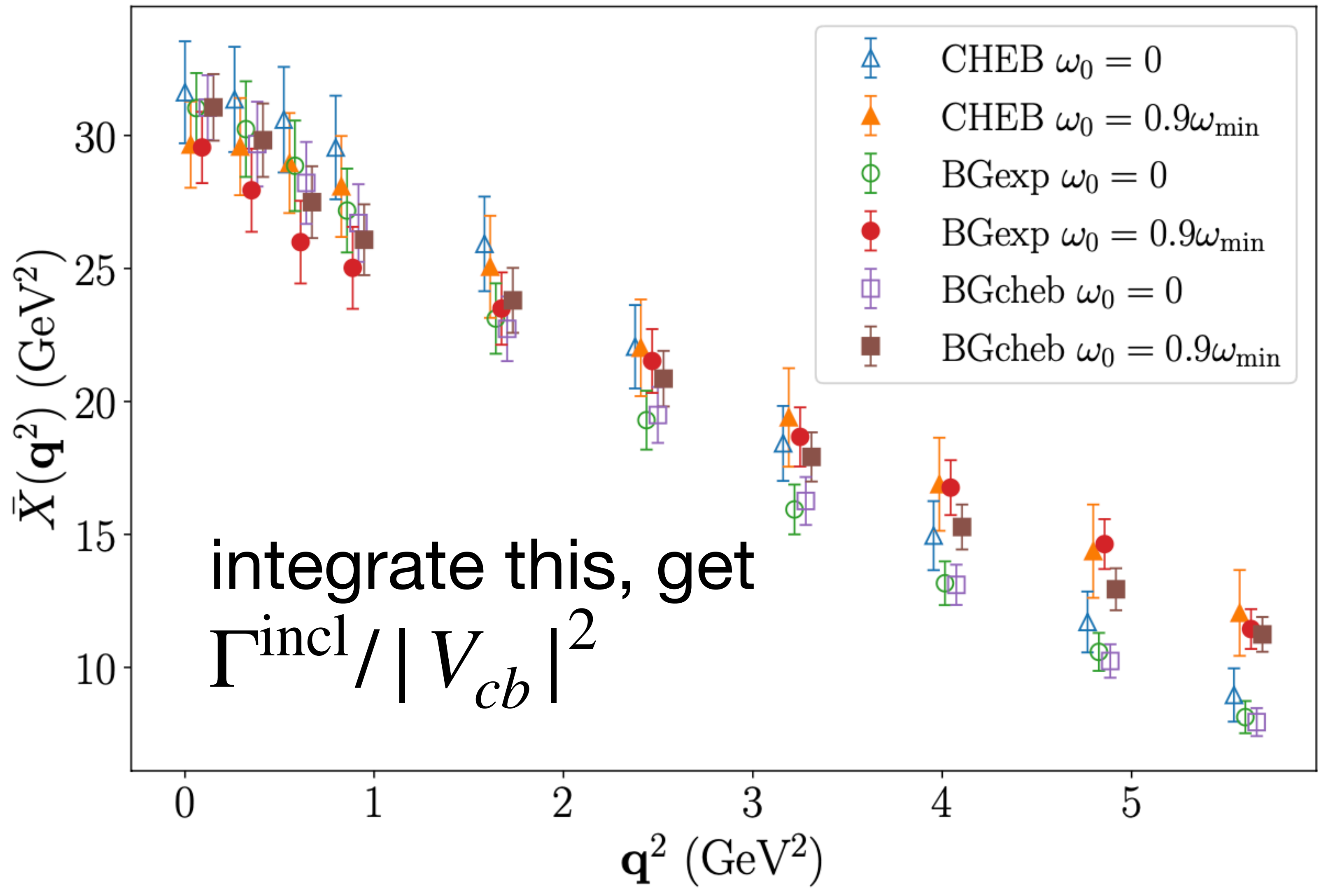
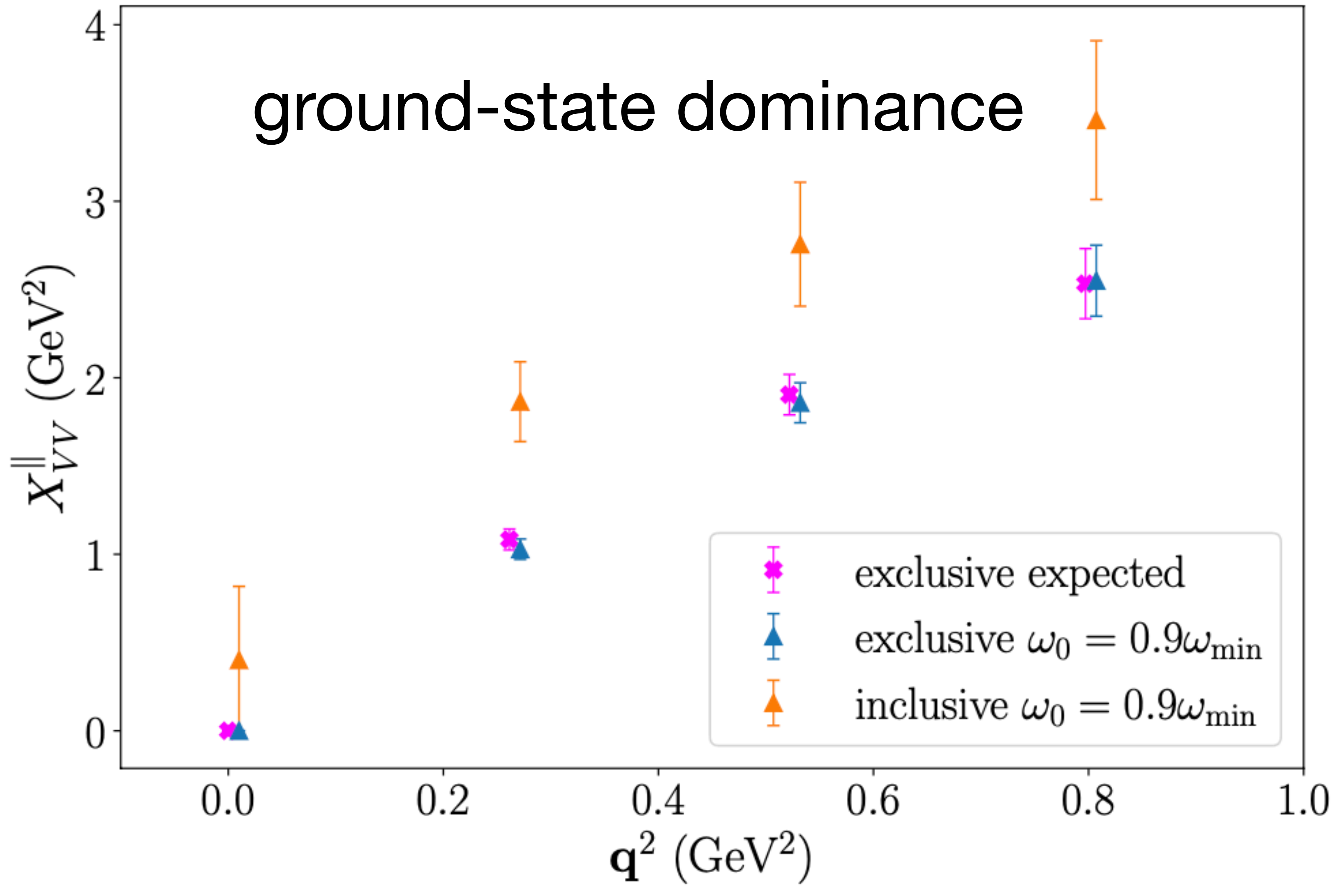
$$\bar{X}(\mathbf{q}) = \sum_k c_{\mu\nu,k}(\mathbf{q})\, C_{\mu\nu}(ak, \mathbf{q})$$



Gambino, Hashimoto, PRL (2020) arXiv:2005.13730
Hansen, Meyer, Robaina, PRD (2017), arXiv:1704.08993

Barone, Hashimoto, AJ, Kaneko, Kellerman arXiv:2305.14092





- PRELIMINARY study
- concentrating on controlling systematics
- could be interesting to define new observables
- test continuum ↔ lattice
- Next: first comprehensive study with error budget and prediction

Summary

- New results for exclusive $b \to u$ and $b \to c$ transitions
- Some tensions in lattice data require attention
- new ideas for truncation-independent form factor fit
- inclusive decays on the lattice exciting new area of research

Other interesting work I would have liked to cover:
$B \to \rho$, $B \to K^*$ beyond narrow-width approximation (Leskovec, **Lattice22 talk**)

BACKUP

Examples: $B_s \to K\ell\nu$

pheno results

K_+	K_0	$f(q^2=0)$	$R^{\rm impr}_{B_s\to K}$	$R_{B_s\to K}$	$\frac{\Gamma^\tau}{\lvert V_{ub}\rvert^2}\,[\frac{1}{\rm ps}]$	$\frac{\Gamma^\mu}{\lvert V_{ub}\rvert^2}\,[\frac{1}{\rm ps}]$	$V^{\rm low}_{\rm CKM}$	$V^{\rm high}_{\rm CKM}$	$V^{\rm full}_{\rm CKM}$
2	2	0.208(25)	1.524(37)	0.727(25)	4.51(45)	6.23(76)	0.00383(47)	0.00352(35)	0.00363(37)
2	3	0.226(34)	1.511(41)	0.704(39)	4.67(49)	6.67(97)	0.00361(53)	0.00344(34)	0.00349(38)
3	2	0.233(27)	1.609(58)	0.733(27)	4.44(45)	6.08(77)	0.00368(45)	0.00367(37)	0.00367(38)
3	3	0.293(41)	1.592(57)	0.664(40)	4.84(51)	7.3(1.1)	0.00310(44)	0.00349(35)	0.00333(36)
3	4	0.293(56)	1.593(60)	0.667(59)	4.85(58)	7.4(1.4)	0.00313(55)	0.00349(37)	0.00338(40)
4	3	0.294(42)	1.594(60)	0.663(40)	4.85(52)	7.4(1.1)	0.00309(44)	0.00348(36)	0.00332(36)
4	4	0.285(92)	1.593(60)	0.677(88)	4.83(62)	7.3(1.7)	0.00328(86)	0.00350(38)	0.00346(42)
5	5	0.277(88)	1.595(62)	0.685(85)	4.81(62)	7.2(1.7)	0.00333(85)	0.00351(38)	0.00348(42)
6	6	0.277(88)	1.592(63)	0.685(86)	4.79(63)	7.2(1.7)	0.00335(88)	0.00350(38)	0.00348(43)
7	7	0.282(89)	1.592(60)	0.680(87)	4.82(64)	7.3(1.7)	0.00332(89)	0.00350(38)	0.00347(43)
8	8	0.283(88)	1.594(61)	0.679(85)	4.83(64)	7.3(1.7)	0.00330(85)	0.00351(37)	0.00347(41)
9	9	0.289(91)	1.594(62)	0.674(88)	4.85(64)	7.4(1.8)	0.00327(89)	0.00350(38)	0.00347(42)
10	10	0.293(95)	1.593(60)	0.670(91)	4.87(67)	7.5(1.9)	0.00325(92)	0.00349(38)	0.00346(42)

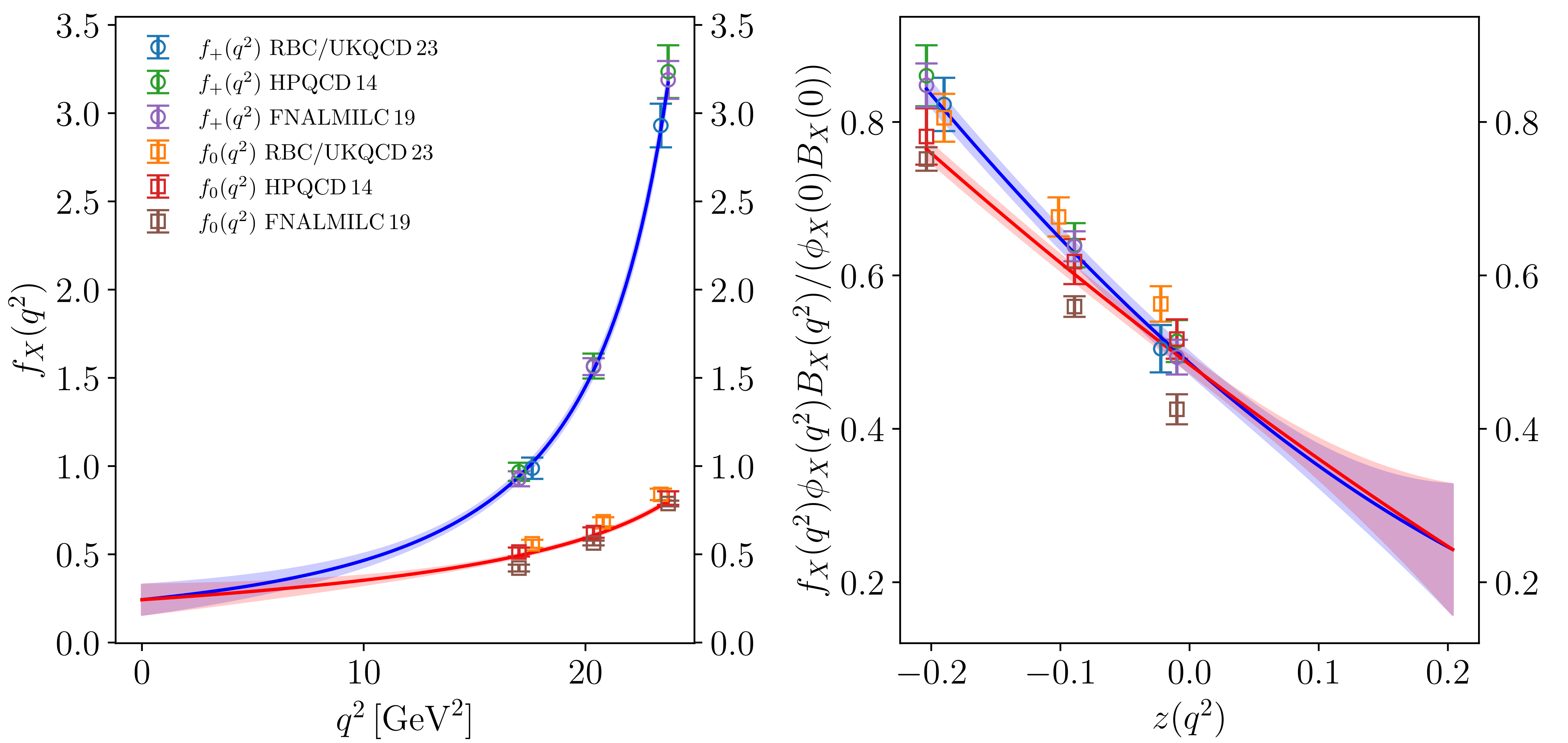
K_+	K_0	$I[\mathcal{A}^\tau_{\rm FB}]\,[\frac{1}{\rm ps}]$	$I[\mathcal{A}^\mu_{\rm FB}]\,[\frac{1}{\rm ps}]$	$\bar{\mathcal{A}}^\tau_{\rm FB}$	$\bar{\mathcal{A}}^\mu_{\rm FB}$	$I[\mathcal{A}^\tau_{\rm pol}]\,[\frac{1}{\rm ps}]$	$I[\mathcal{A}^\mu_{\rm pol}]\,[\frac{1}{\rm ps}]$	$\bar{\mathcal{A}}^\tau_{\rm pol}$	$\bar{\mathcal{A}}^\mu_{\rm pol}$
2	2	1.22(13)	0.0278(51)	0.2708(37)	0.00443(34)	0.74(15)	6.15(75)	0.164(29)	0.98767(96)
2	3	1.26(14)	0.0314(70)	0.2709(38)	0.00465(44)	0.81(18)	6.59(96)	0.173(31)	0.9872(12)
3	2	1.23(13)	0.0319(59)	0.2780(43)	0.00524(51)	0.46(19)	5.99(76)	0.103(40)	0.9852(15)
3	3	1.36(15)	0.045(10)	0.2814(48)	0.00612(66)	0.53(20)	7.2(1.1)	0.110(40)	0.9830(18)
3	4	1.37(17)	0.046(14)	0.2814(50)	0.00611(83)	0.53(22)	7.3(1.3)	0.109(41)	0.9830(22)
4	3	1.37(15)	0.046(10)	0.2815(50)	0.00616(71)	0.53(22)	7.2(1.1)	0.109(42)	0.9829(20)
4	4	1.36(19)	0.046(21)	0.2810(69)	0.0060(15)	0.53(21)	7.2(1.7)	0.109(42)	0.9834(41)
5	5	1.35(19)	0.044(20)	0.2806(67)	0.0058(15)	0.53(22)	7.1(1.6)	0.109(44)	0.9837(39)
6	6	1.35(20)	0.044(20)	0.2803(69)	0.0058(15)	0.53(22)	7.1(1.7)	0.111(44)	0.9838(39)
7	7	1.35(20)	0.045(20)	0.2806(69)	0.0059(15)	0.53(21)	7.2(1.7)	0.111(43)	0.9835(39)
8	8	1.36(20)	0.045(20)	0.2808(69)	0.0059(15)	0.53(22)	7.2(1.7)	0.109(44)	0.9835(39)
9	9	1.36(20)	0.047(21)	0.2812(71)	0.0060(15)	0.53(22)	7.3(1.7)	0.109(44)	0.9832(40)
10	10	1.37(21)	0.048(23)	0.2815(72)	0.0061(15)	0.53(22)	7.4(1.8)	0.109(43)	0.9831(41)

$B_s \to K\ell\nu$ — continued

Easy to combine independent or correlated data sets:

K_+	K_0	$a_{+,0}$	$a_{+,1}$	$a_{+,2}$	$a_{+,3}$	$a_{+,4}$	p	χ^2/N_{dof}	N_{dof}
2	2	0.02641(58)	-0.0824(26)	-	-	-	0.00	5.15	14
2	3	0.02668(68)	-0.0811(31)	-	-	-	0.00	5.50	13
3	2	0.02477(68)	-0.0829(26)	0.054(12)	-	-	0.00	3.95	13
3	3	0.02534(73)	-0.0792(31)	0.062(12)	-	-	0.00	3.89	12
3	4	0.02534(73)	-0.0781(34)	0.067(14)	-	-	0.00	4.19	11
4	3	0.02535(73)	-0.0776(38)	0.074(20)	0.023(30)	-	0.00	4.19	11
4	4	0.02592(97)	-0.033(50)	0.69(69)	2.1(2.3)	-	0.00	4.53	10
5	5	0.0266(10)	0.052(65)	2.21(97)	11.1(5.6)	17.2(15.1)	0.00	5.04	8

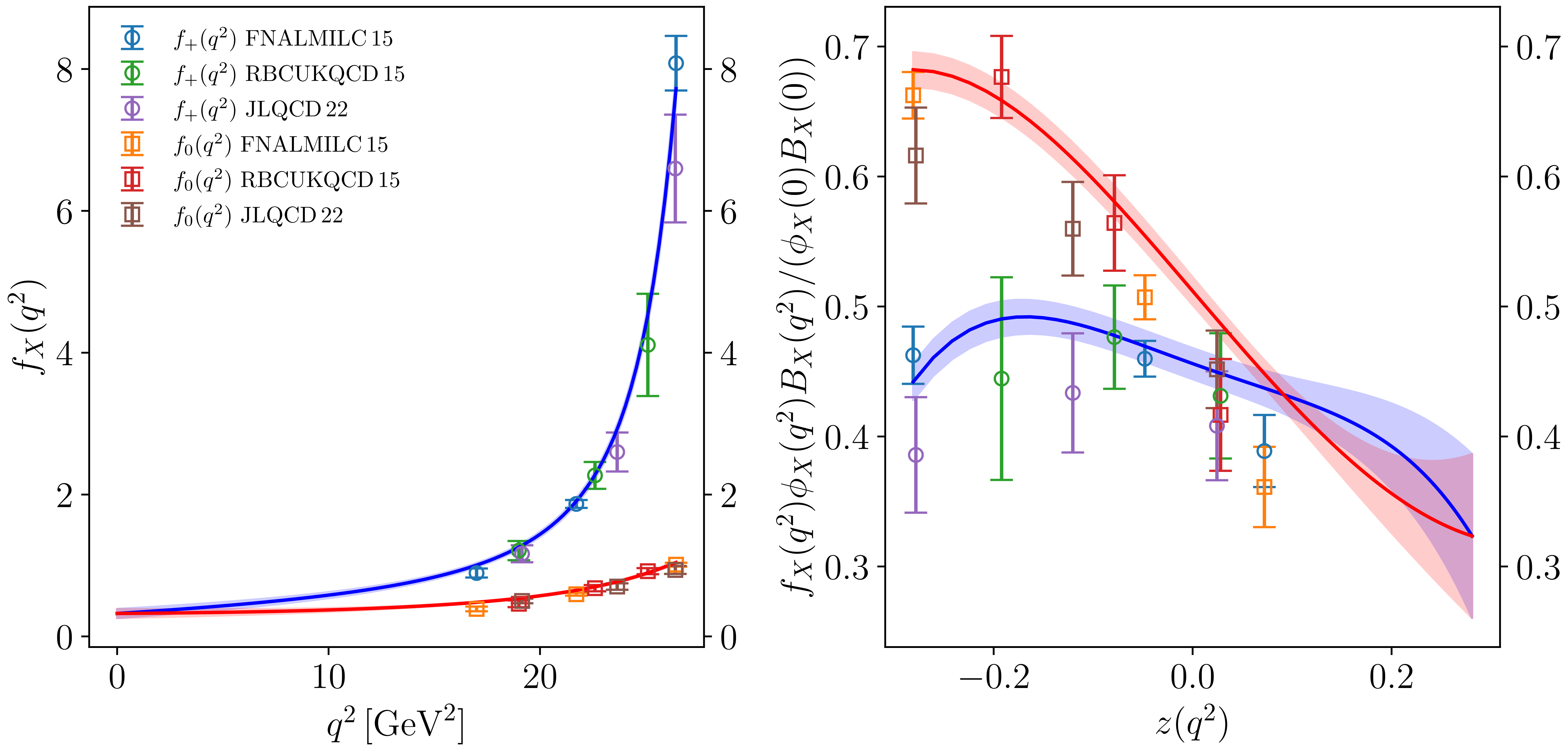
K_+	K_0	$a_{0,0}$	$a_{0,1}$	$a_{0,2}$	$a_{0,3}$	$a_{0,4}$	p	χ^2/N_{dof}	N_{dof}
2	2	0.0854(17)	-0.2565(75)	-	-	-	0.00	5.15	14
2	3	0.0856(18)	-0.2527(91)	0.021(27)	-	-	0.00	5.50	13
3	2	0.0858(18)	-0.2501(77)	-	-	-	0.00	3.95	13
3	3	0.0864(18)	-0.2379(95)	0.061(28)	-	-	0.00	3.89	12
3	4	0.0869(19)	-0.231(13)	0.067(29)	-0.08(10)	-	0.00	4.19	11
4	3	0.0869(19)	-0.229(15)	0.091(48)	-	-	0.00	4.19	11
4	4	0.0887(27)	-0.08(17)	2.2(2.4)	7.0(7.9)	-	0.00	4.53	10
5	5	0.0887(28)	0.07(20)	6.1(3.3)	41.5(19.0)	93.3(44.0)	0.00	5.04	8



Bayesian and frequentist provide complementary information — consider both simultaneously!

Conclusion: World lattice data for $B_s \to K\ell\nu$ is in quite bad shape…

$B \to \pi \ell \nu$

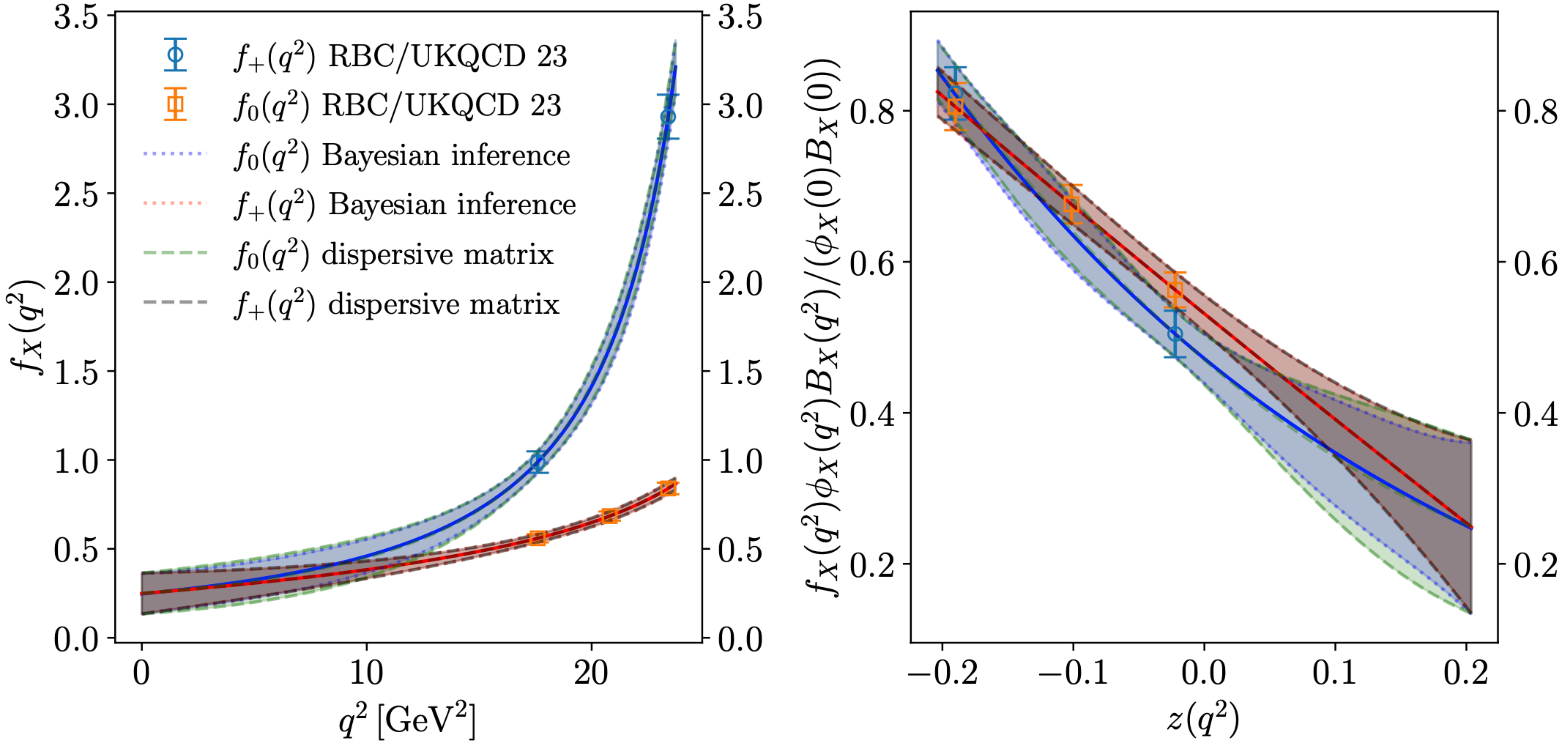


Bayesian and frequentist provide complementary information — consider both simultaneously!

Conclusion: World lattice data for $B \to \pi \ell \nu$ is in quite bad shape…

Relation to dispersive-matrix method?

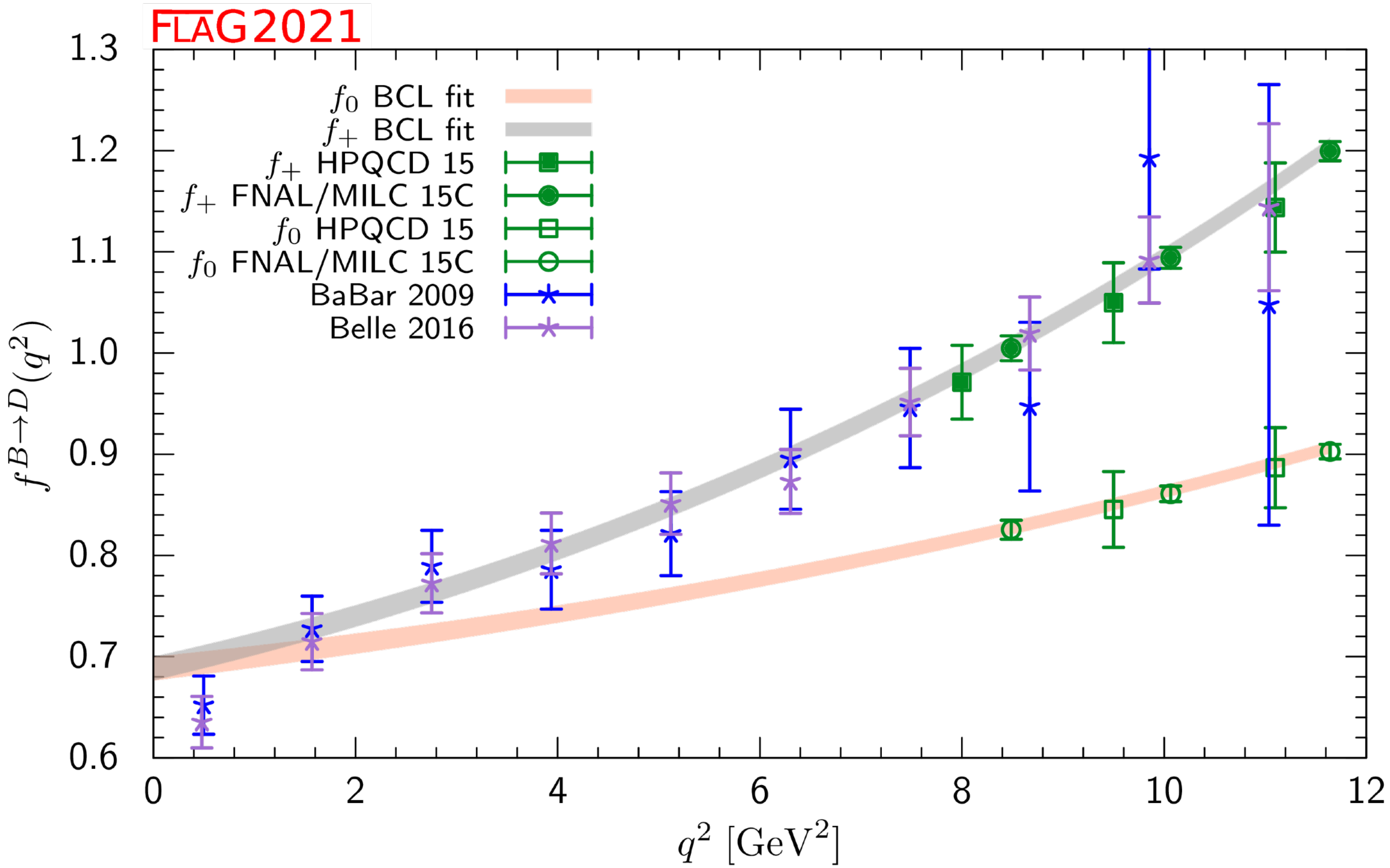
Di Carlo, Martinelli, Naviglio et al. PRD 104 (2021) 054502



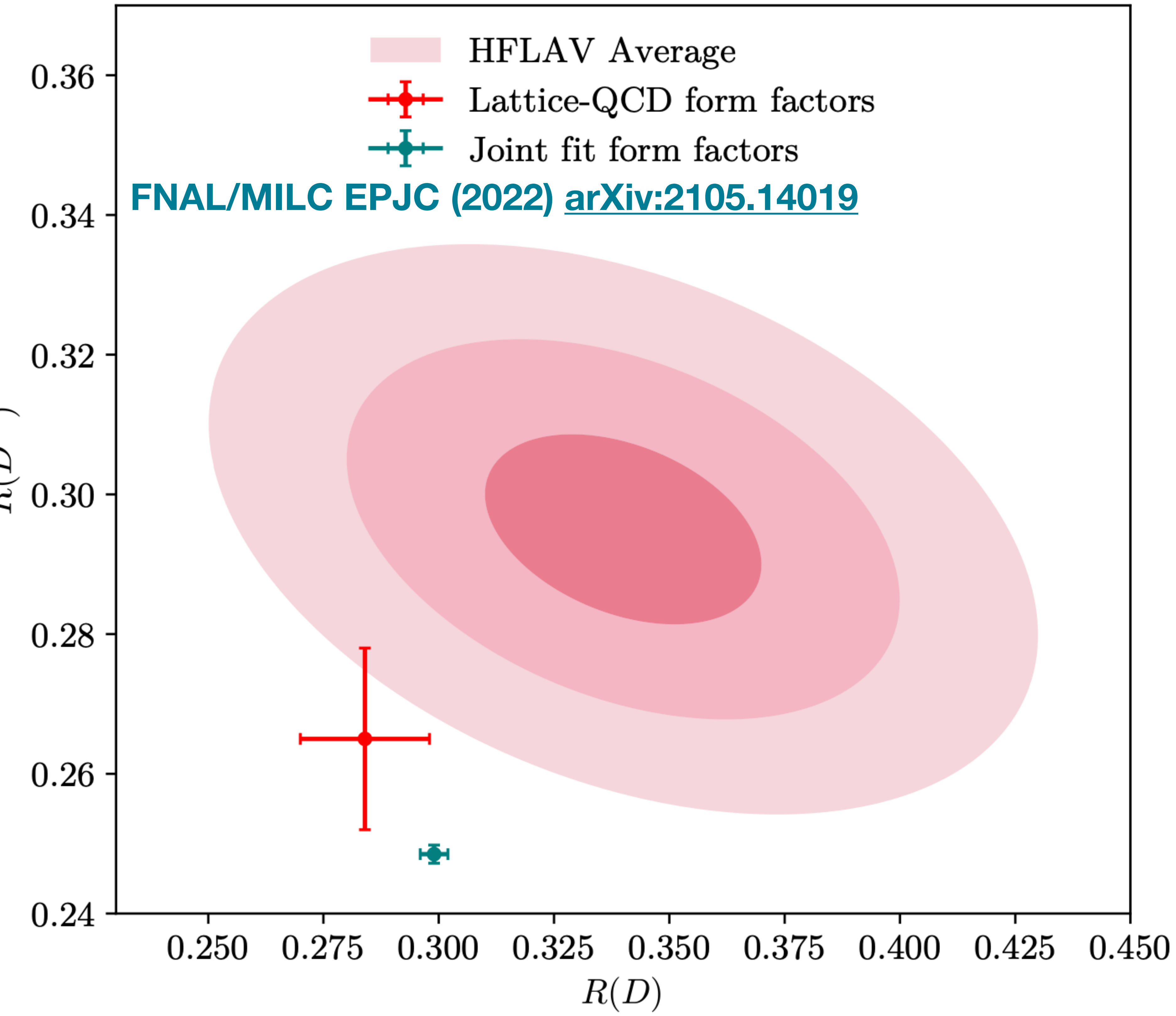
The methods produce essentially the same results. Clear practical advantages of Bayesian inference:

- kinematical constraints exactly and cleanly implemented
- simultaneous fit over various (correlated) data sets possible
- clean statistical underpinning

$B \to D$



FLAG2021
f_0 BCL fit
f_+ BCL fit
f_+ HPQCD 15
f_+ FNAL/MILC 15C
f_0 HPQCD 15
f_0 FNAL/MILC 15C
BaBar 2009
Belle 2016
$f^{B\to D}(q^2)$
q^2 [GeV^2]
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0
2
4
6
8
10
12



FNAL/MILC 0.265(13)
HPQCD 0.279(13)