

# Investigating the strong interaction between hadrons and light nuclei with femtoscopy and hypernuclei measurements with ALICE

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# Many-body systems in nuclear physics

- Properties of nuclei and hypernuclei cannot be described satisfactorily with two-body forces only

*L.E. Marcucci et al., Front. Phys. 8:69 (2020)*

- NNN interaction contributes ~10% to the binding energies of  $^3\text{H}$  and  $^4\text{He}$

- Many-body scattering requires three-body calculations (e.g. **neutron-deuteron**)

*L. Girlanda et al., PRC 102, 064003 (2020)*

## $^3\text{H}$ and $^4\text{He}$ binding energies and n-d scattering length

Potential(NN)	$^3\text{H}$ [MeV]	$^4\text{He}$ [MeV]	$^2a_{nd}$ [fm]
AV18	7.624	24.22	1.258
CDBonn	7.998	26.13	
N3LO-Idaho	7.854	25.38	1.100

Potential(NN+NNN)	$^3\text{H}$ [MeV]	$^4\text{He}$ [MeV]	$^2a_{nd}$ [fm]
AV18/UIX	8.479	28.47	0.590
CDBonn/TM	8.474	29.00	
N3LO-Idaho/N2LO	8.474	28.37	0.675
Exp.	8.48	28.30	0.645±0.010

Nuclei    Hypernuclei



Ordinary nucleus



Single A-hypernucleus

$\rho_0$

Baryon density

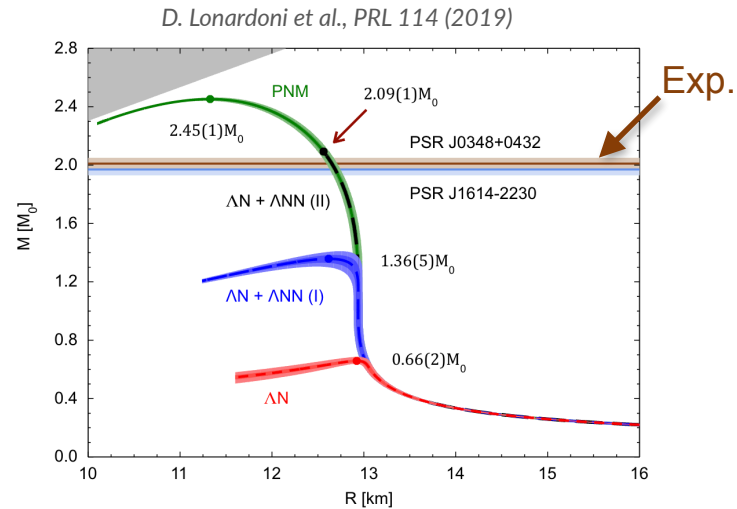


# Many-body systems in astrophysics

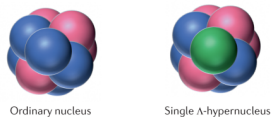
- Production of hyperons energetically favourable in neutron stars (NS) around  $2-3 \rho_0$
- Only two-body  $\Lambda N$ 
  - Too soft EoS, incompatible with measured heavy NS
  - Large improvement in 2-body YN with femtoscopy

ALICE Coll., PLB 833 (2022), ALICE Coll., Nature 588 (2020),  
L. Fabbietti et al., Ann.Rev.Nucl.Part.Sci. 71 (2021)

- Introduction of three-body  $\Lambda NN$  forces
  - Stiffens EoS, model-dependent
  - Need for additional experimental constraints

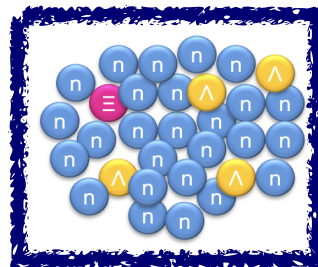


Nuclei      Hypernuclei



$\rho_0$

$3\rho_0$

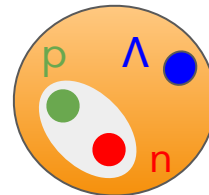


Neutron Stars

Baryon density

# ${}^3_{\Lambda}\text{H}$ lifetime and $\Lambda$ separation energy

- Hypertriton ( ${}^3_{\Lambda}\text{H}$ ):  $\Lambda$ pn bound state
  - ➔ powerful probe for investigating the  $\Lambda$ N interaction



- Weakly bound state
  - ➔ ALICE measured with unprecedented precision the  ${}^3_{\Lambda}\text{H}$  lifetime and the energy ( $B_{\Lambda}$ ) required to separate the  $\Lambda$  from the deuteron

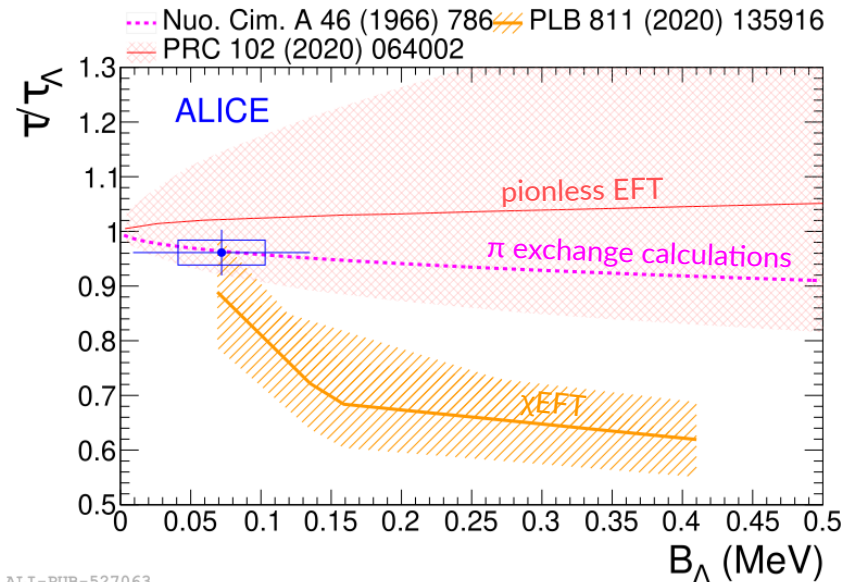
ALICE Coll., arXiv:2209.07360

$$B_{\Lambda} = [72 \pm 63 \text{ (stat.)} \pm 36 \text{ (syst.)}] \text{ keV}$$

- ➔ the very low  $B_{\Lambda}$  of  $\sim 100$  keV corresponds to a large radius of  $\sim 5$  fm

Hildenbrand et al., Phys. Rev. C, 100(3), 034002 (2019)

How does this reflect on its production?



ALI-PUB-527063

# ${}^3_{\Lambda}\text{H}$ production yield in Pb–Pb

- Weakly bound state ( $B_{\Lambda} \sim 100$  keV)
  - ${}^3_{\Lambda}\text{H} / \Lambda$ : large separation between statistical hadronization model (SHM) and coalescence predictions at low charged-particle multiplicity density

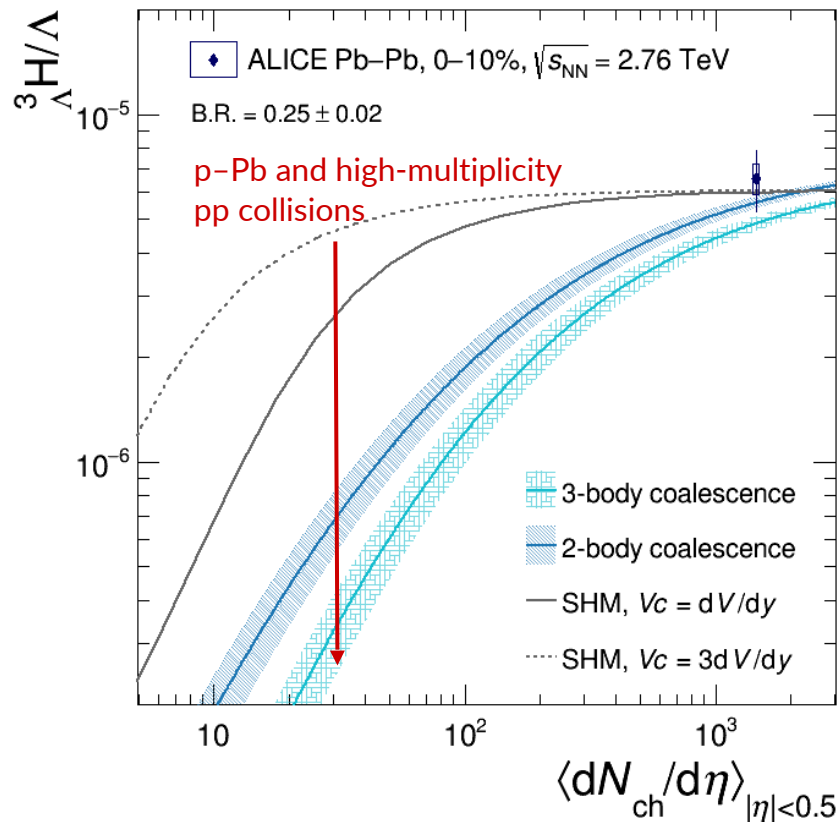
Vovchenko et al., *Phys. Lett., B* 785, 171–174, (2018)

Sun et al., *Phys. Lett. B* 792, 132–137, (2019)

- coalescence is sensitive to the interplay between the size of the collision system and the spatial extension of the nucleus wave function

- ${}^3_{\Lambda}\text{H}$  production in pp and p–Pb: a key to understand the nuclear production mechanism at the LHC

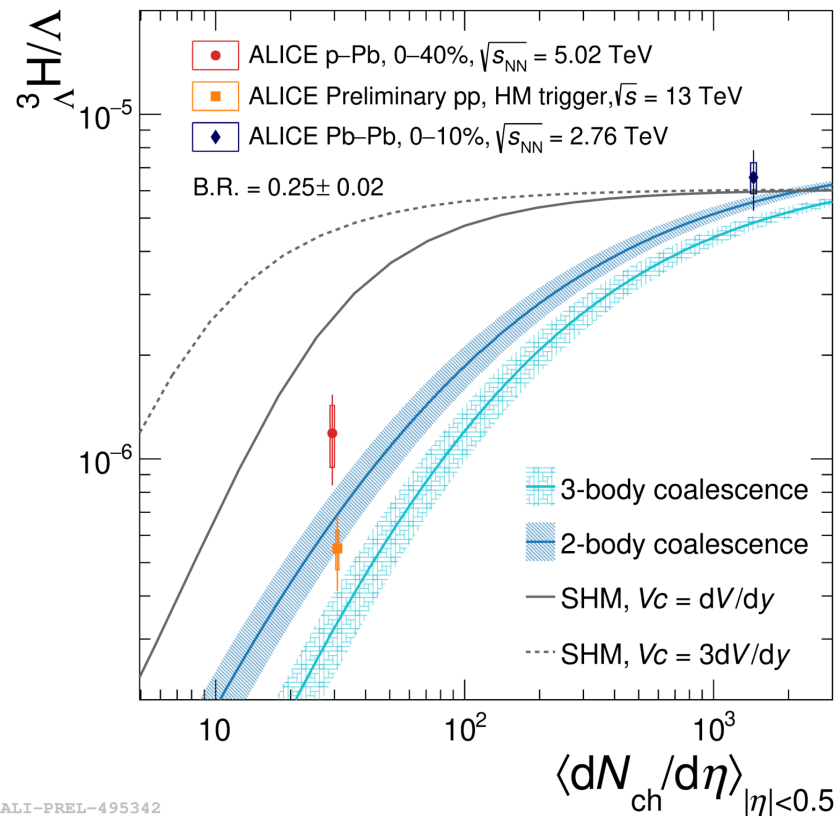
ALICE Coll., *Phys. Lett. B* 754, 360–372, (2016)



# ${}^3_{\Lambda}\text{H}$ production yield in pp and p-Pb

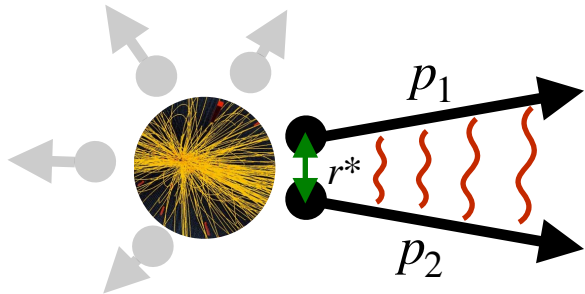
- First measurements of  ${}^3_{\Lambda}\text{H}$  production in pp and p-Pb collisions
  - ➔ good agreement with 2-body coalescence
  - ➔ tension with SHM at low charged-particle multiplicity density
    - First significant constraint to SHM possible configurations
- Coalescence quantitatively describes the  ${}^3_{\Lambda}\text{H}$  suppression in small systems
  - ➔ the nuclear size matters at low charged-particle multiplicity

ALICE Coll., Phys. Rev. Lett. 128, (2022) 252003



ALI-PREL-495342

# Two-body femtoscopy



## Emission source $S(r^*)$

ALICE Coll. PLB 811 (2020) 135849

ALICE Collaboration

pΛ: PLB 833 (2022) 137272

pΣ<sup>0</sup>: PLB 805 (2020) 135419

pΞ/pΩ: Nature 588 (2020) 232-238

pφ: PRL 127 (2021) 172301

Kρ: PRL 124 (2020) 09230

PLB 822 (2021) 136708

arXiv 2205.15176

pp, pΛ, ΛΛ: PRC 99 (2019) 2, 024001

ΛΛ: PLB 797 (2019) 134822

pΞ: PRL 123 (2019) 112002

ΛK: PRC 103 (2021) 5, 055204

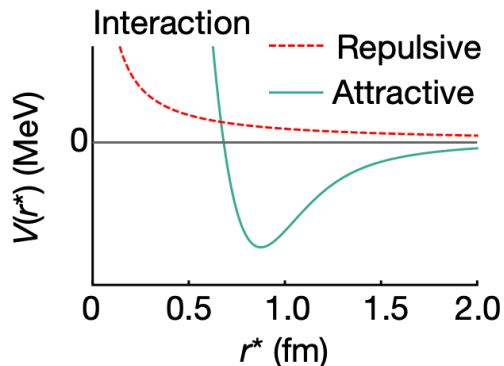
pD: PRD 106 (2022) 052010

ΛΞ: arXiv 2204.10258

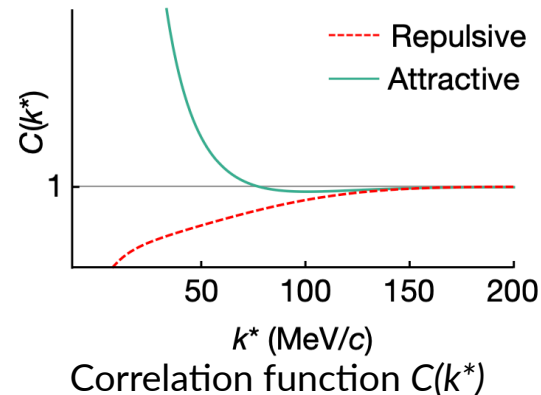
- Two-particle correlation function:

$$C(k^*) = \mathcal{N} \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)} = \int S(r^*) |\psi(\mathbf{k}^*, \mathbf{r}^*)|^2 d^3r^*$$

- Measurements in small colliding systems (~1-2 fm)  
→ Access to the strong interaction and short-range dynamics



Schrödinger equation  
Two-particle wave function  
 $\psi(\mathbf{k}^*, \mathbf{r}^*)$



Correlation function  $C(k^*)$

# Proton-deuteron correlations

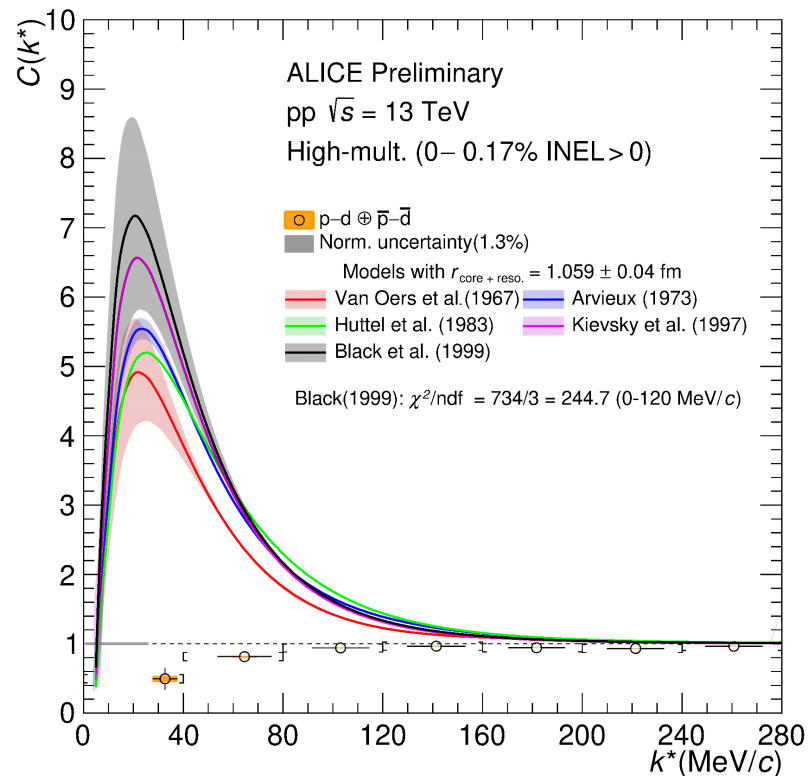
Point-like particle models anchored to scattering experiments

	$S = 1/2$		$S = 3/2$	
	$f_0(\text{fm})$	$d_0(\text{fm})$	$f_0(\text{fm})$	$d_0(\text{fm})$
Van Oers et al. (1967)	$-1.30^{+0.20}_{-0.20}$	—	$-11.40^{+1.20}_{-1.80}$	$2.05^{+0.25}_{-0.25}$
Arvieux (1973)	$-2.73^{+0.10}_{-0.10}$	$2.27^{+0.12}_{-0.12}$	$-11.88^{+0.10}_{-0.40}$	$2.63^{+0.01}_{-0.02}$
Huttel et al. (1983)	-4.0	—	-11.1	—
Kievsky et al. (1997)	-0.024	—	-13.7	—
Black et al. (1999)	$0.13^{+0.04}_{-0.04}$	—	$-14.70^{+2.30}_{-2.30}$	—

W. T. H. Van Oers, & K. W. Brockman Jr, *NPA* 561 (1967);  
 J. Arvieux et al., *NPA* 221 (1973); E. Huttel et al., *NPA* 406 (1983);  
 A. Kievsky et al., *PLB* 406 (1997); T. C. Black et al., *PLB* 471 (1999);

- Coulomb + strong interaction using the Lednický model  
*Lednický, R. Phys. Part. Nuclei* 40, 307–352 (2009)
- Only s-wave interaction
- Source radius evaluated using the hadron-hadron universal  $m_T$  scaling

Point-like particle description doesn't work for p-d



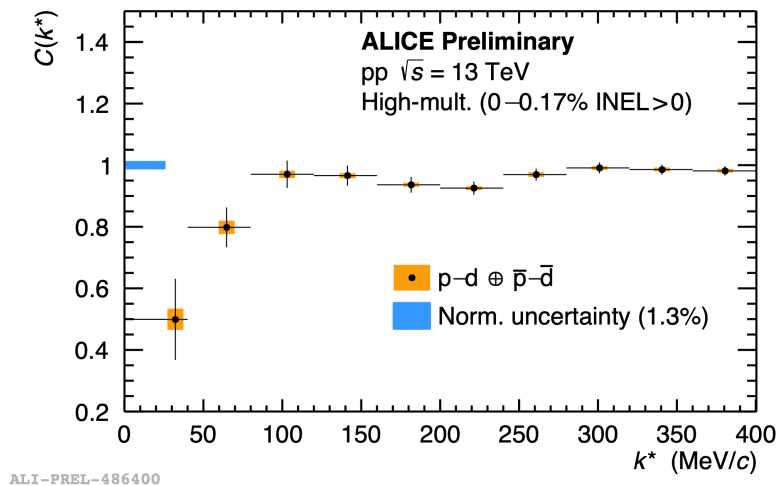
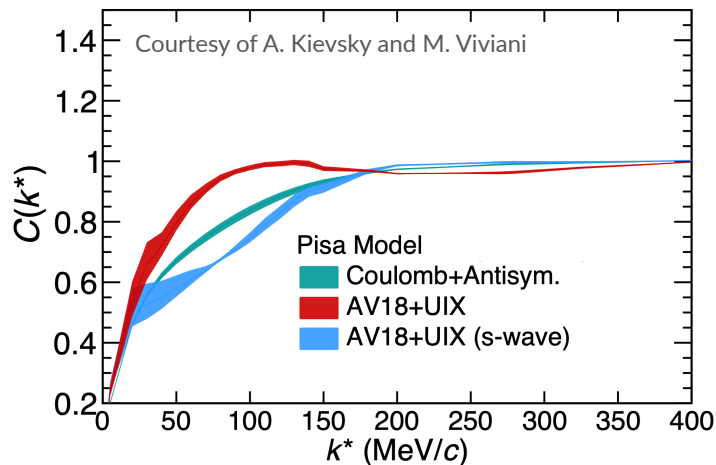
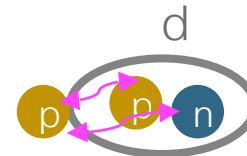
ALI-PREL-501009



# Proton-deuteron correlations

Deuteron treated as a composite object:

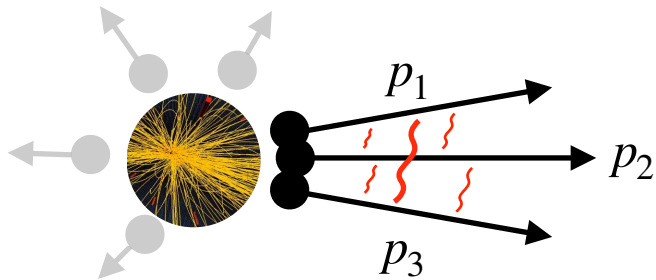
- Coulomb + strong interaction (NN and NNN) + Quantum Statistics



The measured p-d correlation function reflects the full three-nucleon dynamics (not the p-d int.)

- ➔ Sensitivity to the short inter-particle distances
- ➔ Sensitivity to the details of the wave function

# Three-body femtoscopy

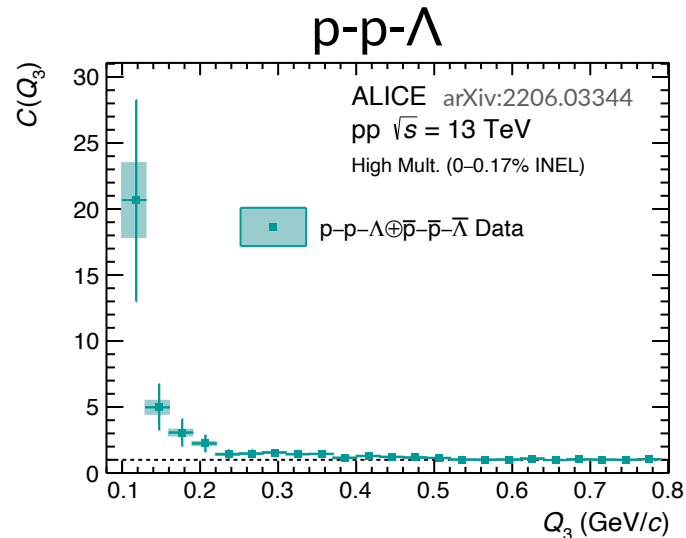
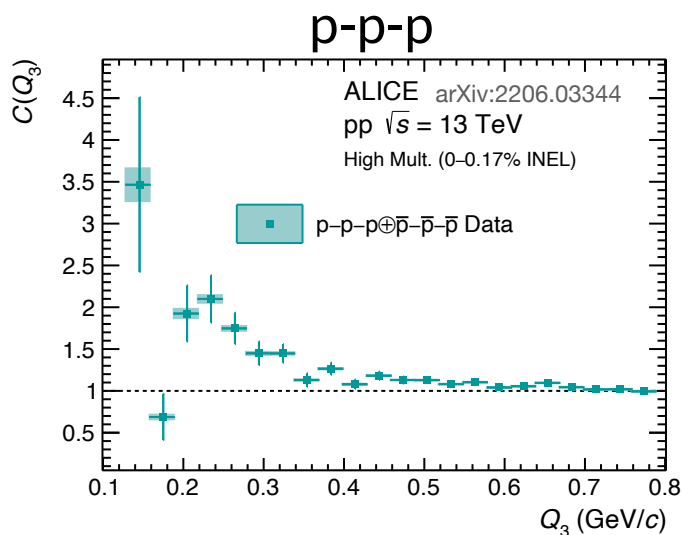


- Three-particle correlation function:

$$C(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \iiint S_3(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \left| \psi_{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) \right|^2 d^3x_1 d^3x_2 d^3x_3 = \mathcal{N} \cdot \frac{N_{\text{same}}(Q_3)}{N_{\text{mixed}}(Q_3)}$$

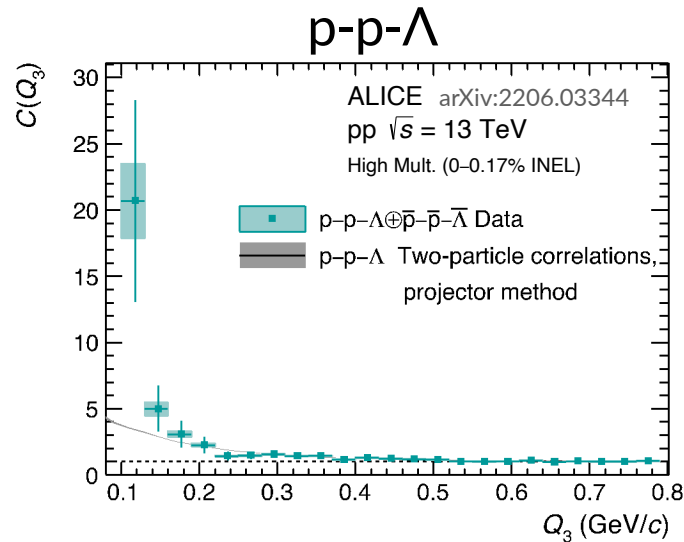
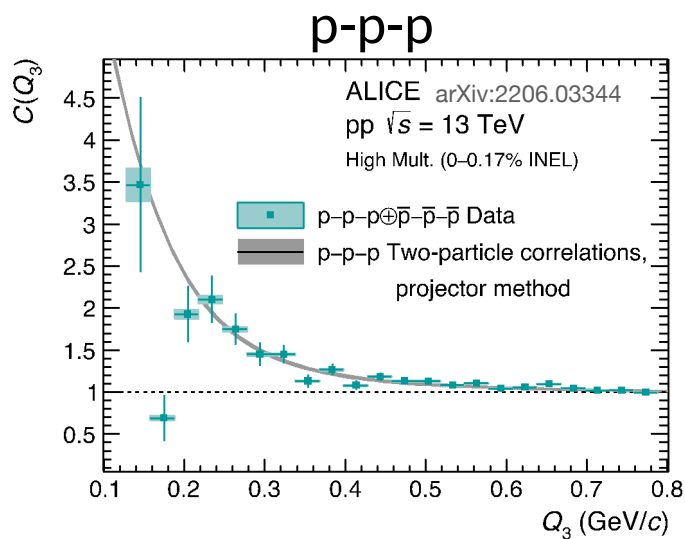
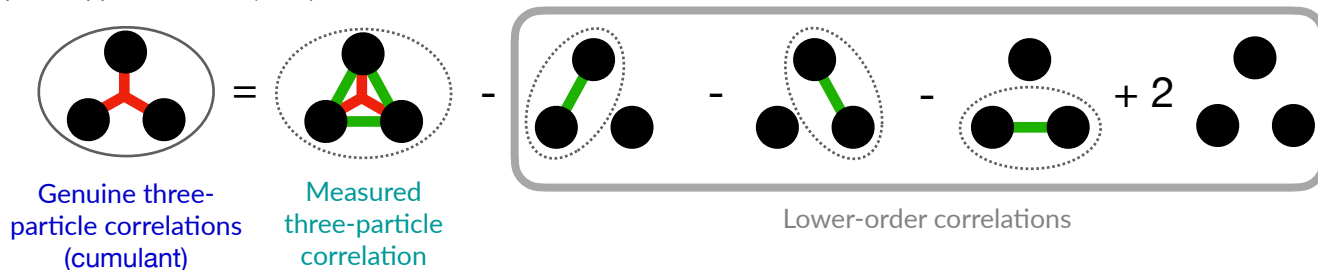
- Lorentz-invariant  $Q_3$  is defined as:

$$Q_3 = \sqrt{-q_{12}^2 - q_{23}^2 - q_{31}^2} \quad q_{ij}^\mu = 2 \left( \frac{m_j E_i}{m_i + m_j} - \frac{m_i E_j}{m_i + m_j}, \frac{m_j}{m_i + m_j} \mathbf{p}_i - \frac{m_i}{m_i + m_j} \mathbf{p}_j \right)$$

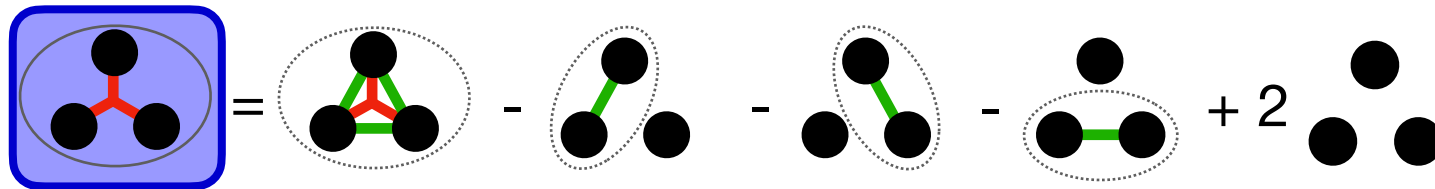


# Cumulants in femtoscopy

R. Kubo, J. Phys. Soc. Jpn. 17, pp. 1100-1120 (1962)



# p-p-p cumulant




Negative cumulant for p-p-p

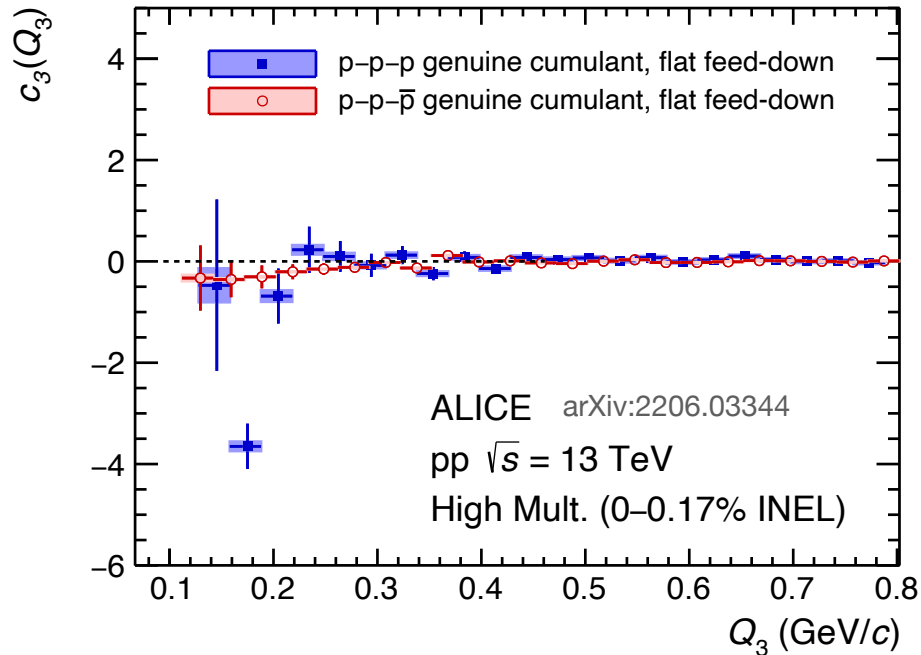
Possible effects at play:

- Pauli blocking at the three-particle level
- three-body strong interaction

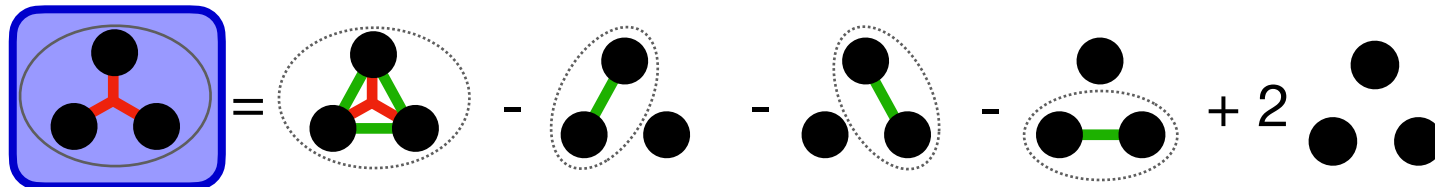
Statistical significance:

$n_{\sigma} = 6.7$  for  $Q_3 < 0.4 \text{ GeV}/c$

 Test with mixed-charge particles, cumulant negligible.



# p-p- $\Lambda$ cumulant



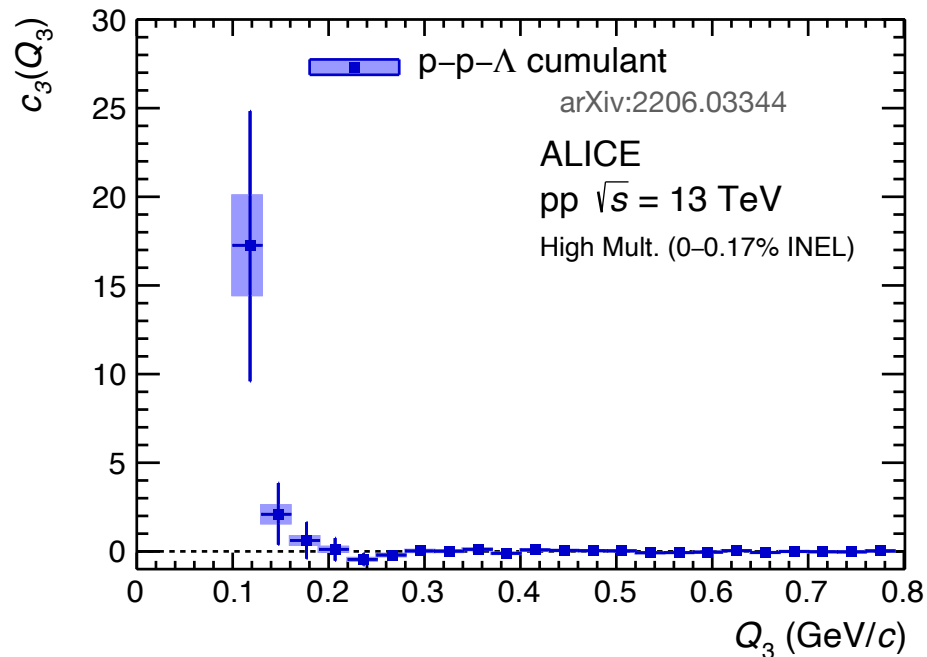
## Positive cumulant for p-p- $\Lambda$

- Only two identical and charged particles
  - ✓ Main expected contribution from three-body strong interaction
- Relevant measurement for equation of state of neutron stars

## Statistical significance:

$n_\sigma = 0.8$  for  $Q_3 < 0.4$  GeV/c

*In Run 3, two orders of magnitude gain in statistics expected!*

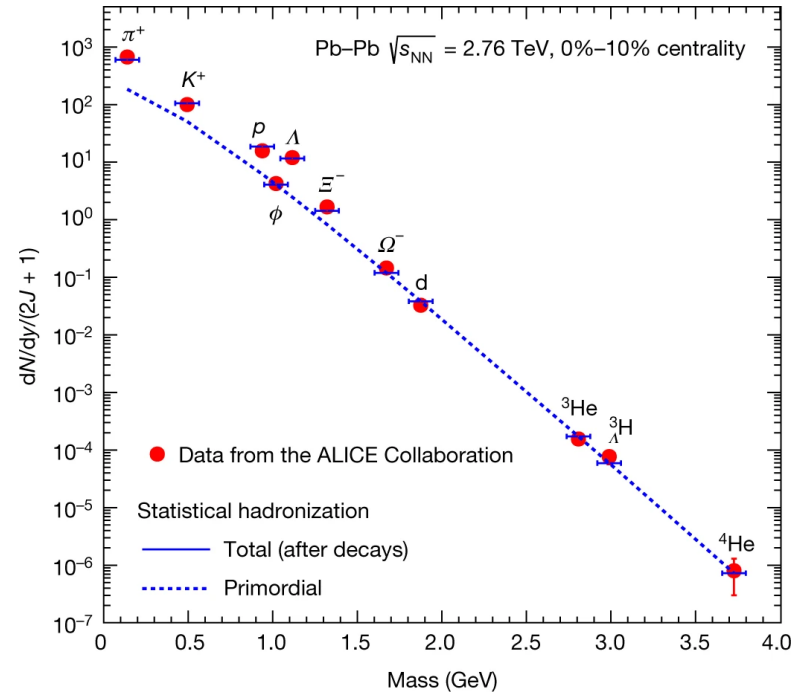


- New precise measurements of the lifetime,  $\Lambda$  separation energy and production yields of the hypertriton ( $^3_\Lambda\text{H}$ ):
  - **Weakly bound state  $B_\Lambda \sim 100$  keV** corresponds to large radii  $\sim 5$  fm
  - $^3_\Lambda\text{H} / \Lambda$  ratio in pp and p-Pb favours coalescence expectation
    - **nuclear size matters at low-charged particle multiplicity**
- Femtoscopic correlations to probe many-body dynamics
  - **p-d** correlation can not be described without including the **full three-nucleon dynamics**
  - **p-p-p** correlation function shows a **significant deviation** from the simple description in terms of mutual two-body interactions
  - **p-p- $\Lambda$**  correlation exhibits **no significant deviation**
    - Precision measurements to come in LHC Run 3 and Run 4!


# Backup slides

# Nucleosynthesis at the LHC

- (Hyper)Nuclei: unique probes to study the interactions between hadrons
  - at the LHC (hyper)nuclei formation occurs at extremely high temperatures ( $T \sim 100$  MeV)
- Two theoretical models available to describe nuclear production
  - Statistical Hadronisation Models (SHM) <sup>1, 2</sup>: yields described by filling the available phase-space after the collision
    - No microscopic description of nuclei formation



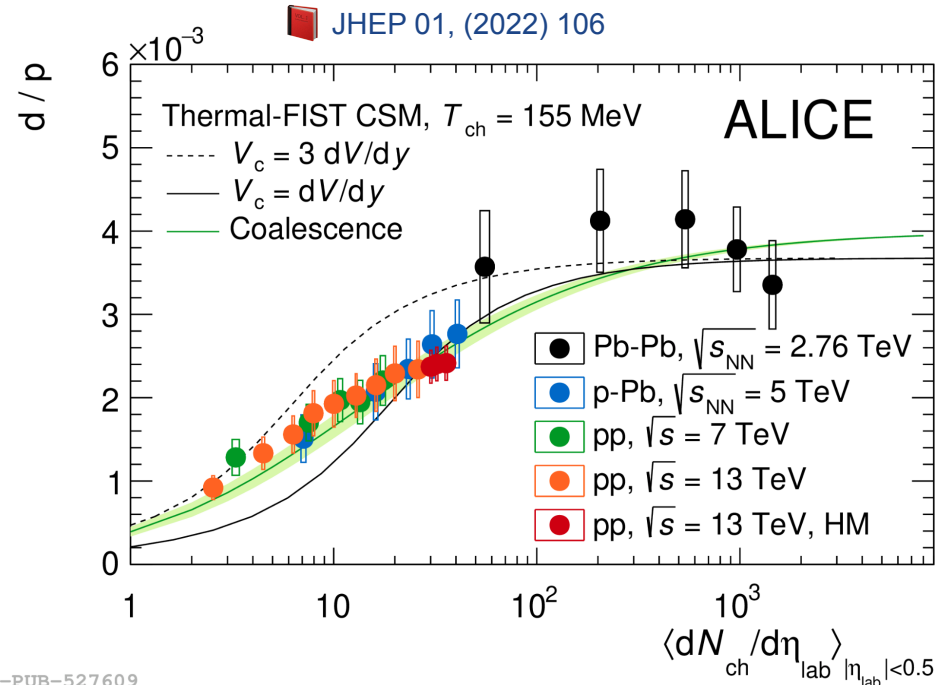
<sup>2</sup>  A. Andronic et al., Nature 561, (2018) 3210


<sup>1</sup>  Vovchenko et al., Phys. Lett. B 785, (2018) 171



# Nucleosynthesis at the LHC

- (Hyper)Nuclei: unique probes to study the interactions between hadrons
  - at the LHC (hyper)nuclei formation occurs at extremely high temperatures ( $T \sim 100$  MeV)
- Two theoretical models available to describe nuclear production
  - SHM<sup>1</sup>
  - **Coalescence**<sup>2</sup>: nuclei arise from the overlap of the nucleons in the phase space
    - microscopic description
    - yield predictions only relative to the nucleon ones

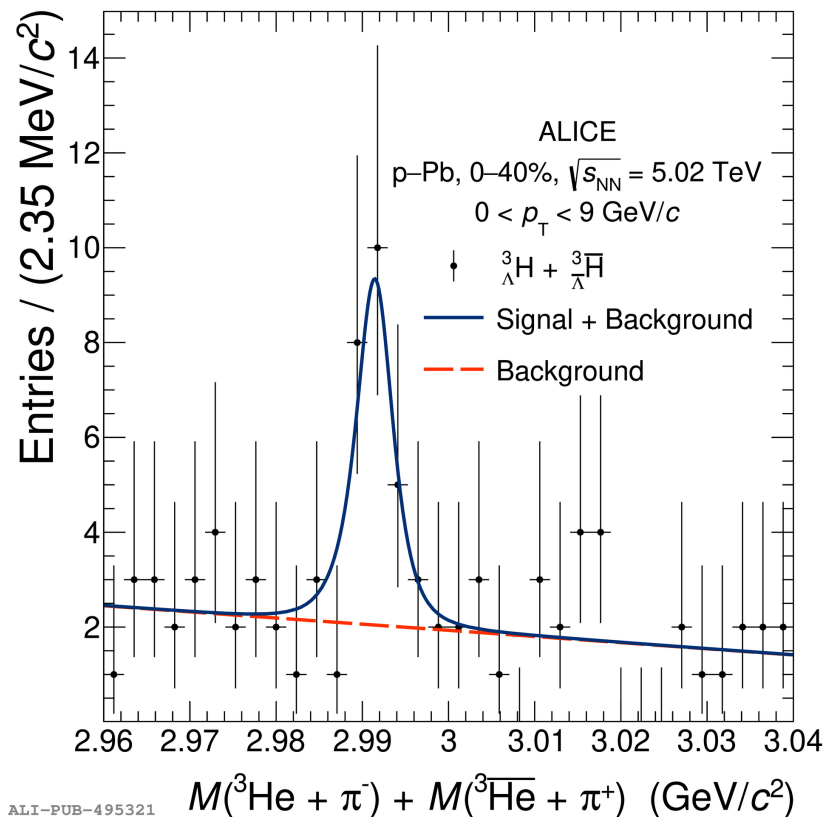


<sup>1</sup>  Vovchenko et al., Phys. Lett. B 785, (2018) 171

<sup>2</sup>  Sun et al., Phys. Lett. B 792, (2019) 132

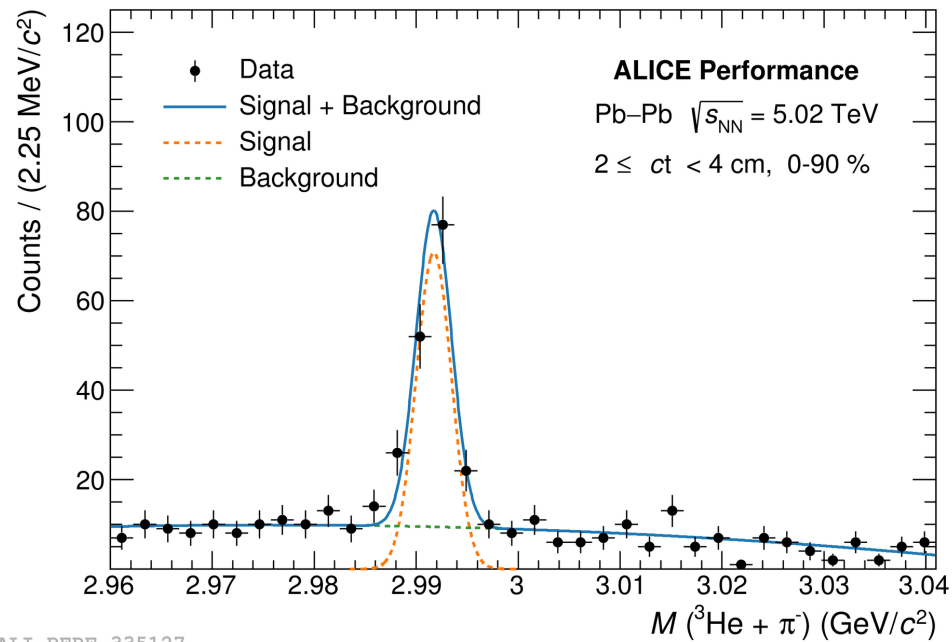
# ${}^3_{\Lambda}\text{H}$ selection in pp and p-Pb collisions

- Data samples:
  - pp collisions at  $\sqrt{s} = 13$  TeV and p-Pb collisions at  $\sqrt{s_{\text{NN}}} = 5.02$  TeV collected during Run 2
- ${}^3_{\Lambda}\text{H}$  selection in pp: **trigger on high multiplicity events using V0 detectors** + topological selections on triggered events
- ${}^3_{\Lambda}\text{H}$  selection in p-Pb: 40% most central collisions + BDT Classifier
- **Significance  $> 4\sigma$  both in pp and p-Pb**



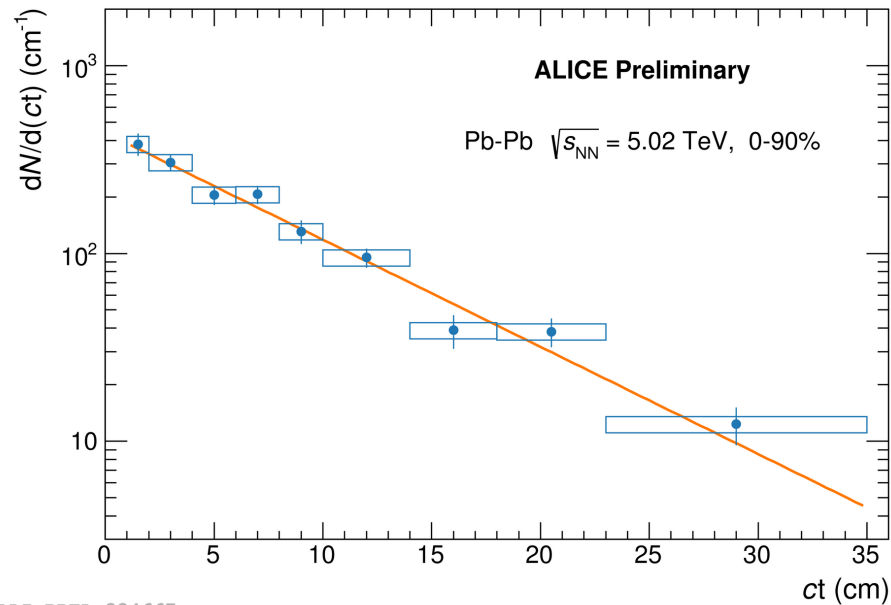
 Phys. Rev. Lett. 128, (2022), 252003

- Signal extracted with a fit to the invariant mass spectrum of the selected candidates
- high significance over a wide range
  - 9 ct bins from 1 to 35 cm



ALI-PERF-335127

- Corrected ct spectrum fitted with an exponential function
- Lifetime value from the fit
  - Statistical uncertainty  $\sim 6\%$
  - Systematic uncertainty  $\sim 7\%$
- Most precise measurement of the lifetime ever done so far



ALI-PREL-334667

# Proton-deuteron wave function

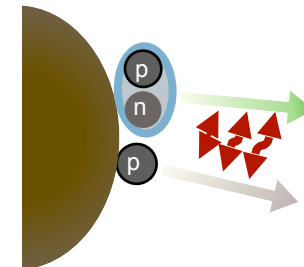
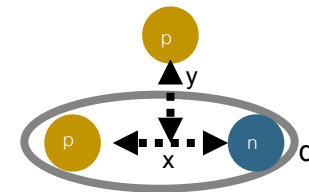
The three body wave function with proper treatment of 2N and 3N interaction at very short distances goes to a p-d state.

- Three-body wavefunction for p-d:  $\Psi_{m_2, m_1}(x, y)$  describing three-body dynamics, anchored to p-d scattering observables.
  - $x$  = distance of p-n system within the deuteron
  - $y$  = p-d distance
  - $m_2$  and  $m_1$  deuteron and proton spin

- $\Psi_{m_2, m_1}(x, y)$  three-nucleon wave function asymptotically behaves as p-d state:

$$\Psi_{m_2, m_1}(x, y) = \underbrace{\Psi_{m_2, m_1}^{(\text{free})}}_{\text{Asymptotic form}} + \sum_{LSJ}^{J \leq \bar{J}} \underbrace{\sqrt{4\pi} i^L \sqrt{2L+1} e^{i\sigma_L} (1m_2 \frac{1}{2} m_1 |SJ_z)(LOSJ_z | JJ_z) \tilde{\Psi}_{LSJJ_z}}_{\text{Strong three-body interaction}}$$

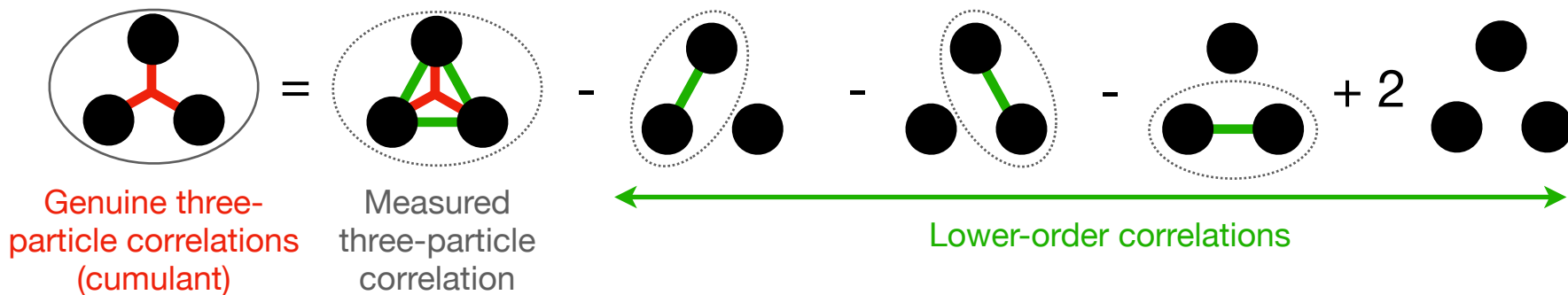
- ~
- $\Psi_{LSJJ_z}$  describe the configurations where the three particles are close to each other
- $\Psi_{m_1, m_2}^{(\text{free})}$  an asymptotic form of p-d wave function



Kievsky et al, Phys. Rev. C 64 (2001) 024002  
 Kievsky et al, Phys. Rev. C 69 (2004) 014002  
 Deltuva et al, Phys. Rev. C 71 (2005) 064003

# Cumulants in femtoscopy

Genuine three-particle correlations are obtained from the total three-particle correlations by subtracting the lower-order contribution [1]:



In terms of correlation functions:

$$c_3(Q_3) = C(Q_3) - C_{12}(Q_3) - C_{23}(Q_3) - C_{31}(Q_3) + 2$$

How to estimate lower-order contributions?

[1] R. Kubo, J. Phys. Soc. Jpn. 17, pp. 1100-1120 (1962)

# Kubo's cumulant expansion method

- $X_i$  denotes the general  $i$ -th stochastic variable
- The most general decomposition of 2-particle correlation is:

$$\langle X_1 X_2 \rangle = \langle X_1 \rangle \langle X_2 \rangle + \langle X_1 X_2 \rangle_c$$

- By definition, the 2<sup>nd</sup> term on the right is the 2-particle cumulant
- Cumulants cannot be measured directly, however:

$$\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$$

# Kubo's cumulant expansion method

- The most general decomposition of 3-particle correlation is:

$$\begin{aligned} \langle X_1 X_2 X_3 \rangle &= \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle \\ &+ \langle X_1 X_2 \rangle_c \langle X_3 \rangle + \langle X_1 X_3 \rangle_c \langle X_2 \rangle + \langle X_2 X_3 \rangle_c \langle X_1 \rangle \\ &+ \langle X_1 X_2 X_3 \rangle_c \end{aligned}$$

- Using the 2-particle cumulant:  $\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$
- Working recursively from higher to lower orders, we have 3-particle cumulant expressed in terms of the measured 3-, 2-, and 1-particle averages:

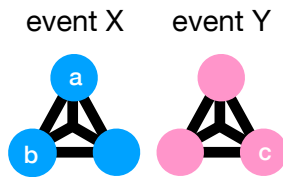
$$\begin{aligned} \langle X_1 X_2 X_3 \rangle_c &= \langle X_1 X_2 X_3 \rangle \\ &- \langle X_1 X_2 \rangle \langle X_3 \rangle - \langle X_1 X_3 \rangle \langle X_2 \rangle - \langle X_2 X_3 \rangle \langle X_1 \rangle \\ &+ 2 \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle \end{aligned}$$



# Lower-order contributions

## Data-driven method

- Use event mixing
- Two particles from the same event and one particle from another:



$$C_{ab,c}(Q_3) = \frac{N_2(\mathbf{p}_a, \mathbf{p}_b) N_1(\mathbf{p}_c)}{N_1(\mathbf{p}_a) N_1(\mathbf{p}_b) N_1(\mathbf{p}_c)}$$

- Lorentz-invariant scalar  $Q_3$  calculated using the single particle momenta  $\mathbf{p}_a, \mathbf{p}_b, \mathbf{p}_c$

## Projector method

- Use two-particle measured or theoretical correlation function  $C(k_{ab}^*)$
- Perform kinematic transformation:

$$C_{ab,c}(Q_3) = \int \underbrace{C(k_{ab}^*)}_{\text{two-body CF}} \underbrace{W_{ab}(k_{ab}^*, Q_3)}_{\text{projector}} dk_{ab}^*$$

Measured/modeled 2-body correlation functions

Jacobian from 2-body to 3-body coordinates

# Projection onto $Q_3$

- The projection onto  $Q_3$  is performed as follows

$$C_3(Q_3) = \iiint_{(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \in \mathcal{D}} C_3(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \mathcal{N} d^3 \mathbf{p}_1 d^3 \mathbf{p}_2 d^3 \mathbf{p}_3$$

$$\mathcal{D} = \{(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \in \mathcal{S} \mid Q_3 = \text{constant}\}$$

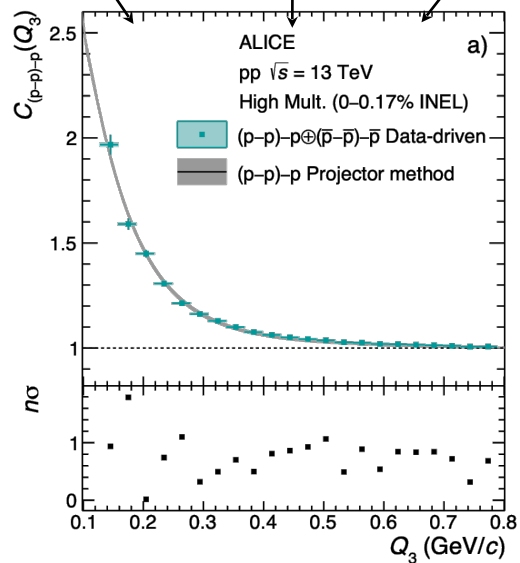
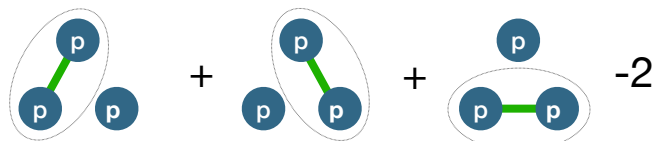
density of states in the phase space  
(uniform)

- In the case of two-body correlations, the projections turns to be

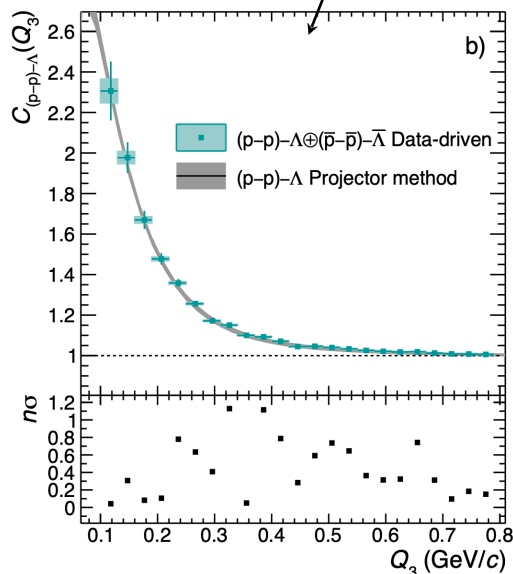
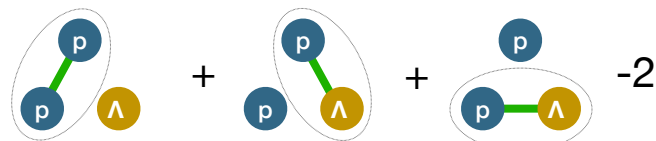
$$C_3(Q_3) = \int_0^{\sqrt{\frac{\gamma}{\alpha\gamma - \beta^2}}} Q_3 \overset{\text{two-body correlation function}}{C_2(k_1)} \overset{\text{projector } W(k_1, Q_3) \text{ ----> phase space density at } Q_3 = \text{constant}}{\left[ \frac{16(\alpha\gamma - \beta^2)^{3/2} k_1^2}{\pi Q_3^4 \gamma^2} \sqrt{\gamma Q_3^2 - (\alpha\gamma - \beta^2) k_1^2} \right]} dk_1$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are constants depending on the particles mass.

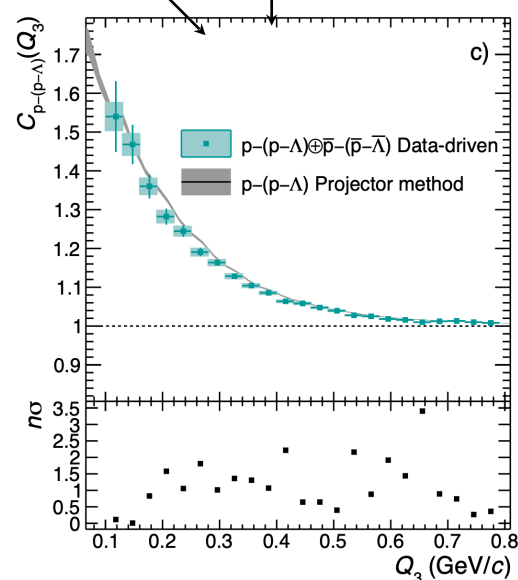
# Lower-order contributions in p-p-p and p-p- $\Lambda$



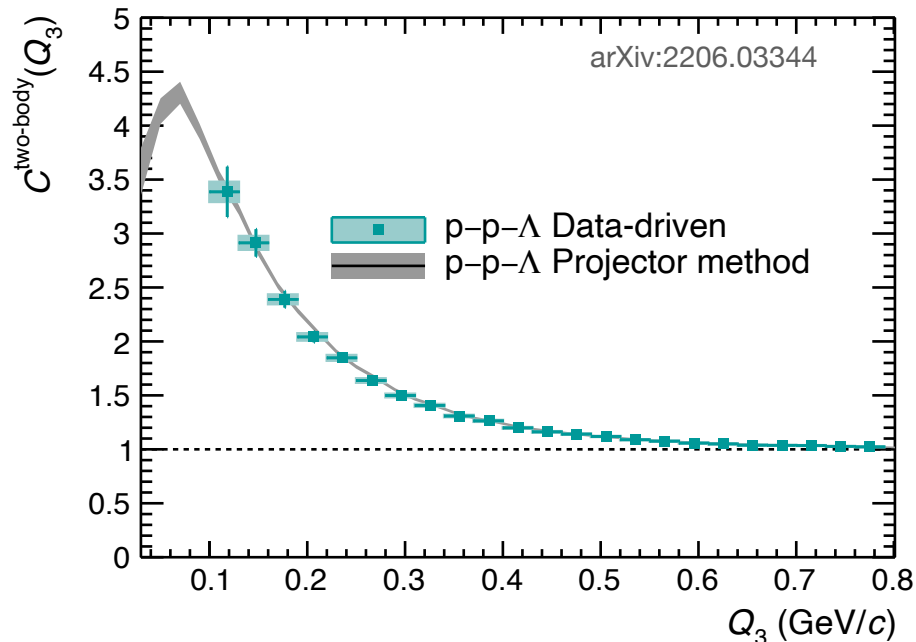
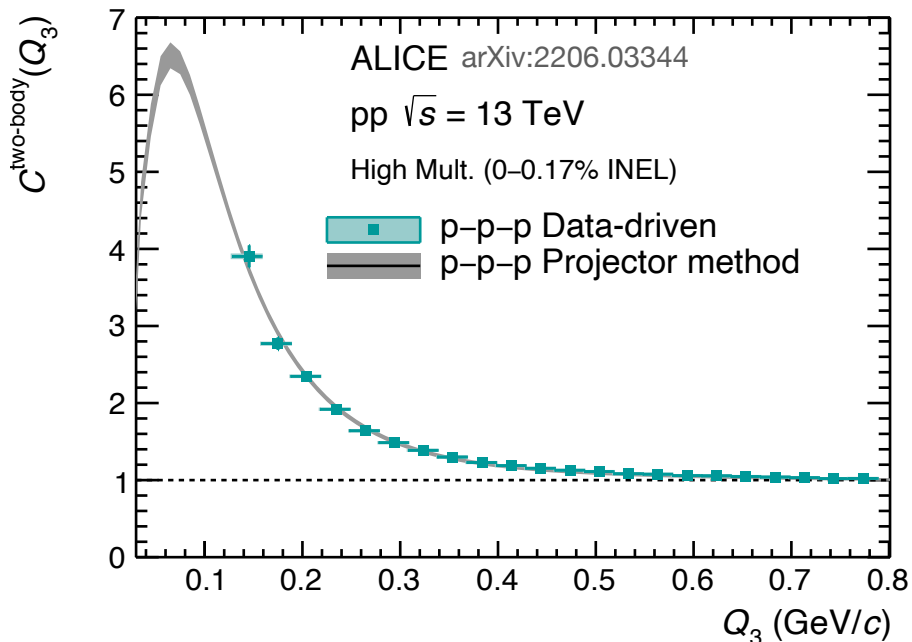
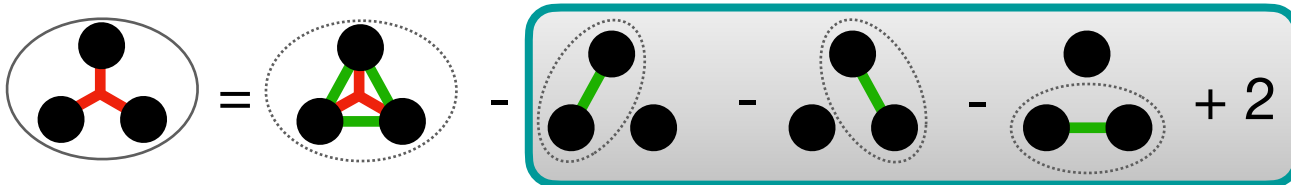
Measured p-p correlation ALICE Coll. PLB 805 (2020) 135419



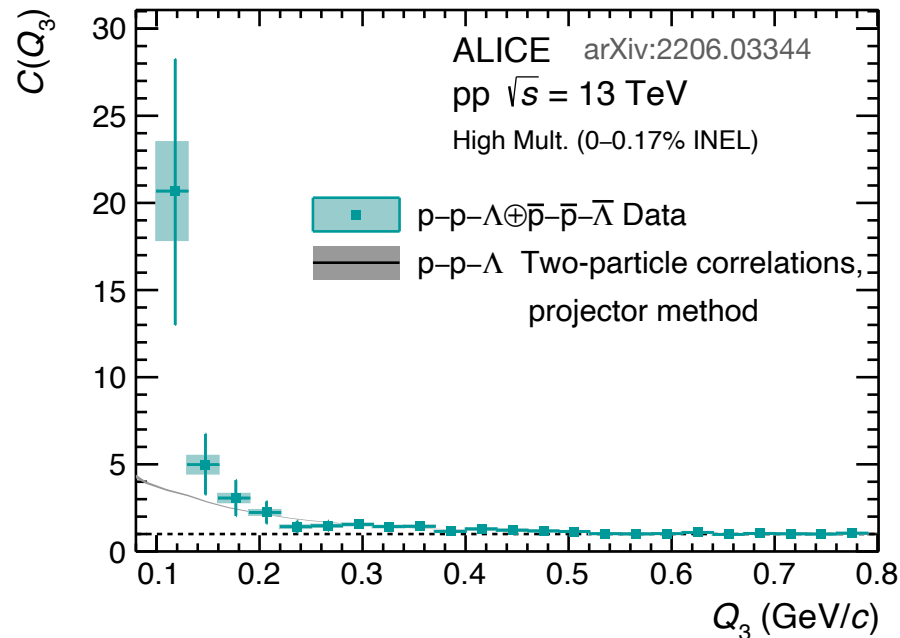
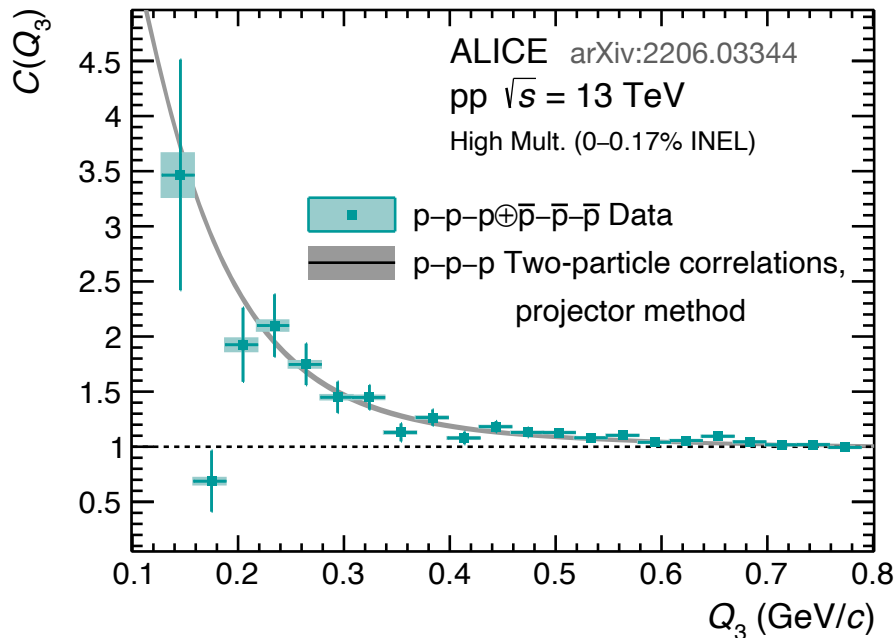
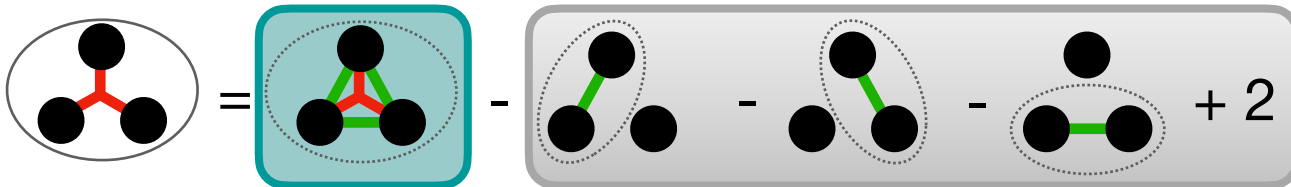
Measured p- $\Lambda$  correlation ALICE Coll. PLB 833 (2022), 137272



# p-p-p and p-p- $\Lambda$ correlation functions

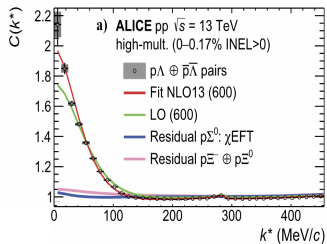


# p-p-p and p-p- $\Lambda$ correlation functions

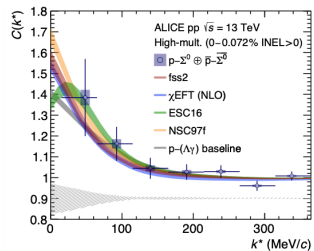


# An example of Equation of State for NS

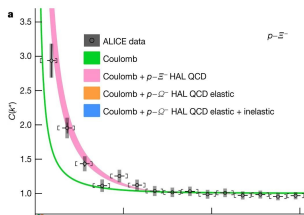
Correlation = two-body interaction



$p\Lambda$

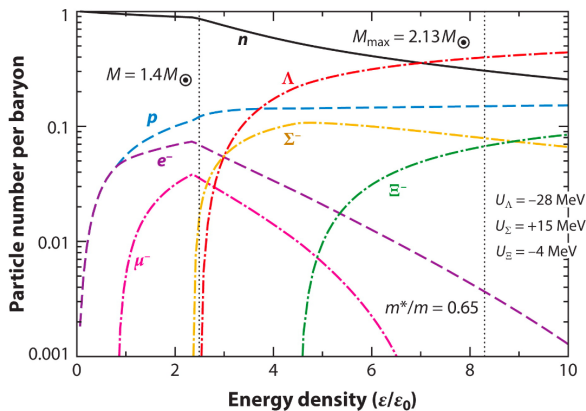


$p\Sigma$



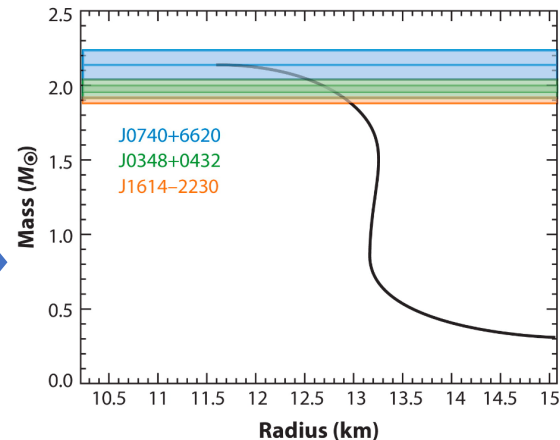
$p\Sigma$

Single-particle potentials = Equation of State



Courtesy J. Schaffner-Bielich 2020

Mass-Radius diagram for hyperon stars



L. Fabbietti et al. Ann.Rev.Nucl.Part.Sci. 71 (2021)

What about the three-body strong interaction?

# Source determination

The first step is “traditional” femtoscopy: known interaction → determine source size

- p-p interaction: Argonne v18 potential
- crosscheck with p- $\Lambda$  ( $\chi$ EFT)

Determine **gaussian “core” radius**

- As a function of pair  $\langle m_T \rangle$
- **Common to all hadron-hadron pairs**



**Effect of strong short-lived resonances**

Adds exponential tail to the source profile

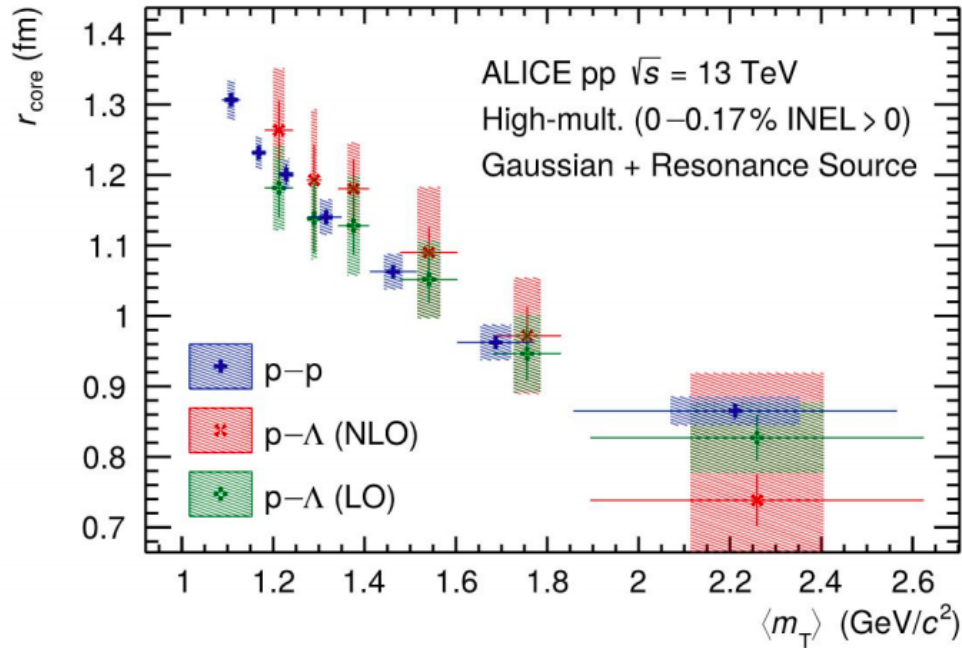
→ Angular distributions from EPOS

→ Production fraction from SHM

	Primordial	Resonances lifetime
p	35.8 %	1.65 fm
$\Lambda$	35.6 %	4.69 fm

[ALICE Coll., Phys. Lett. B 811 (2020) 135849]

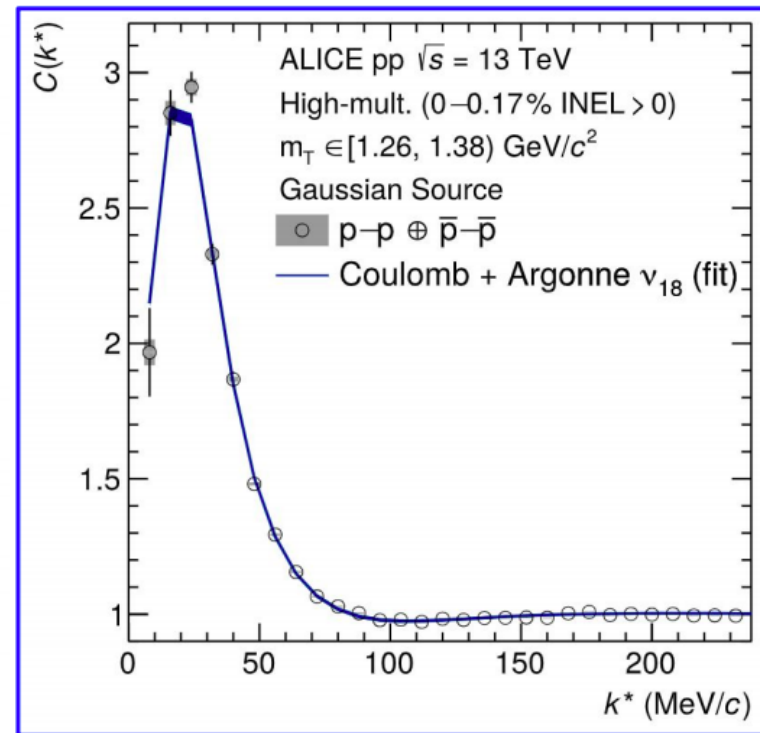
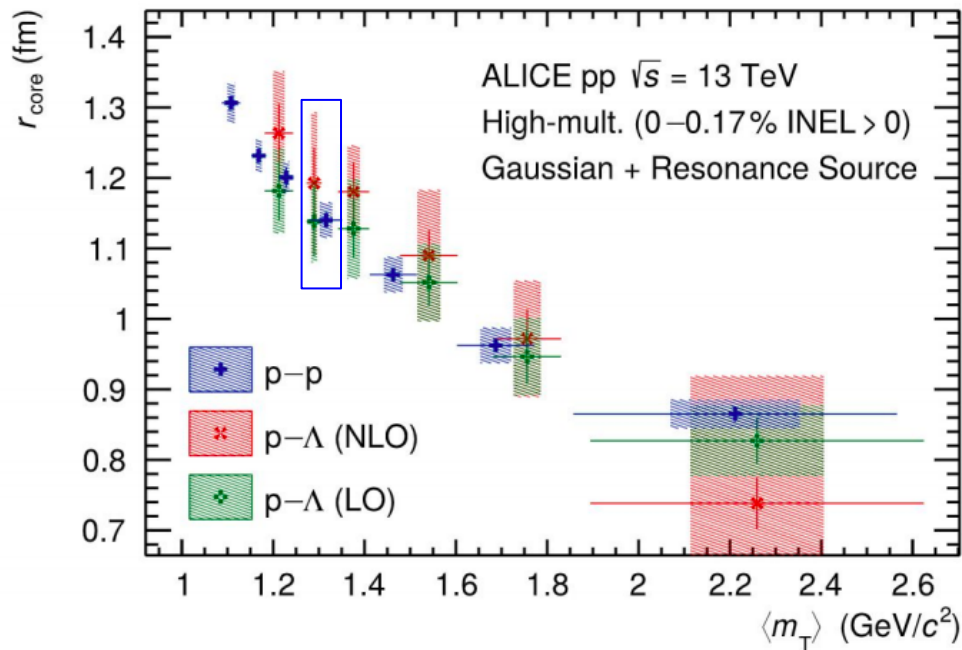
# Source determination



[ALICE Coll., Phys. Lett. B 811 (2020) 135849]

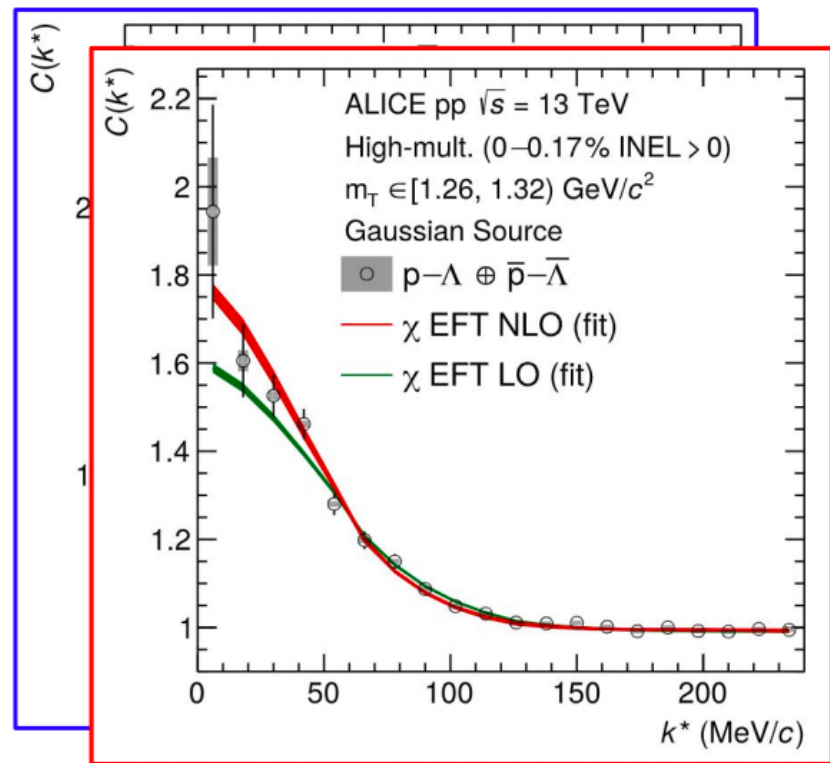
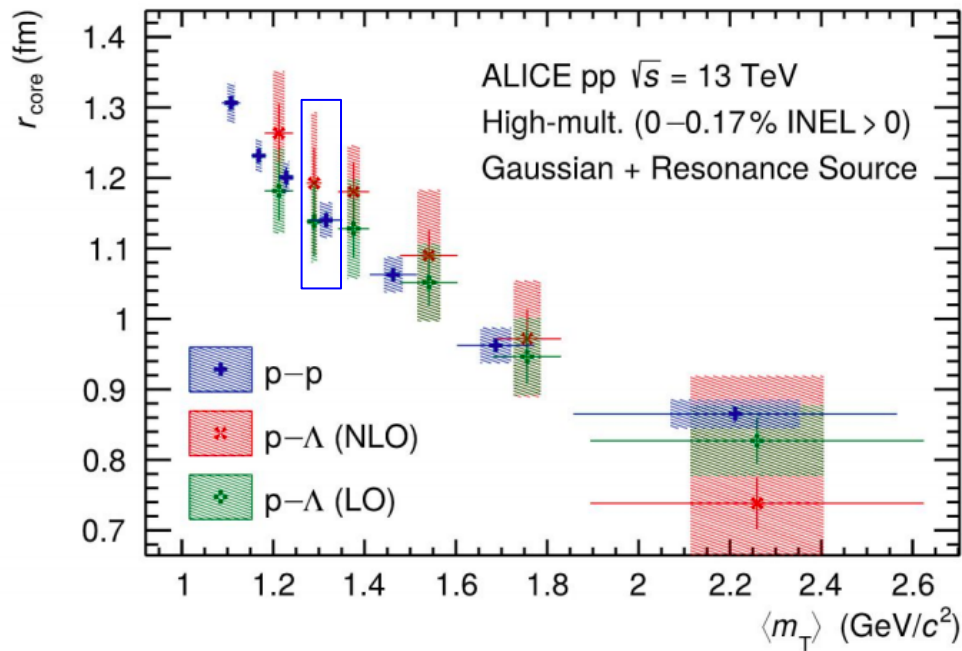


# Source determination



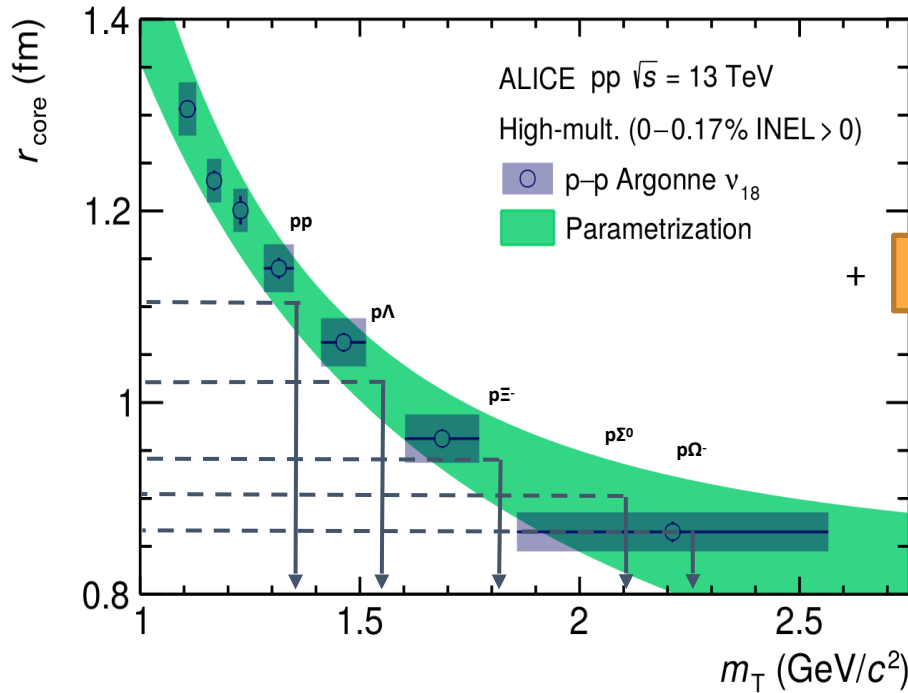
[ALICE Coll., Phys. Lett. B 811 (2020) 135849]

# Source determination



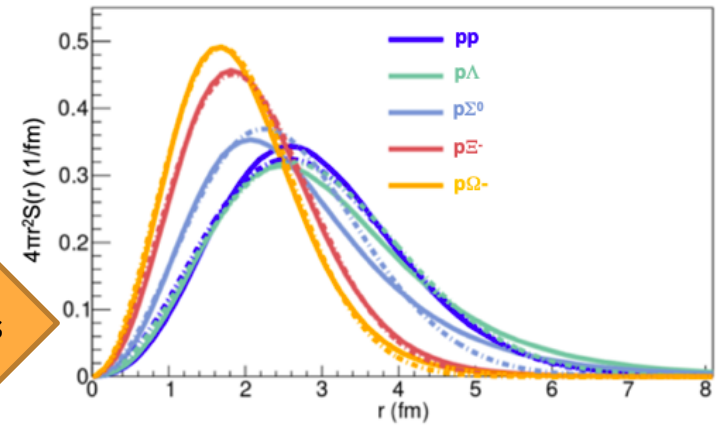
[ALICE Coll., Phys. Lett. B 811 (2020) 135849]

# Gaussian source with resonances



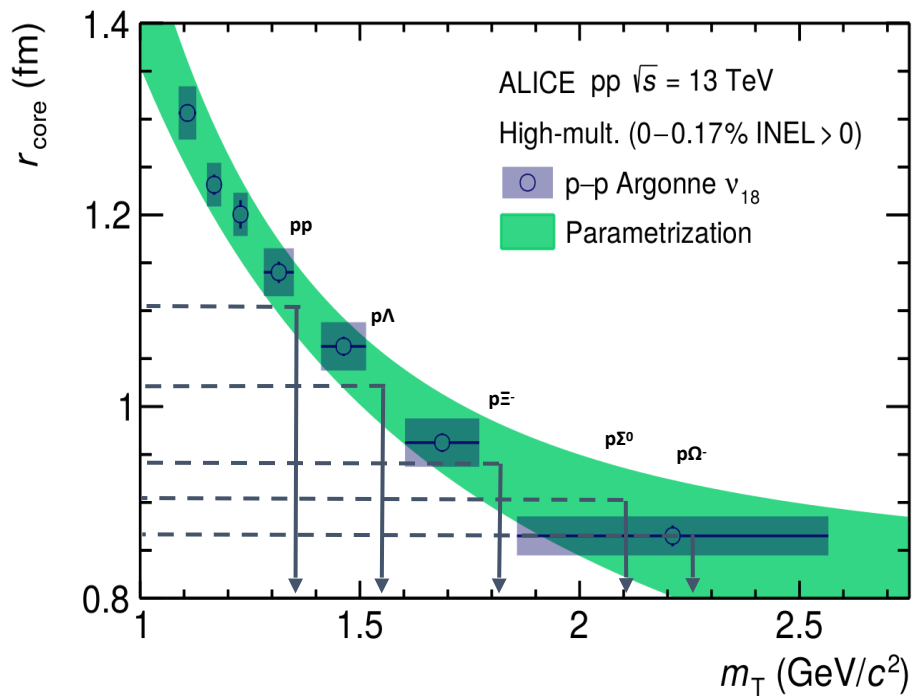
ALICE Coll. PLB 811 (2020)

+ Resonances



Pair	$r_{\text{Core}}$ [fm]	$r_{\text{Eff}}$ [fm]
p-p	1.1	1.2
p- $\Lambda$	1.0	1.3
p- $\Sigma^0$	0.87	1.02
p- $\Xi^-$	0.93	1.02
p- $\Omega^-$	0.86	0.95

# Small particle-emitting sources



ALICE Coll. PLB 811 (2020)

Small particle-emitting source created in pp and p-Pb collisions at the LHC.

