# Flavoured jet algorithms

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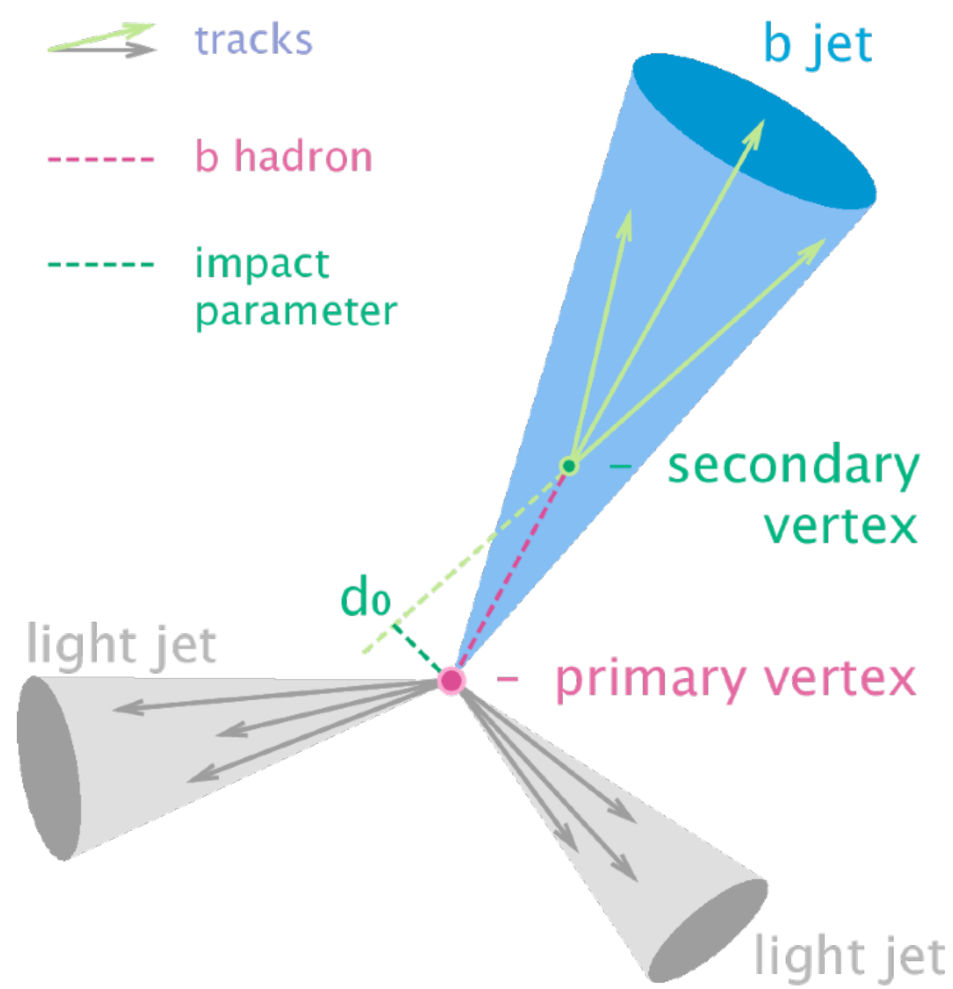
LHCP 2023, Belgrade, 22-26.05.2023

# (Usual) experimental definition of flavoured jet

"An (anti-$k_t$) jet is flavoured if it contains at least one heavy hadron within $\Delta R < R$ with $p_T > p_{T,\mathrm{cut}}$"

This definition is adopted as "true" label in MC samples.

These samples are then used to train ML architectures ("high-level taggers"), which exploit low-level variables as inputs.

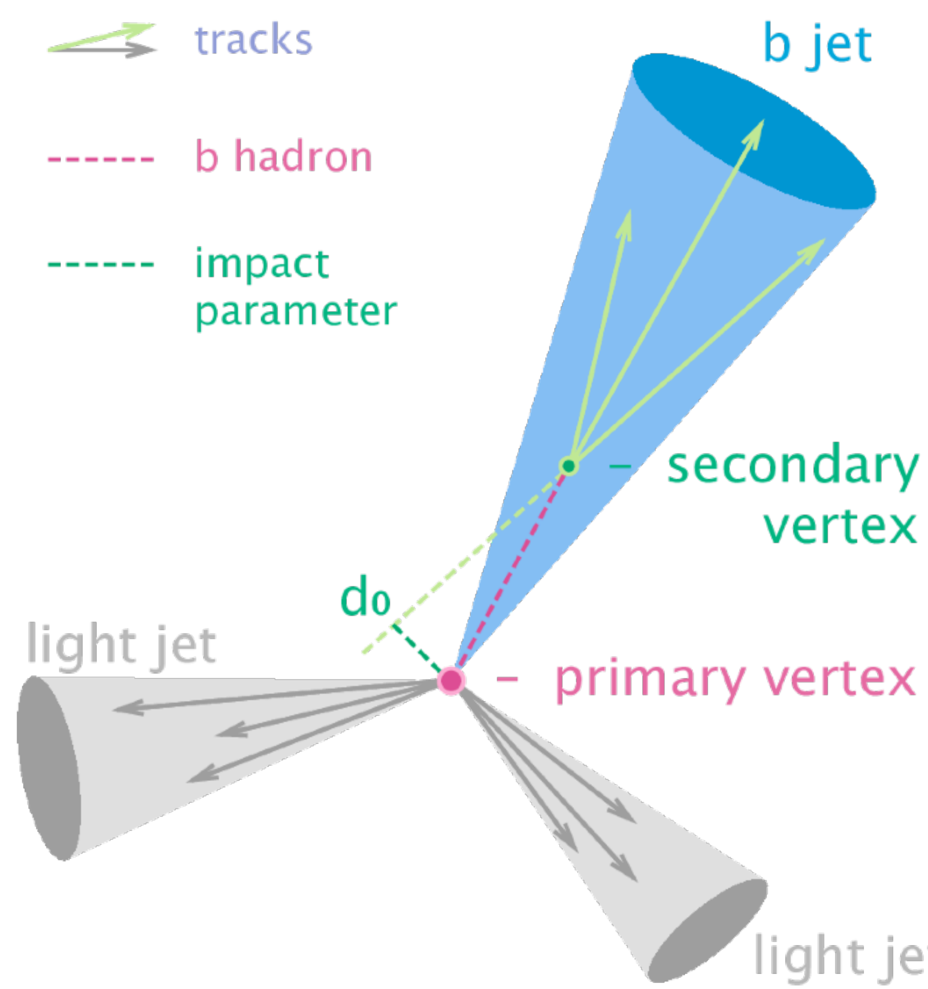


# (Usual) experimental definition of flavoured jet

"An (anti-$k_t$) jet is flavoured if it contains at least one heavy hadron within $\Delta R < R$ with $p_T > p_{T,\text{cut}}$"

This definition is both
**soft and collinear**
**(IRC) unsafe**
(in massless perturbative QCD calculations)

i.e. arbitrary soft and/or collinear emissions alter the flavour of jets



# (Usual) experimental definition of flavoured jet

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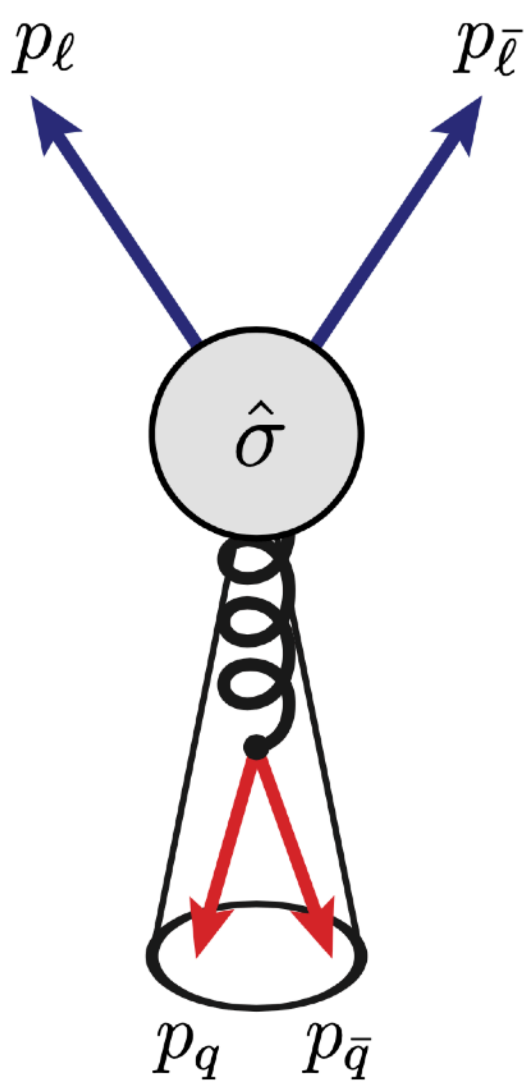


**$g \to q\bar{q}$ is always flavoured even in the collinear limit**

An *even-tag veto* in calculations is enough to fix this issue

# (Usual) experimental definition of flavoured jet

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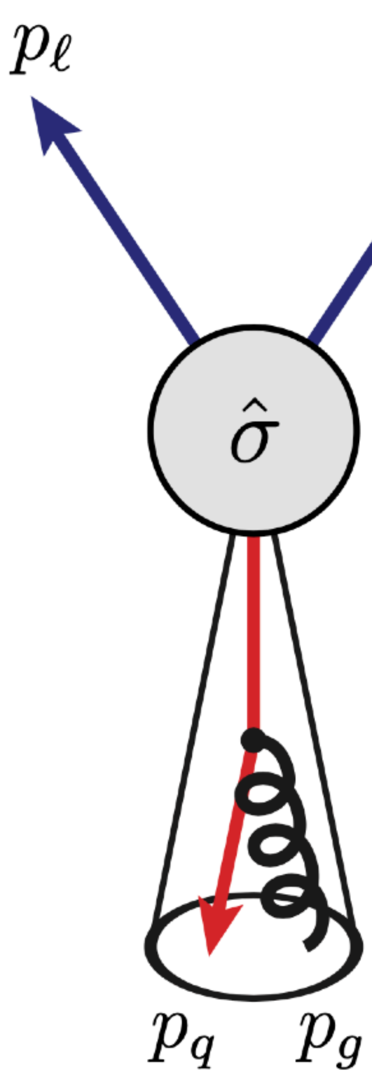


**$q \to qg$ collinear with a hard gluon leads to a flavourless jet**

With $p_{T,\mathrm{cut}}$, it requires a fragmentation function, as we are identifying a particle

Without $p_{T,\mathrm{cut}}$, any IRC safe flavour-agnostic algorithm will recombine the $qg$ pair

# (Usual) experimental definition of flavoured jet

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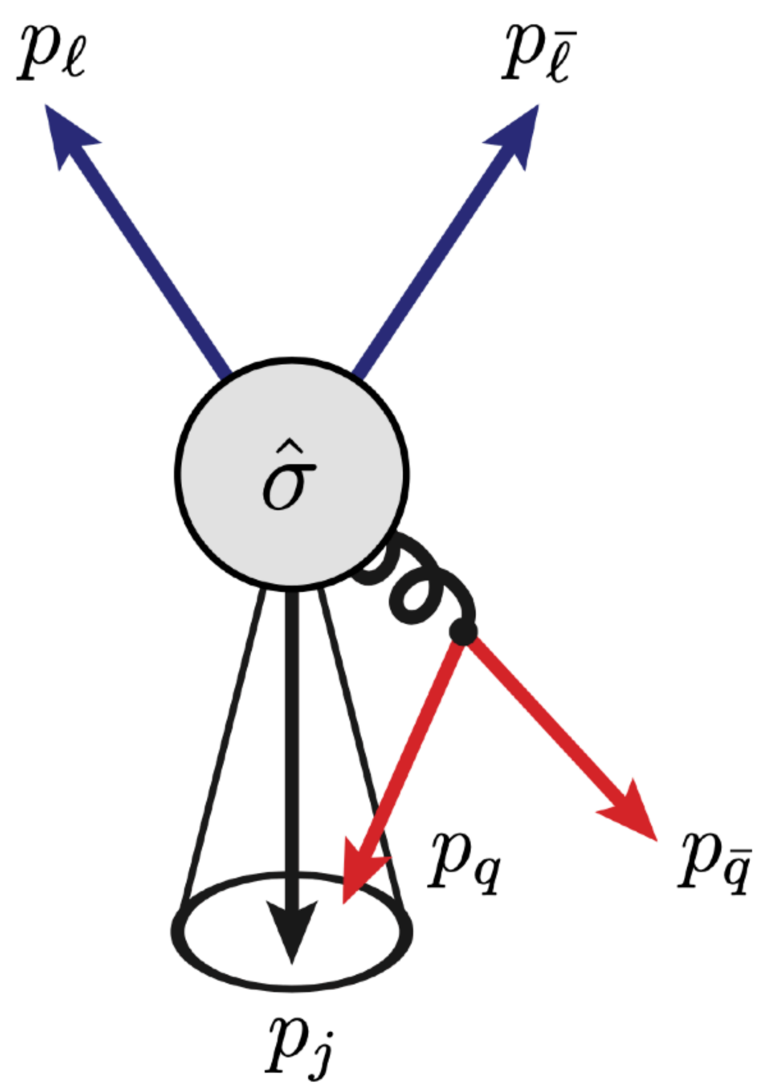



**Soft large-angle $g \to b\bar{b}$**

**polluting the flavour of other jets**

No way of fixing this issue within a flavour-agnostic jet algorithm!

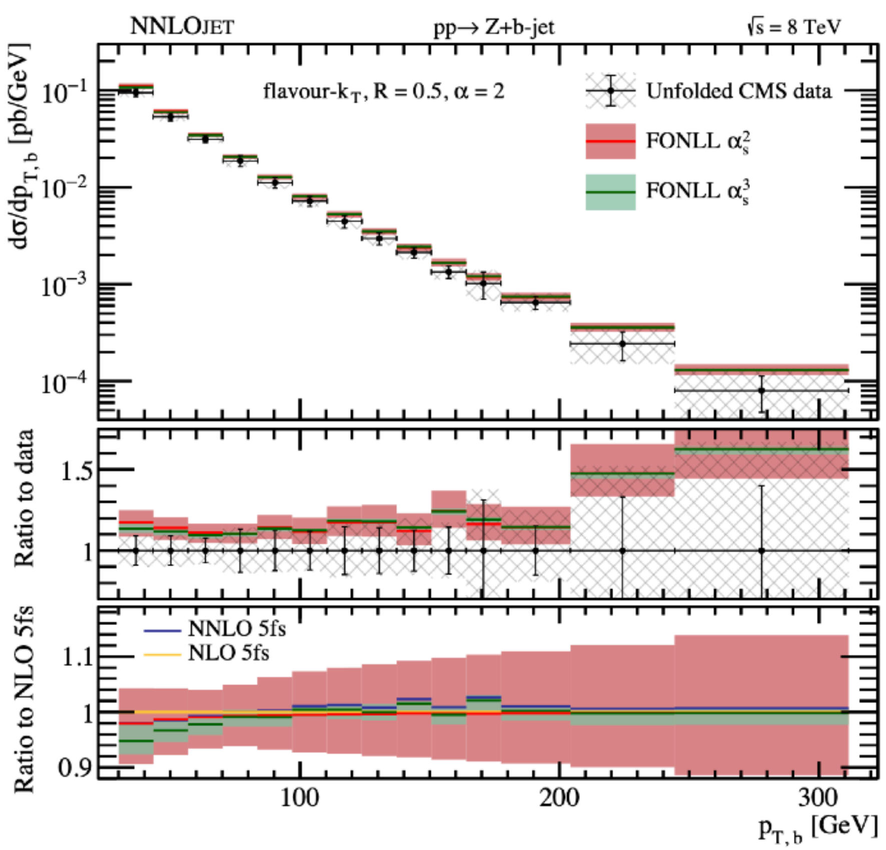
# Solution: the flavour-$k_t$ algorithm

[Banfi, Salam, Zanderighi (hep-ph/0601139)]

Flavour-aware distance:

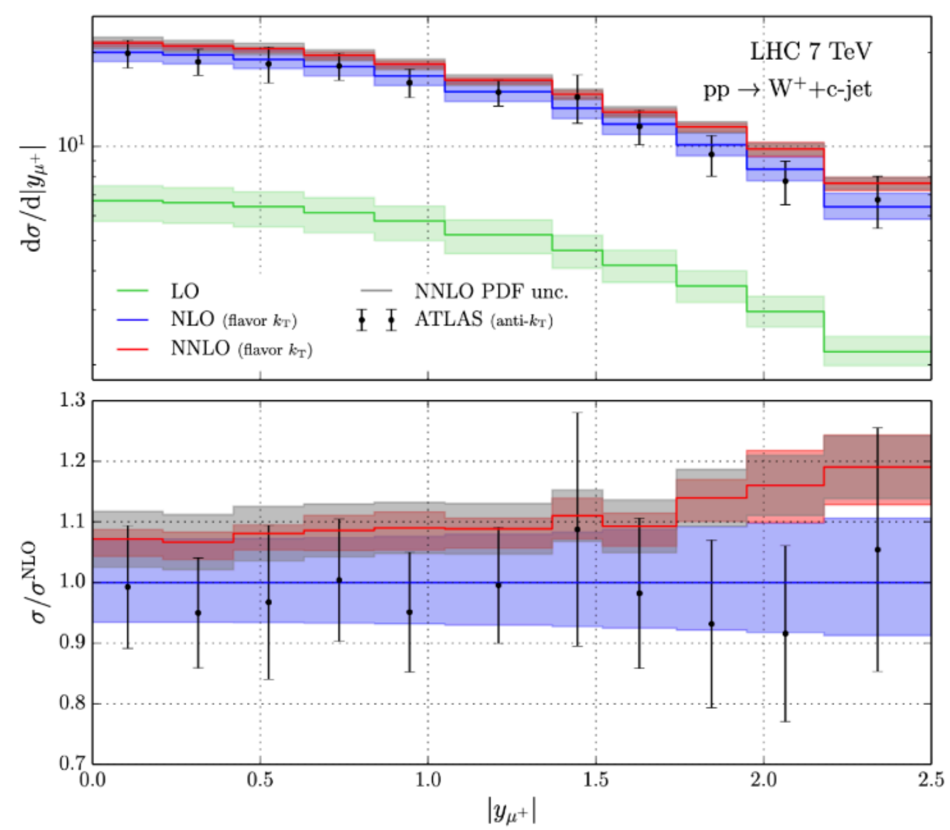
$$d_{ij}^{(F,\alpha)} = \frac{\Delta y_{ij}^2 + \Delta\phi_{ij}^2}{R^2} \times \begin{cases} \max(k_{ti}, k_{tj})^{\alpha} \min(k_{ti}, k_{tj})^{2-\alpha}\,, & \text{softer of } i, j \text{ is flavoured,} \\ \min(k_{ti}^2, k_{tj}^2)\,, & \text{softer of } i, j \text{ is flavourless} \end{cases}$$

at the price of jets with different kinematics i.e. not anti-$k_t$ jets.




[Gauld et al. (2005.03016)]

Comparison with experimental data **not straightforward**




[Czakon et al. (2011.01011)]

## In the past year, several alternative proposals!

[Caletti, Larkoski, Marzani, Reichelt (2205.01109)]

[Caletti, Larkoski, Marzani, Reichelt (2205.01117)]

[Czakon, Mitov, Poncelet (2205.11879)]

[Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler (to appear)]

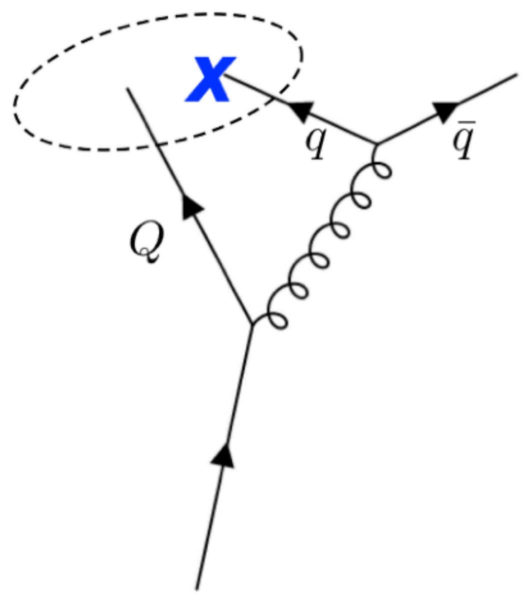
[Gauld, Huss, GS (2208.11138)]

I will briefly introduce them,

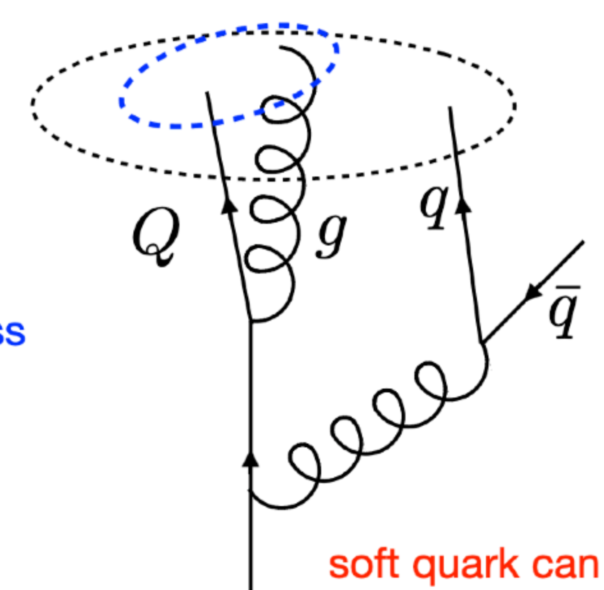
by then focusing on the last one

[Caletti, Larkoski, Marzani, Reichelt (2205.01109)]

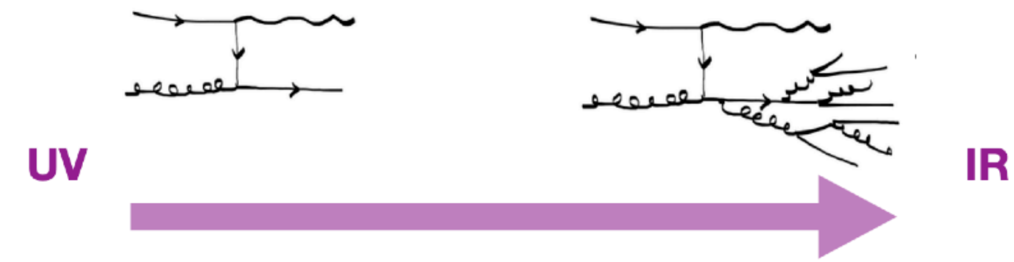
Use Soft Drop to remove soft quarks, by using JADE as reclusters




Known to fail at N3LO

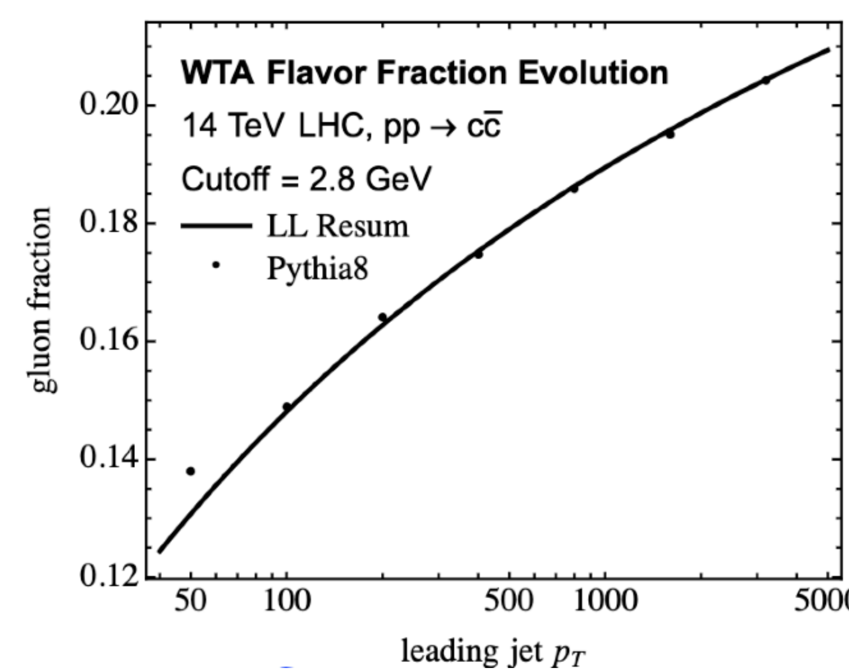



[Caletti, Larkoski, Marzani, Reichelt (2205.01117)]




Flavour of jet = flavour of particle(s) lying along the Winner-Take-All (WTA) axis

Soft safe, but collinear unsafe: requires usage of suited fragmentation functions



**[Czakon, Mitov, Poncelet (2205.11879)]**

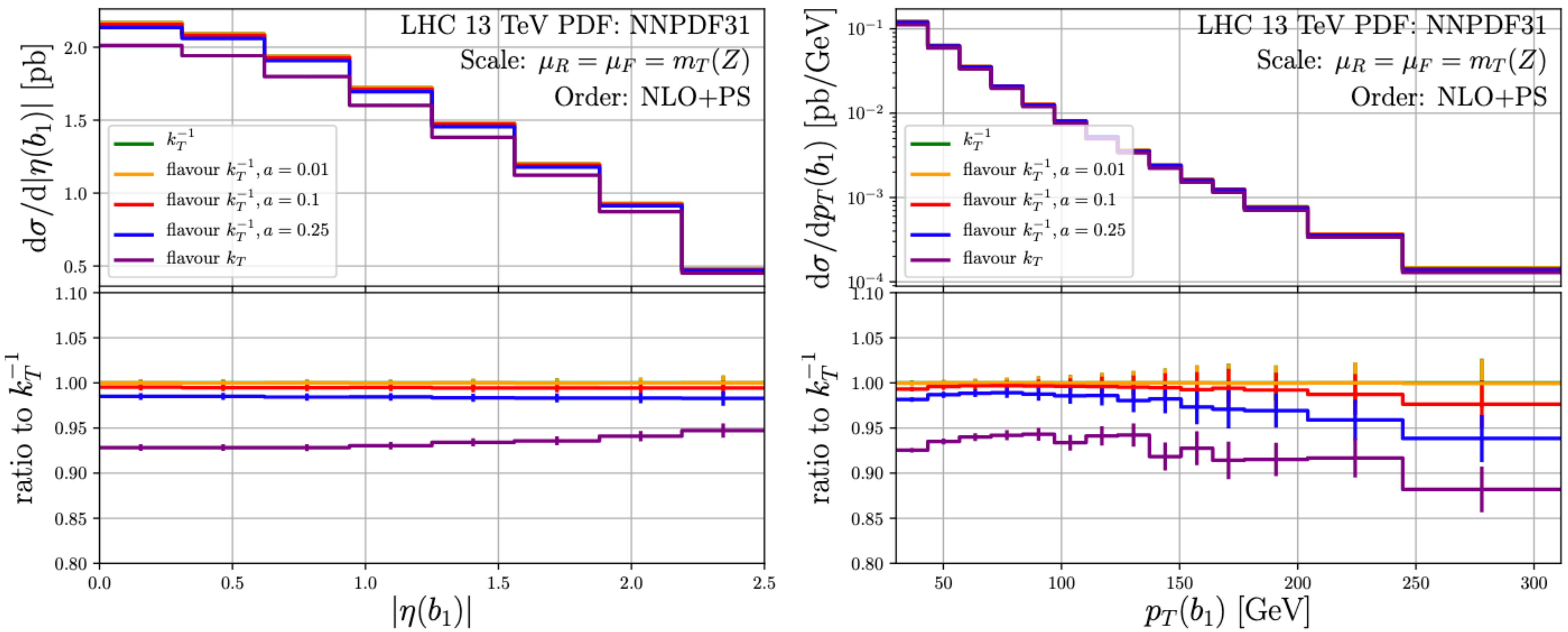
"**Flavour anti-$k_t$**": modify anti-$k_t$ distance when flavoured particles involved

$$d_{ij} = R^2 \min(k_{T,i}^{-2}, k_{T,j}^{-2}) \cdot S_{ij}^a \,, \quad d_B = k_{T,i}^{-2}$$

where $S_{ij} \neq 1$ only when $i$ and $j$ are of opposite flavour

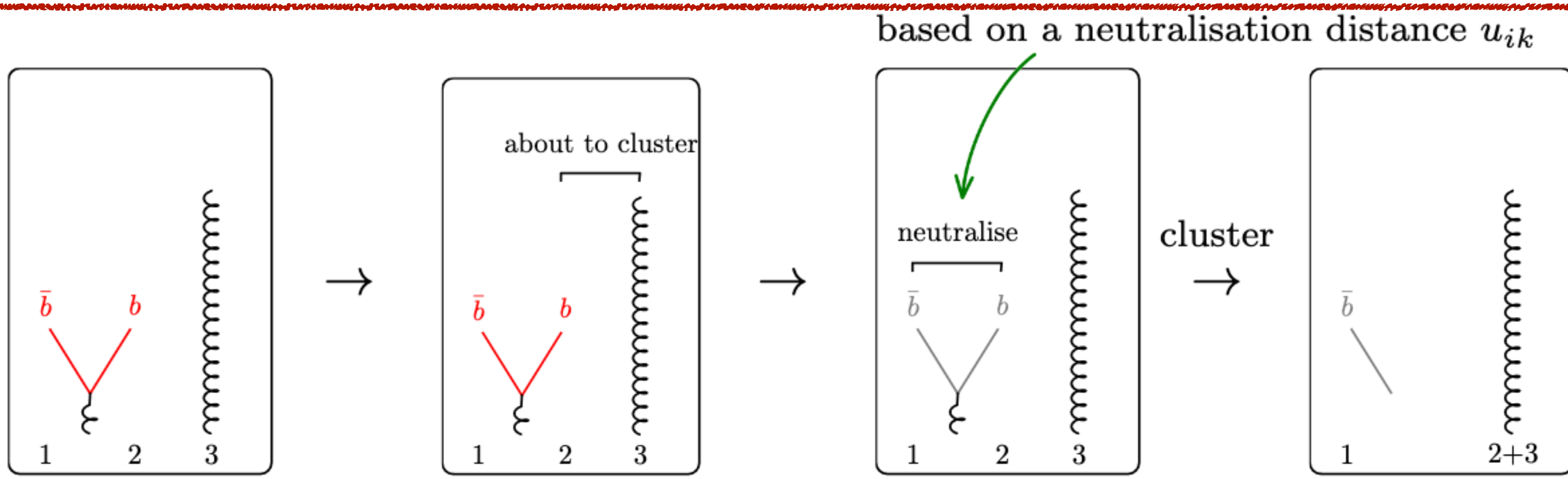
$$S_{ij}^a = 1 - \theta(1-\kappa)\cos\left(\frac{\pi}{2}\kappa\right) \,, \quad \kappa = \frac{1}{a}\frac{k_{T,i}^2 + k_{T,j}^2}{2\,k_{T,\max}^2}$$

One recovers (IRC flavour unsafe) anti-$k_t$ jets when $a \to 0$



[Caola, Grabarczyk, Hutt, Salam, Scyboz, Thaler (to appear)]

## "Flavour neutralisation"




neutralise ≡ remove the (opposite) flavour of both 1 & 2 while maintaining kinematics

- Generic form (with parameters $p$, $q$, and $a$):

$$u_{ik} = \max\left(p_{ti}^2, p_{tk}^2\right)^p \min\left(p_{ti}^2, p_{tk}^2\right)^q \times 2\left[\frac{1}{a^2}\left(\cosh(a\Delta y_{ik}) - 1\right) - \left(\cos\Delta\phi_{ik} - 1\right)\right]$$

from Ludovic Scyboz slides at Moriond QCD 2023

[Gauld, Huss, GS (2208.11138)]

**"Flavour dressing"**
Flavour assignment *factorised* from jet reconstruction
(exact anti-$k_t$ kinematics by construction)

**Inputs**: *flavour-agnostic jets* (jets obtained with any IRC safe algorithm) and *flavour inputs* (e.g. b- or c-quarks, stable heavy-flavour hadrons, …)

**Preliminary step**: we first build flavour clusters to recombine flavour inputs with radiation close in angle, but without touching the soft particles (thanks to a Soft Drop condition [Larkoski, Marzani, Soyez, Thaler 1402.2657]):

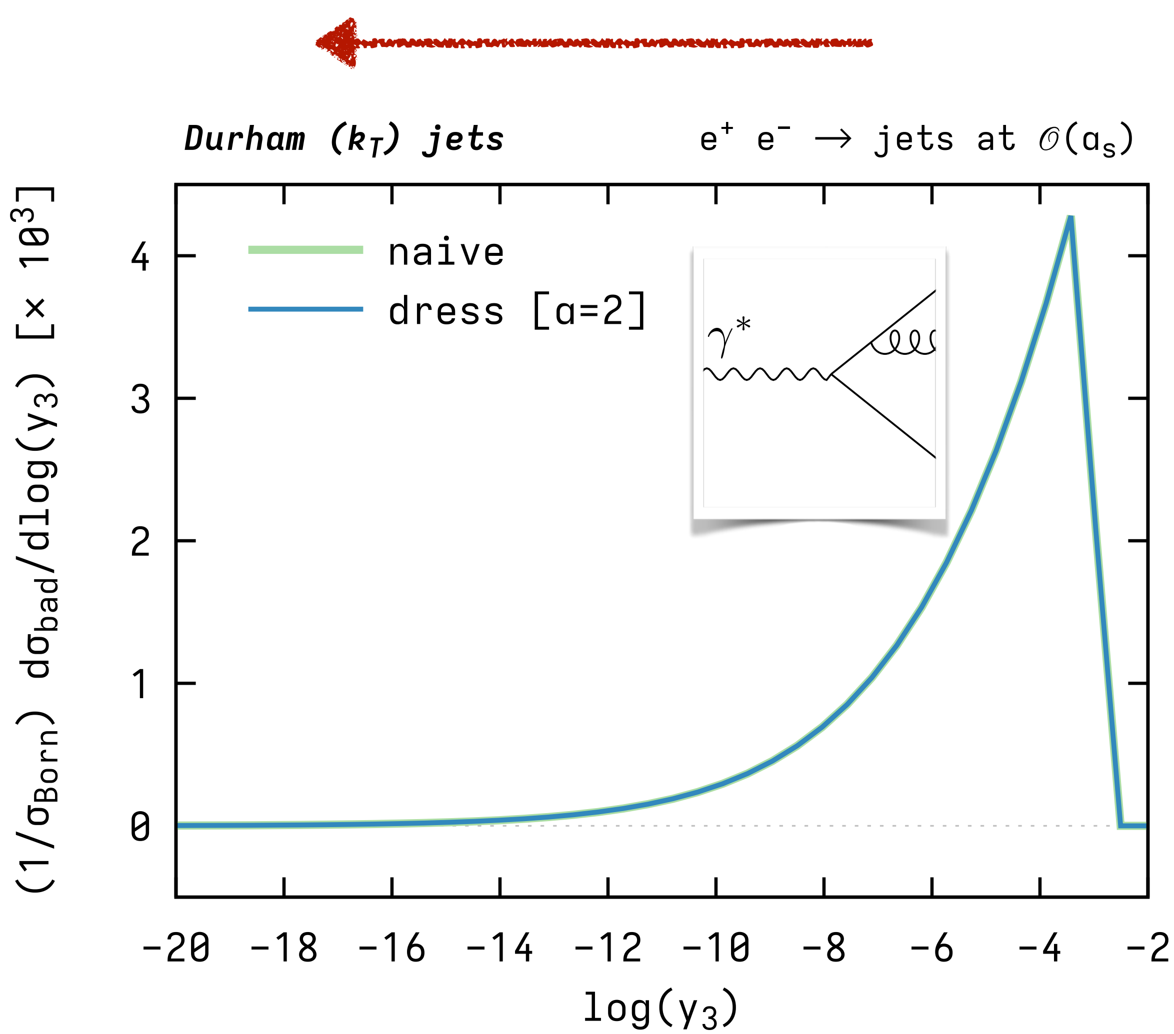
$$\frac{\min(p_{t,a}, p_{t,b})}{(p_{t,a} + p_{t,b})} > z_{\mathrm{cut}} \left( \frac{\Delta R_{ab}}{\delta R} \right)^{\beta}$$

**Dressing step**: in order to assign flavour to jets, we run a sequential recombination algorithm with flavour-$k_t$-like distances between jets and flavour clusters.

# IRC safety test in $e^+e^- \to$ jets

= vanishing "bad" identification of flavours
in the fully unresolved regime

**only soft and/or collinear radiation**




Any gen-$k_t$ algo is safe (no additional flavour in the event)

# IRC safety test in $e^+e^- \to$ jets

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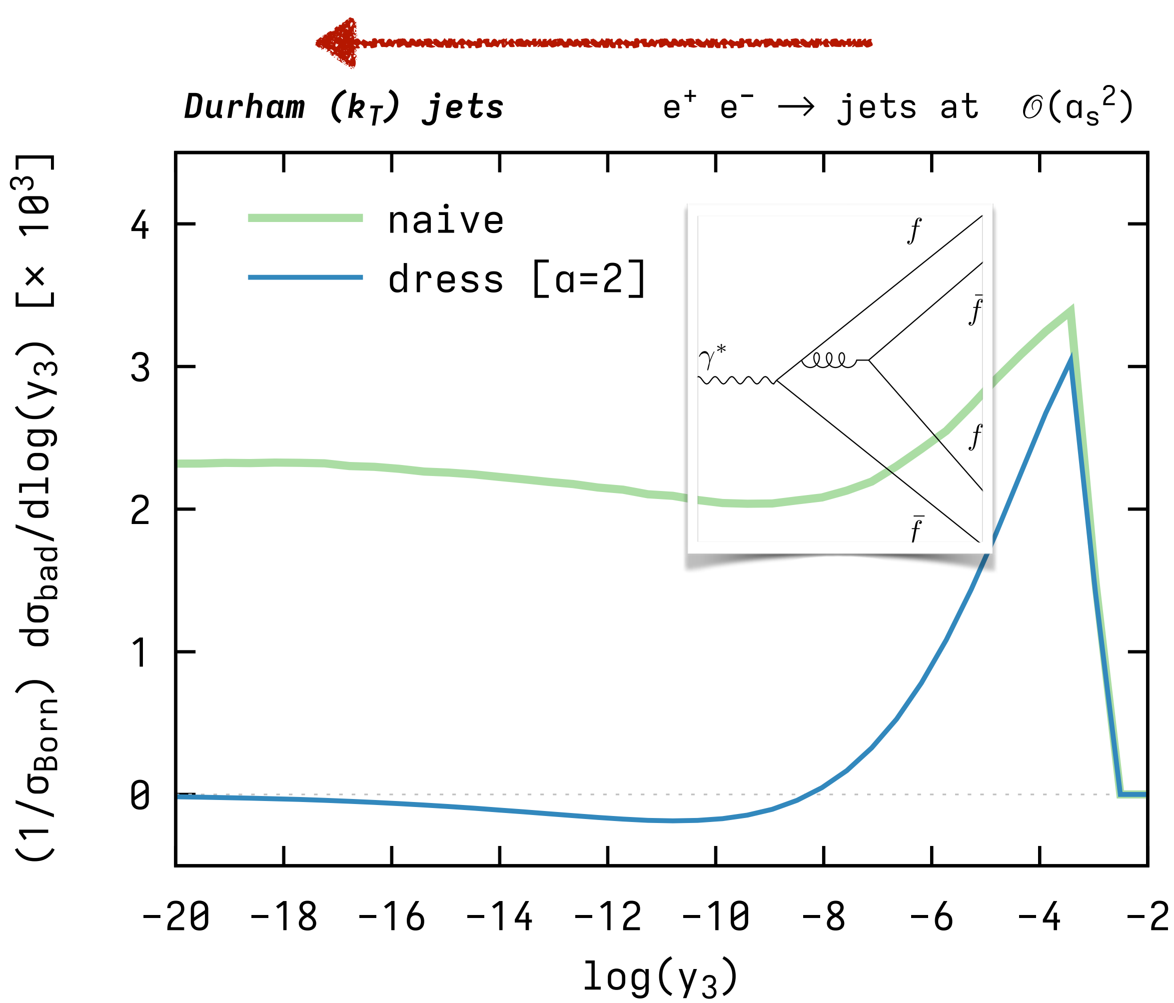


**Naive dressing unsafe, flavour dressing safe!**

# IRC safety test in $e^+e^- \to$ jets

= vanishing “bad” identification of flavours
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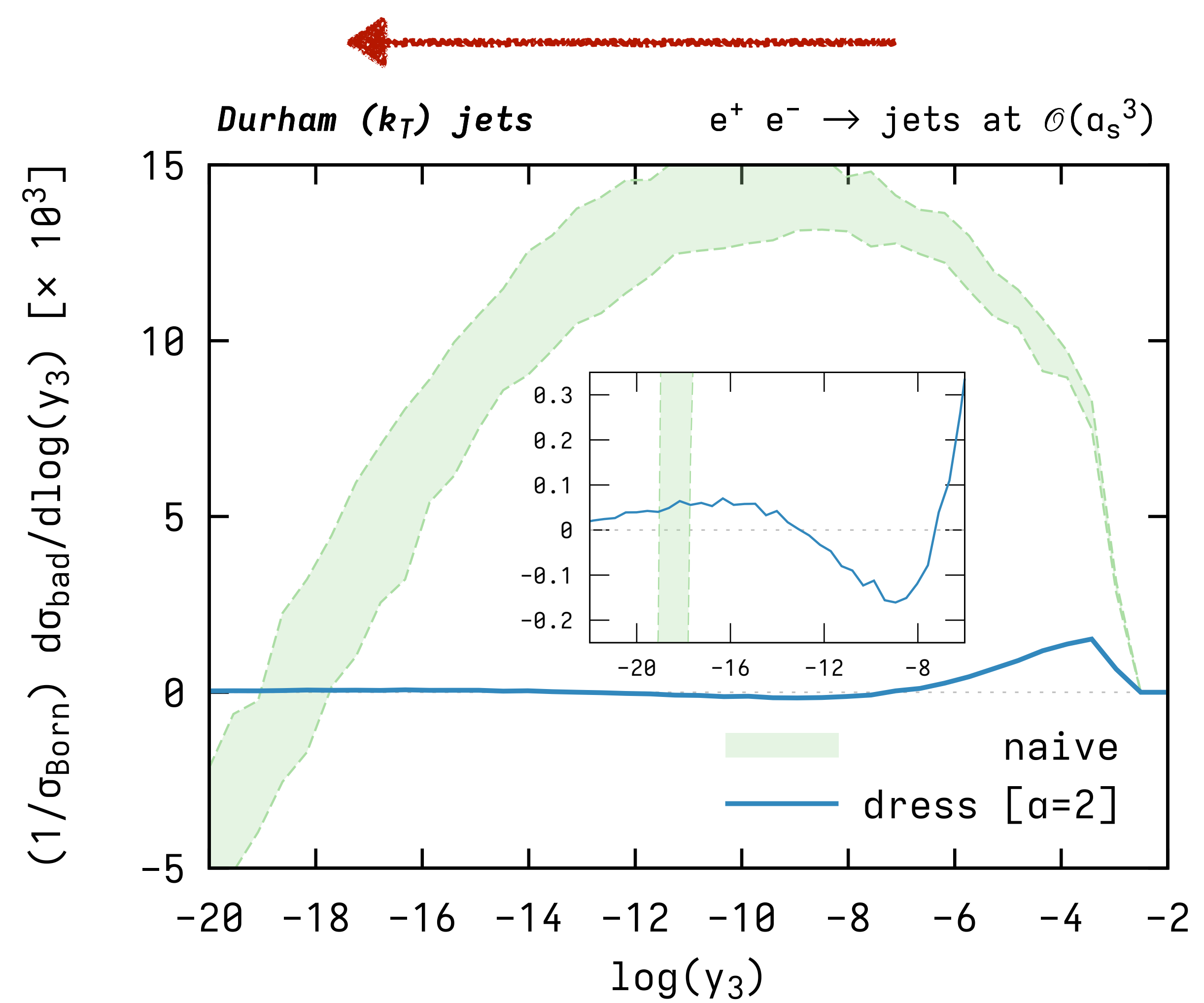
**only soft and/or collinear radiation**



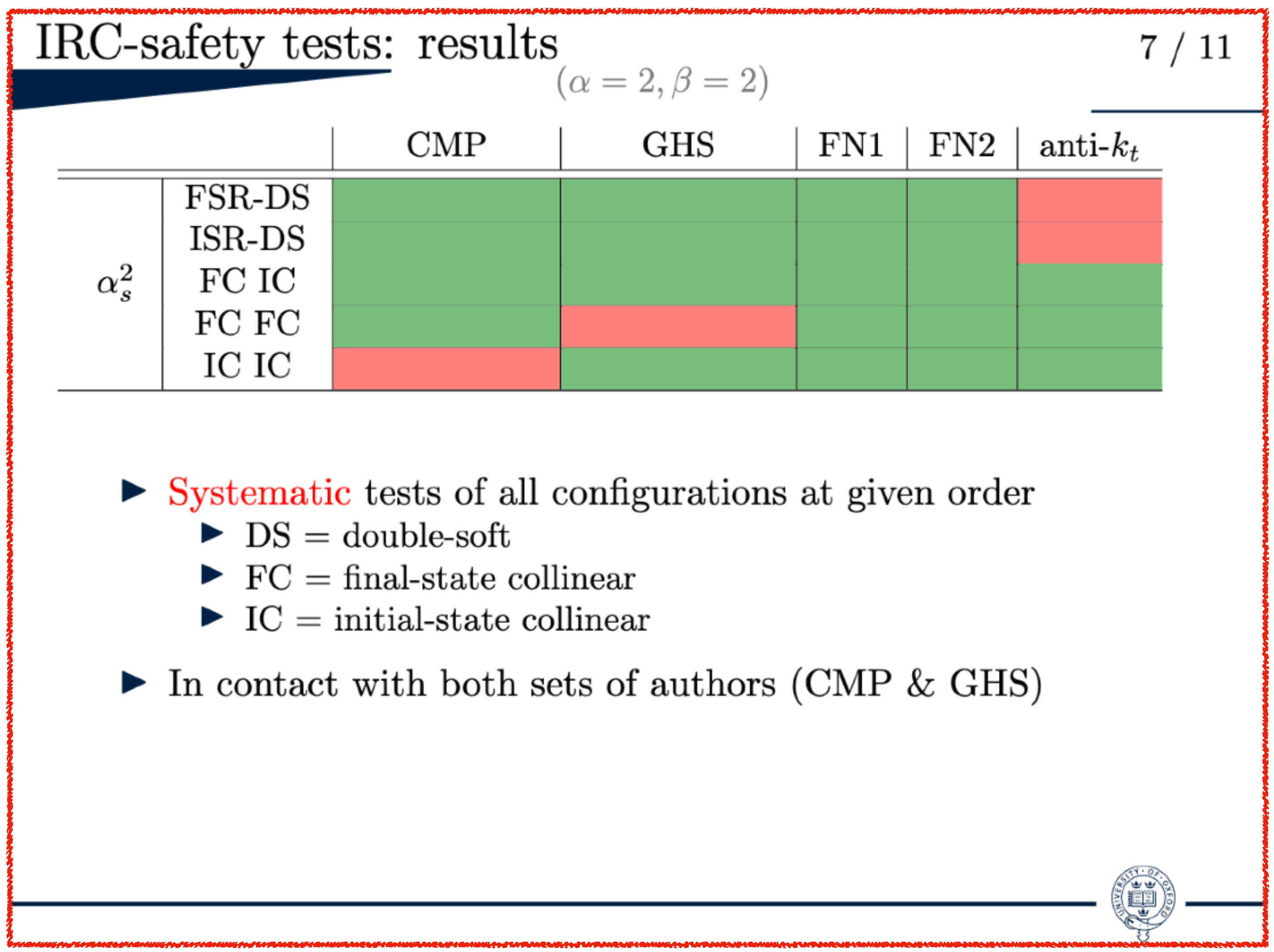

**Naive dressing unsafer, flavour dressing still safe!**

# Systematic IRC safety tests

Numerical framework developed by Caola et al. has allowed to discover potentially problematic configurations at higher orders

(CMP = “flavour anti-$k_t$”; GHS = “flavour dressing”)

→ as for GHS, work in progress to fix them




from Ludovic Scyboz slides at Moriond QCD 2023

# (Massive calculations?)

In principle, massive calculations do not require an IRC safe flavour algorithm (screening effect due to $m_q$).

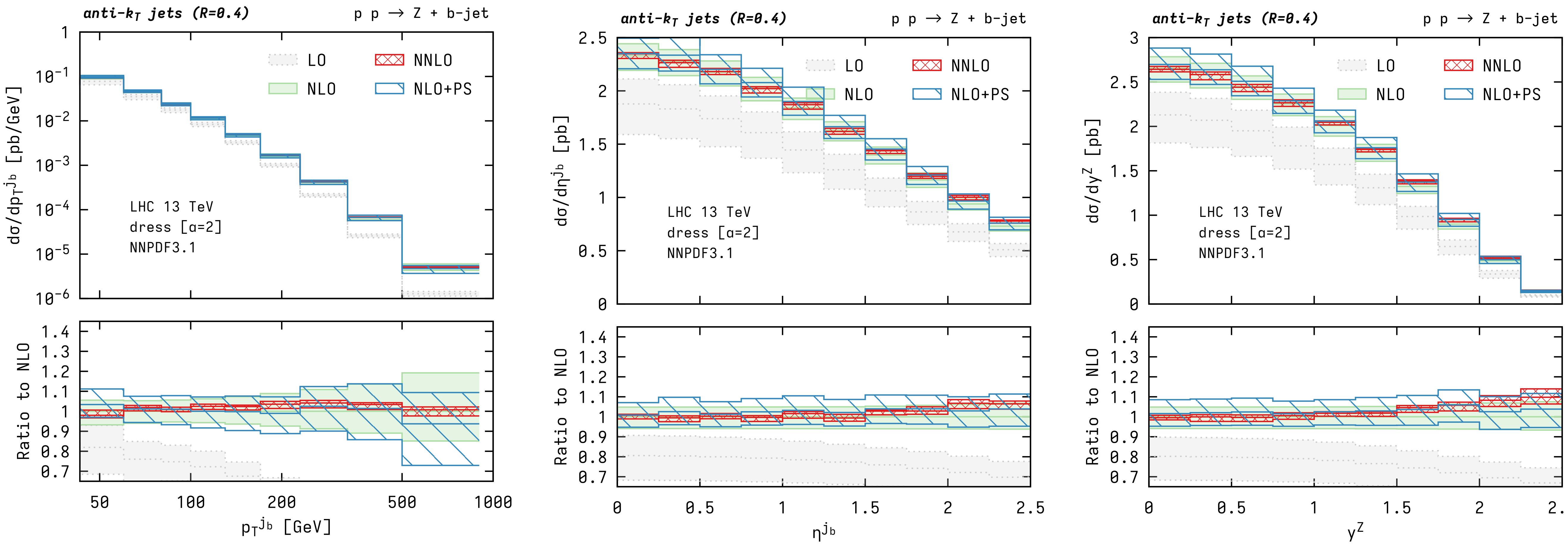
However, presence of large logarithms $\log(Q^2/m_q^2)$, spoiling the convergence of the perturbative series ($\alpha_s \log(m_Z^2/m_c^2) \sim 1$).

Benefits of massless calculations with IRC safe jet tagging:

- in the initial-state, a massless calculation allows for a resummation of $\log(Q^2/m_q^2)$ by PDF evolution (crucial in some cases e.g. when probing non-perturbative charm PDF)
- in the final-state, an IRC safe prescription implies a suppressed sensitivity on $\log(Q^2/m_q^2)$, both in fixed order and resummed calculations / parton showers.

# Test flavour dressing in a realist scenario: $Z + b$-jet

[same setup of Gauld, Gehrmann-De Ridder, Glover, Huss, Majer (2005.03016)]

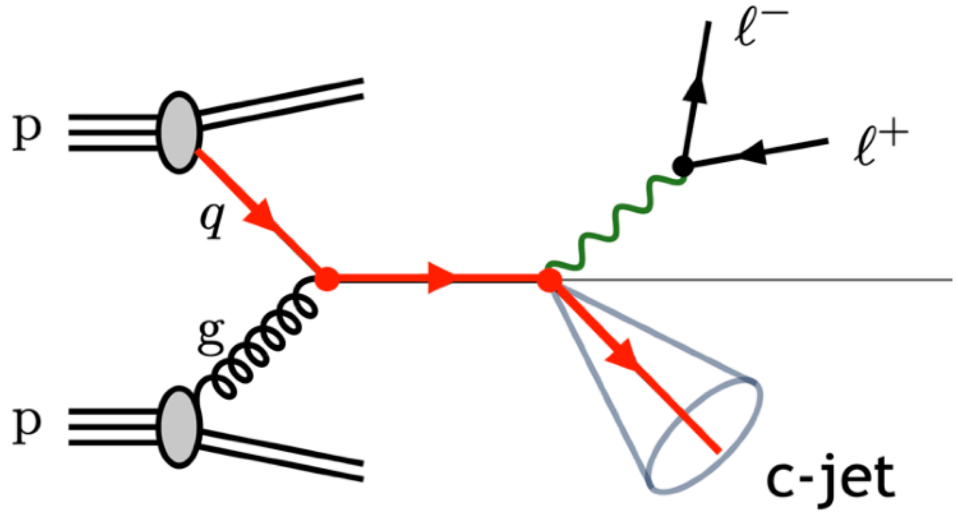


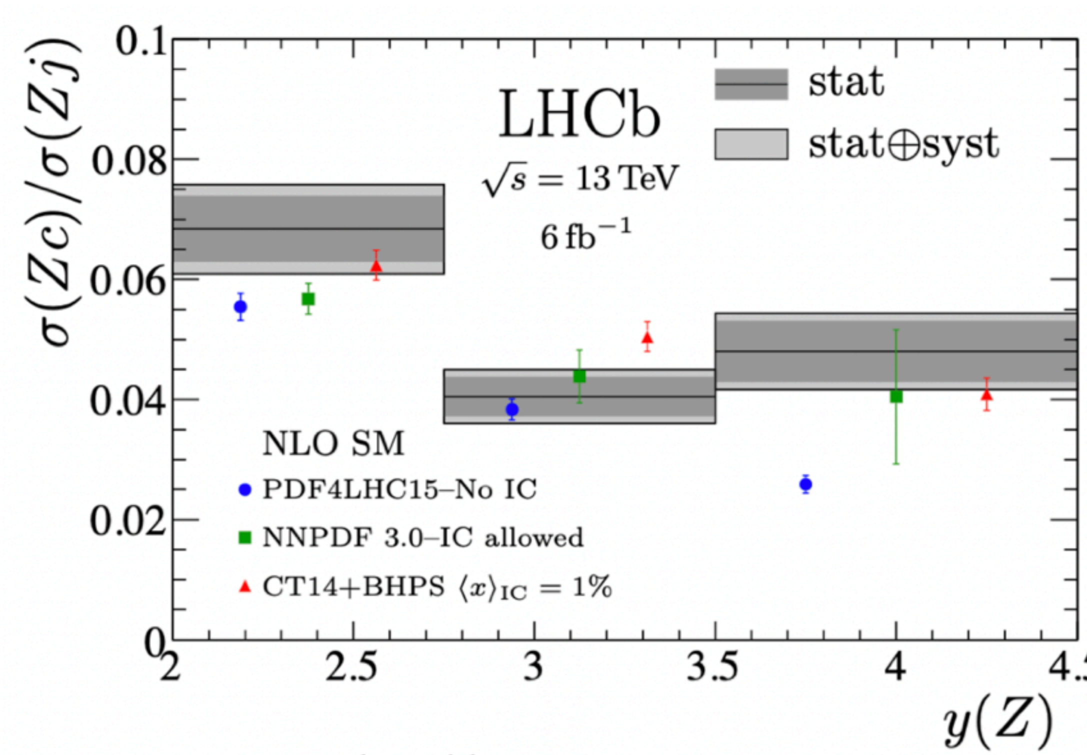

Remarkable agreement between (N)NLO and NLO+PS

→ for most distributions

**largely insensitive to all-order corrections**

# First new result with flavour dressing:
# $Z + c$-jet at LHCb




Measurement sensitive to **intrinsic charm in the proton**

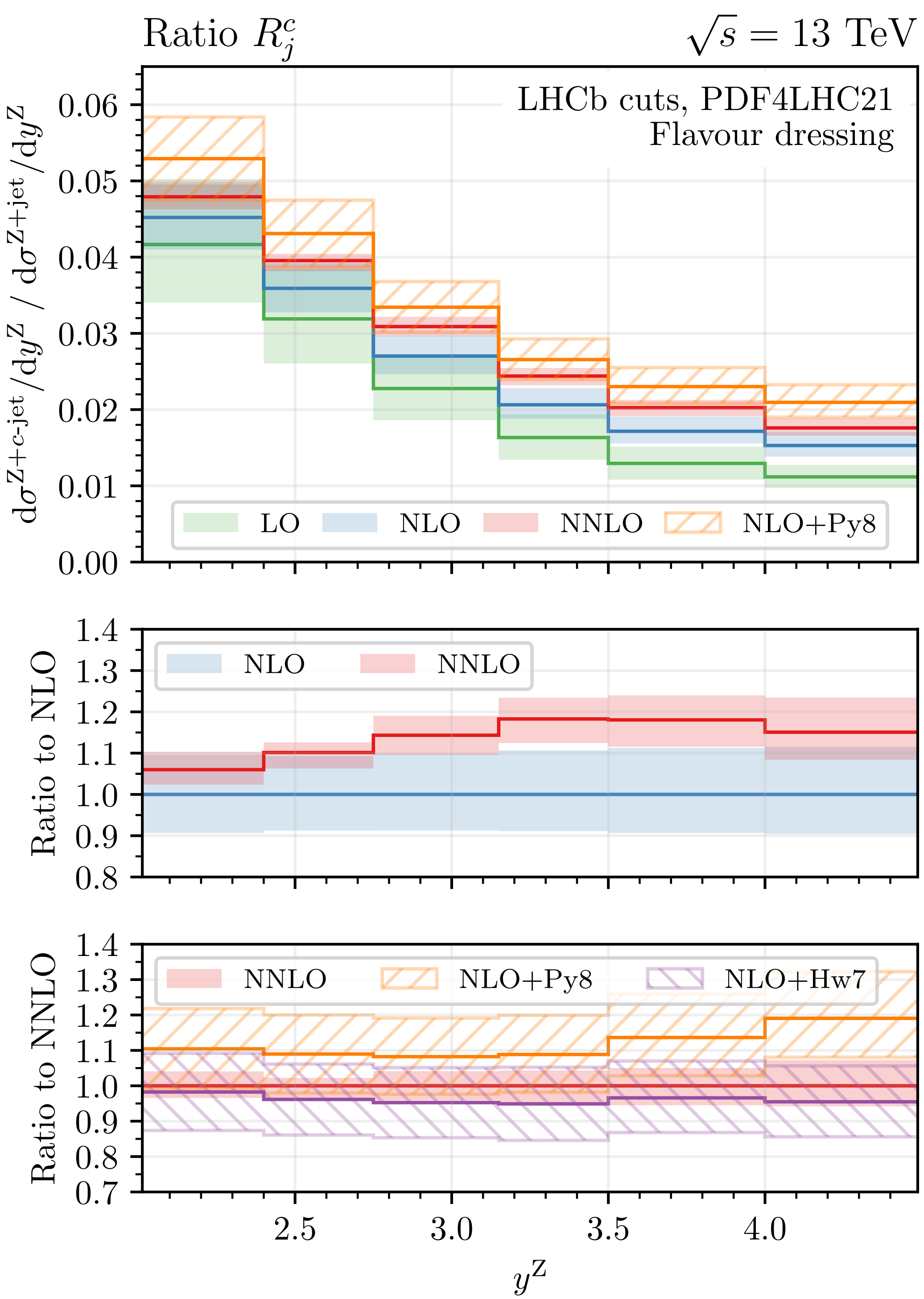
LHCb data at 13TeV for ratio

$(\mathrm{d}\sigma_{Z+c}/dy_Z)$ / $(\mathrm{d}\sigma_{Z+j}/dy_Z)$ [2109.08084]

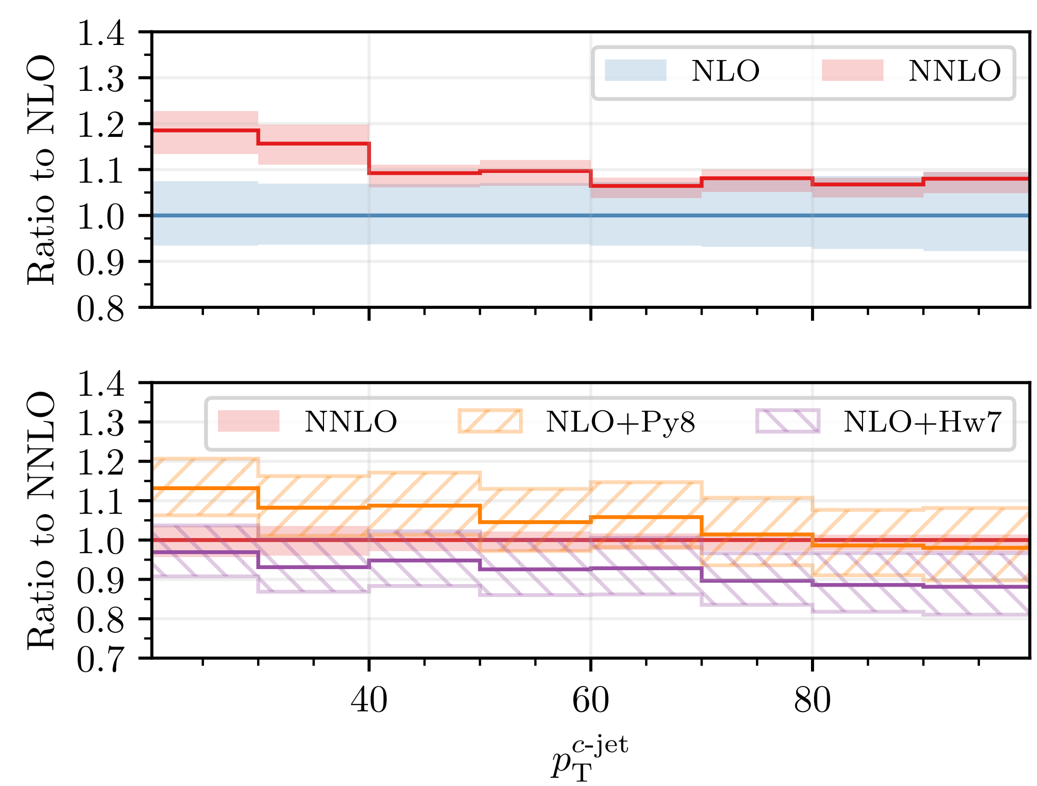
(With flavour dressing, both the numerator and the denominator feature the same sample of anti-$k_t$ jets!)

# Ratio $\sigma(Z + c - \text{jet})$ / $\sigma(Z + \text{jet})$ at NNLO

[Gauld, Gehrmann-De Ridder, Glover, Huss, Rodriguez Garcia, GS (2302.12844)]




**NNLO lies between NLO+PS** predictions with different PS, but **reduction of theory uncertainties** by a factor of 2. Similar for other distributions



# Final remarks

- At lot of recent proposals trying to solve the longstanding issue of a proper definition of flavoured jet

- IRC-safe definition allows for massless fixed-order calculations to be directly compared to experimental data (and a suppressed sensitivity on mass logarithms)

- A comparison between the different approaches would be beneficial, as well as a study of their experimental feasibility

# BACKUP

# LHCb fiducial cuts

Very unique fiducial region of the measurement:

| | |
|---|---|
| $Z$ bosons | $p_{\mathrm{T}}(\mu) > 20\,\mathrm{GeV},\ 2.0 < \eta(\mu) < 4.5,\ 60 < m(\mu^+\mu^-) < 120\,\mathrm{GeV}$ |
| Jets | $20 < p_{\mathrm{T}}(j) < 100\,\mathrm{GeV},\ 2.2 < \eta(j) < 4.2$ |
| Charm jets | $p_{\mathrm{T}}(c \text{ hadron}) > 5\,\mathrm{GeV},\ \Delta R(j, c \text{ hadron}) < 0.5$ |
| Events | $\Delta R(\mu, j) > 0.5$ |

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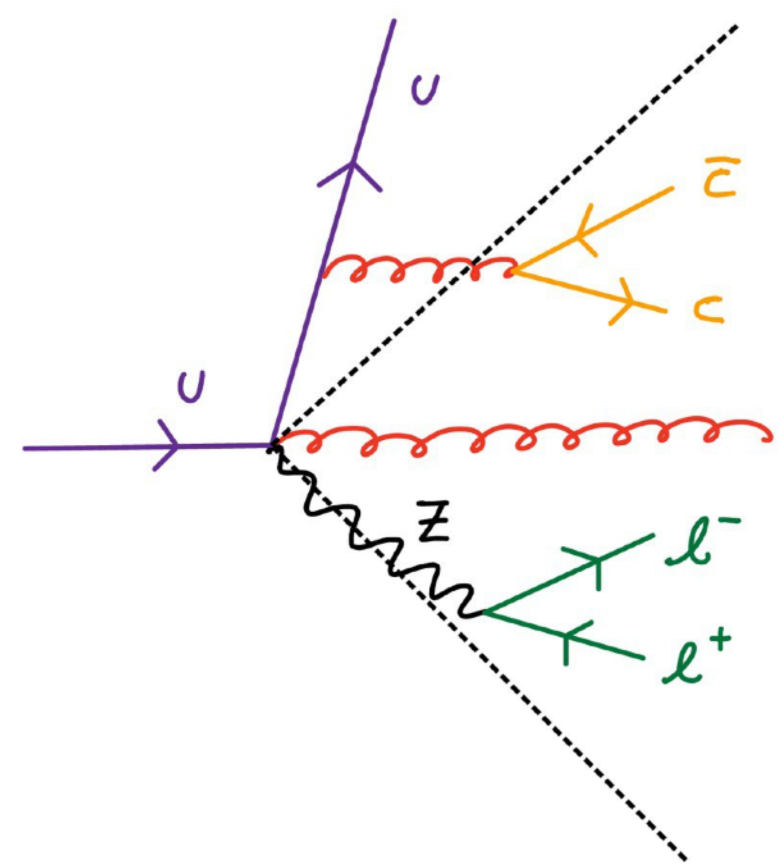
| Events | $\Delta R(\mu, j) > 0.5$ |

We explore a theory-driven cut:

$$p_{\mathrm{T}}(Z + \mathrm{jet}) < \mathrm{p}_{\mathrm{T,jet}}$$

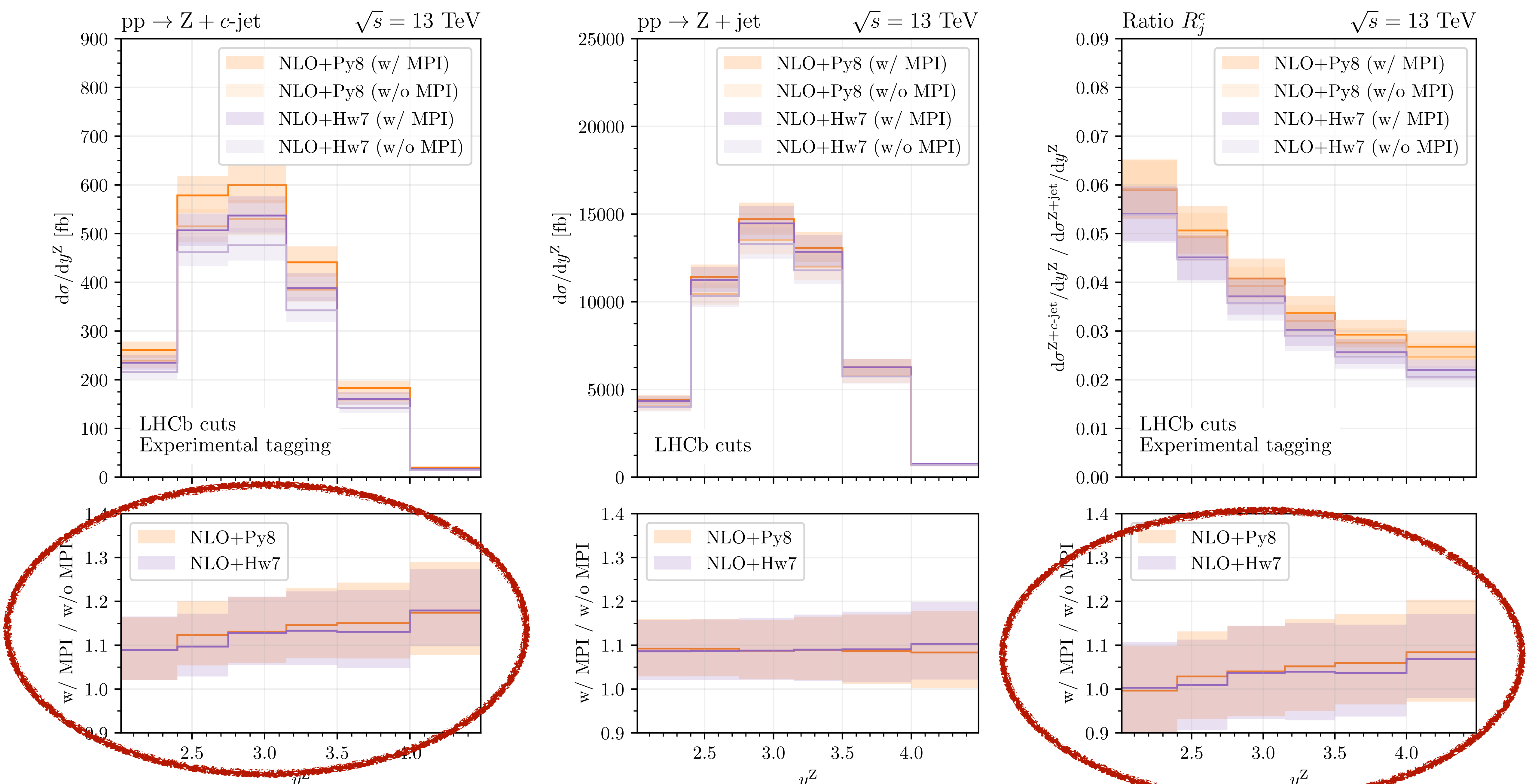
At Born level, the $p_{\mathrm{T}}$ of the $Z$+jet system vanishes, hence the cut limits the hard QCD radiation outside the LHCb acceptance in a dynamical way.



# We refrain from making a comparison to the LHCb data

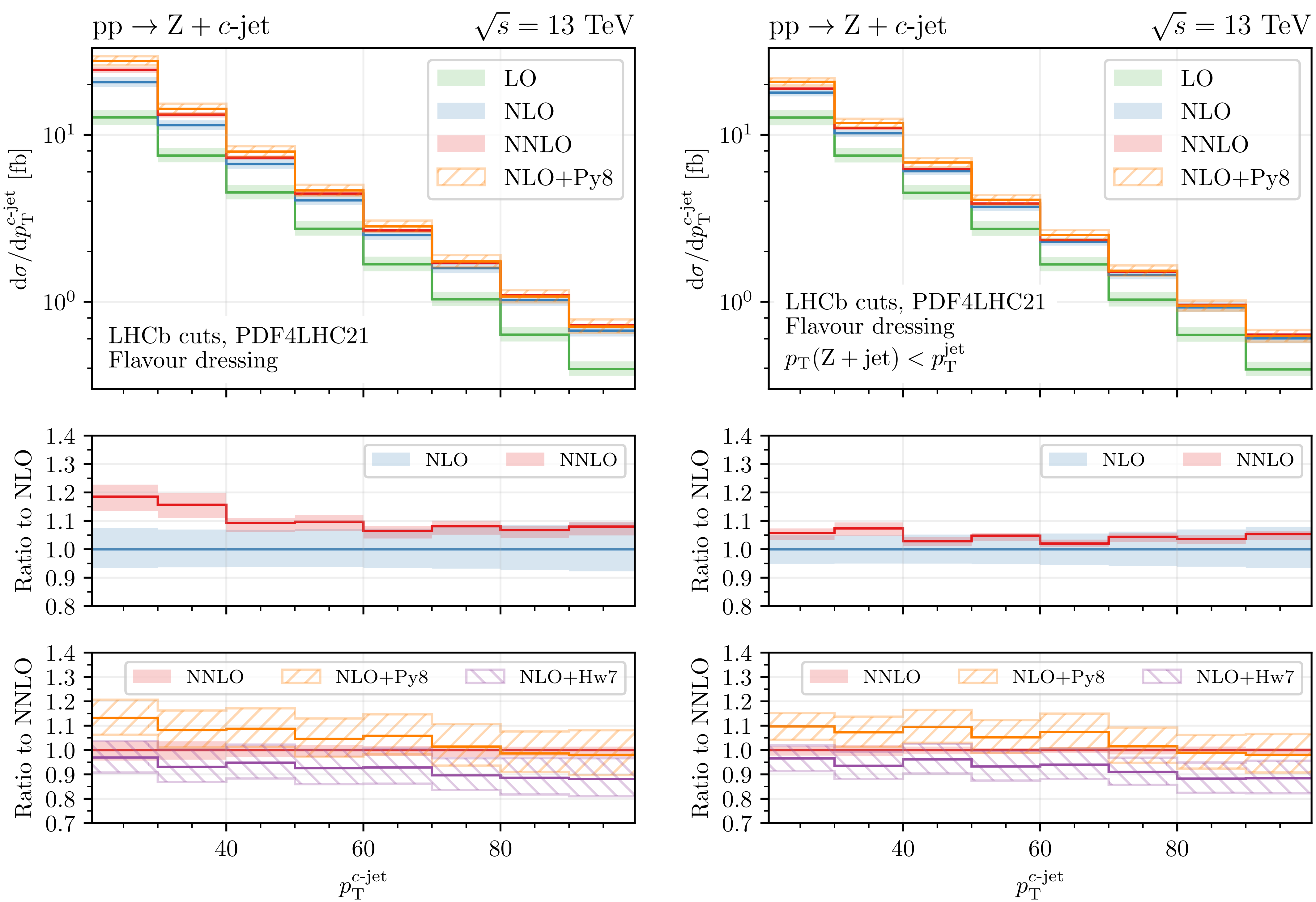
1) definition of flavoured jet not IRC safe

2) significant contamination from MPI



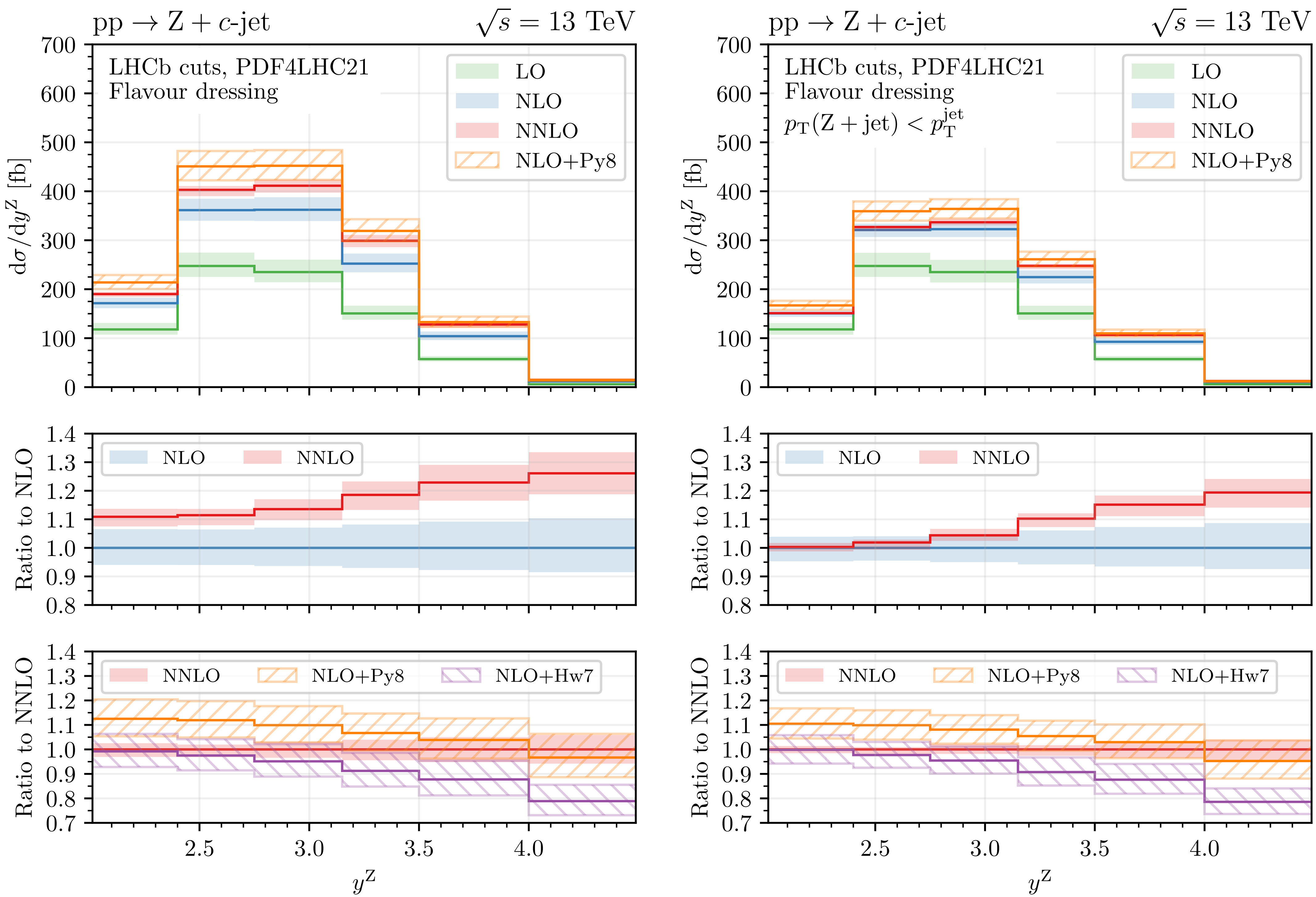
# Results: $p_T^{c-\mathrm{jet}}$

[Gauld, Gehrmann-De Ridder, Glover, Huss, Rodriguez Garcia, GS (2302.12844)]



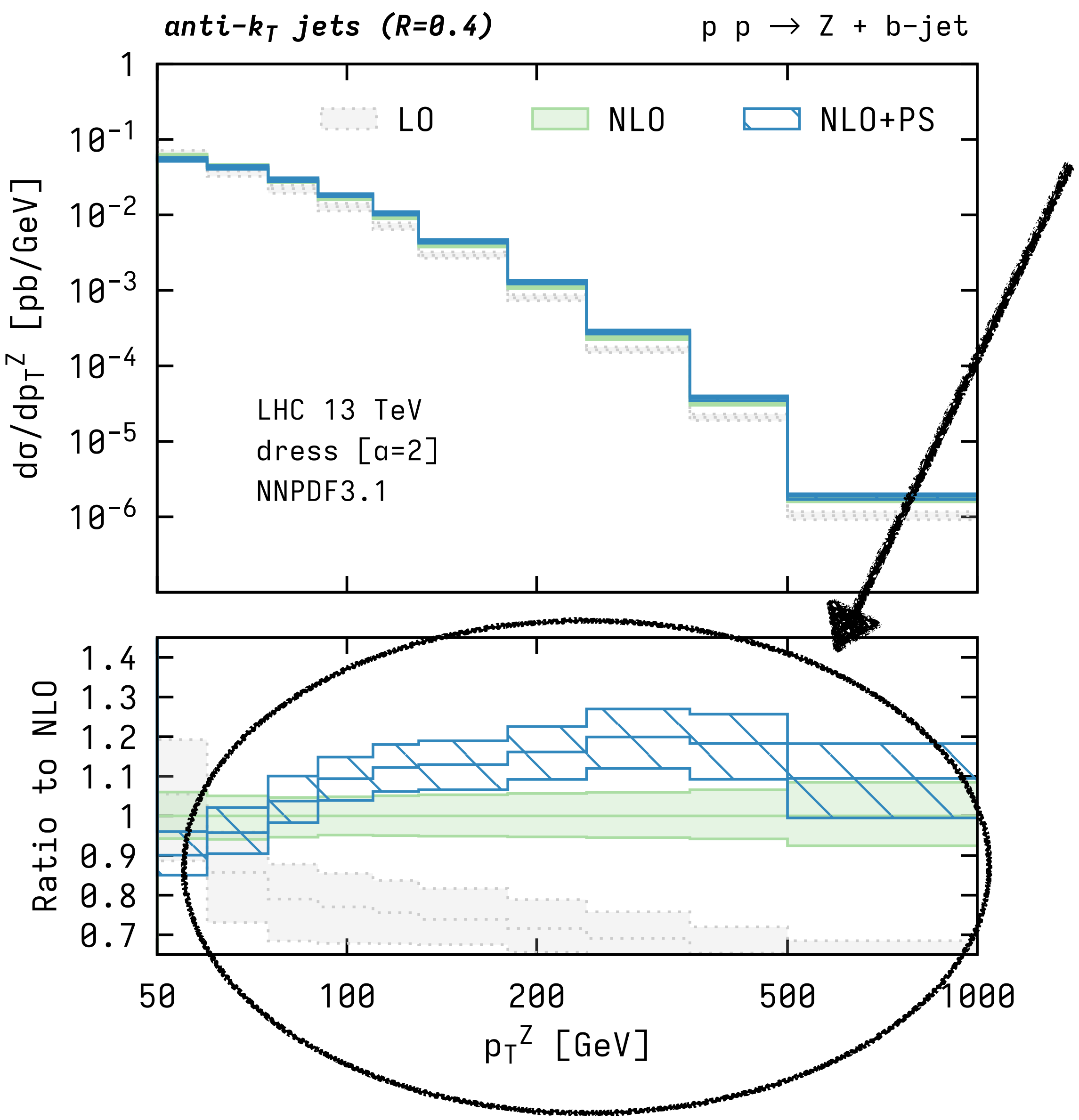
# Results: $y^Z$

[Gauld, Gehrmann-De Ridder, Glover, Huss, Rodriguez Garcia, GS (2302.12844)]

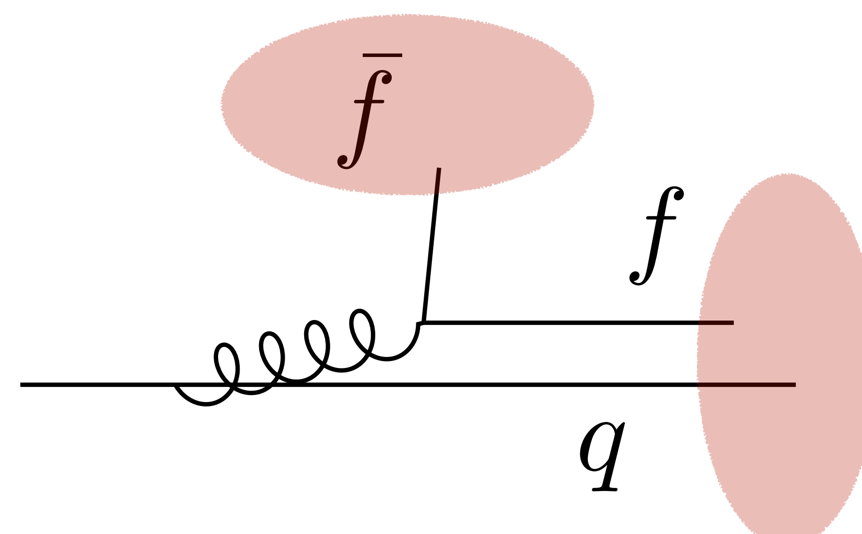


# Test in a realist scenario: $Z + b$-jet

[same setup of Gauld, Gehrmann-De Ridder, Glover, Huss, Majer (2005.03016)]




Some sensitivity observed in $p_T^Z$, likely due to:




Even if IRC finite, it leads to large migration of (unflavoured)-jet into the $b$-jet sample.

# Test in a realist scenario: $Z + b$-jet

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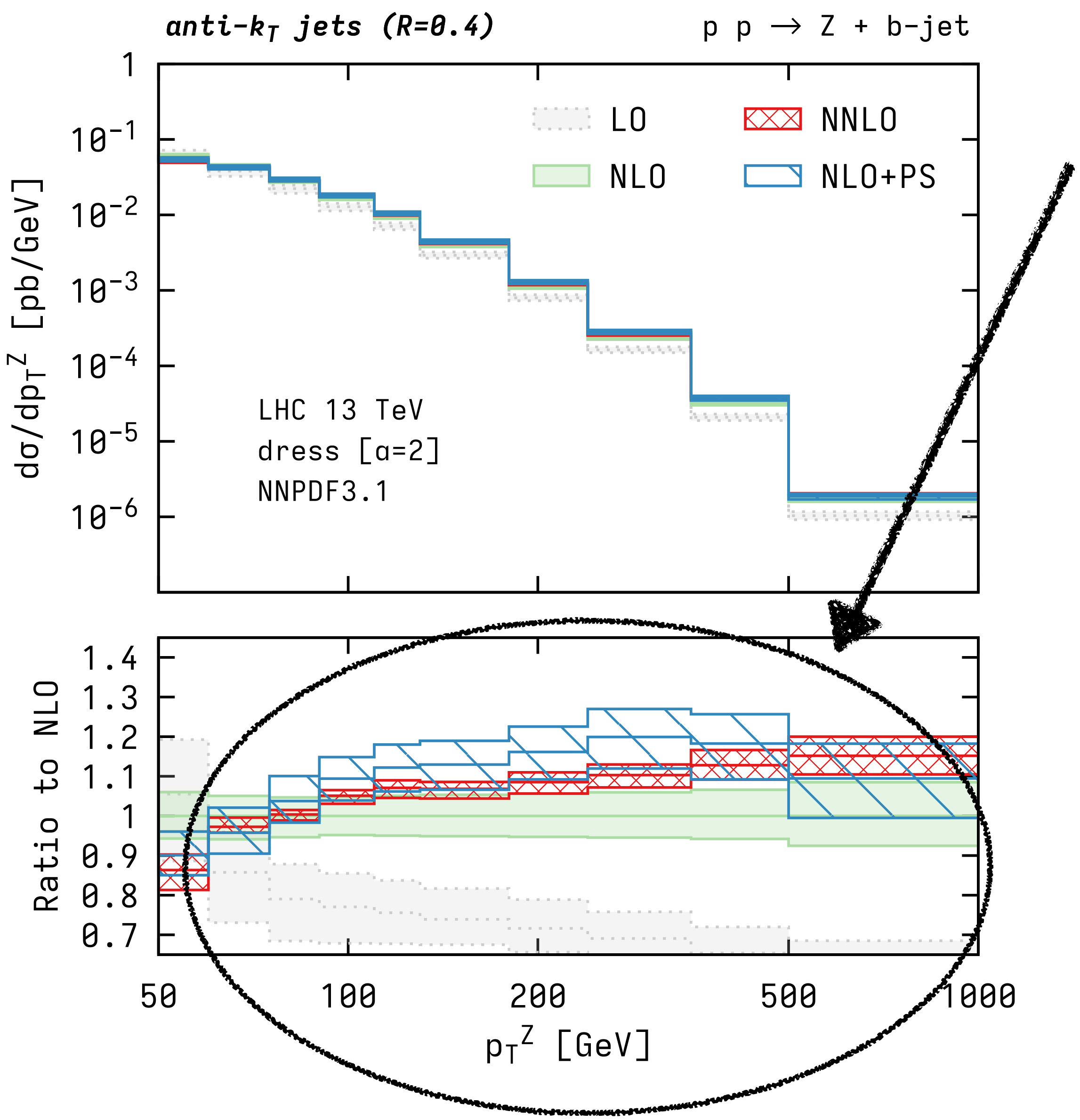


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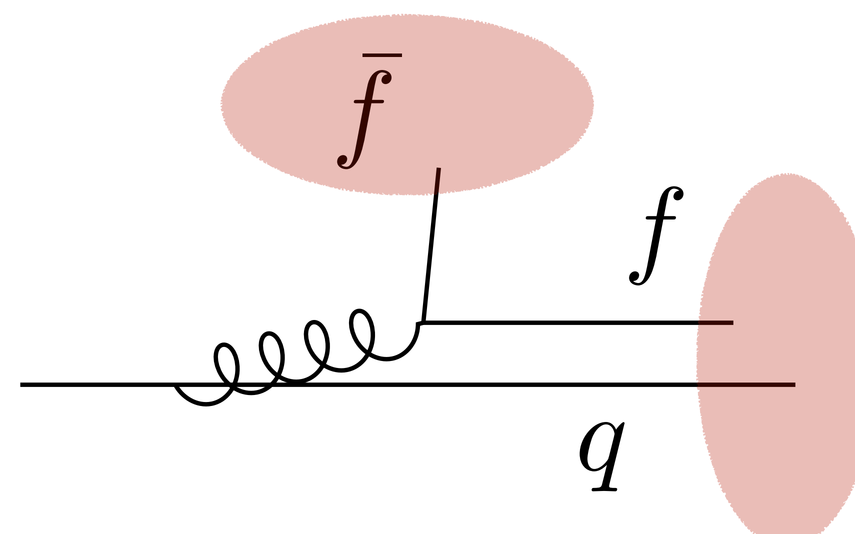
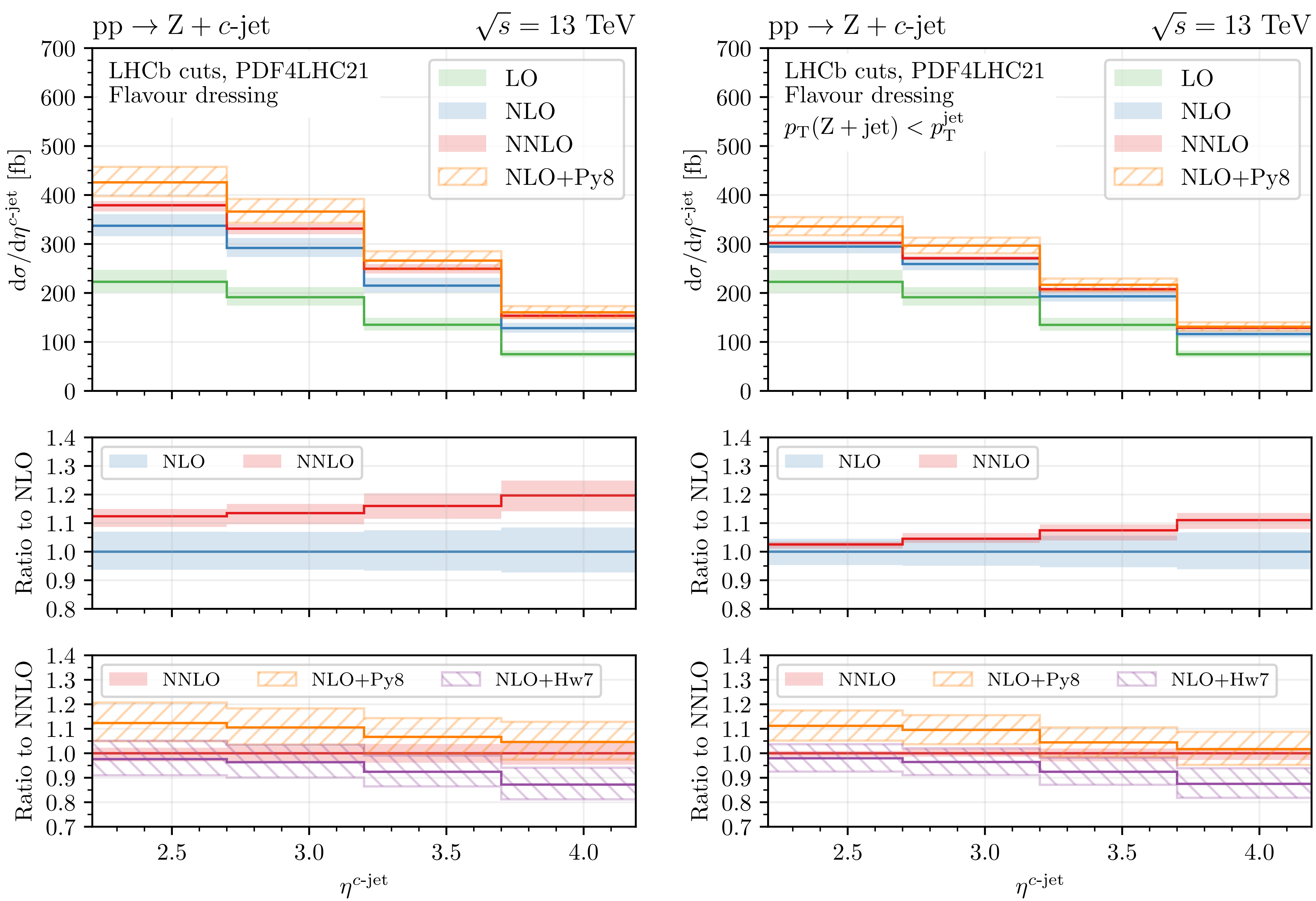


Effect **captured at NNLO**

# Results: $\eta^{\mathrm{c-jet}}$

[Gauld, Gehrmann-De Ridder, Glover, Huss, Rodriguez Garcia, GS (2302.12844)]



# The flavour-$k_t$ algorithm

[Banfi, Salam, Zanderighi (hep-ph/0601139)]

1. Introduce a distance measure $d_{ij}^{(F)}$ between every pair of partons $i$, $j$:

$$d_{ij}^{(F,\alpha)} = (\Delta\eta_{ij}^2 + \Delta\phi_{ij}^2) \times \begin{cases} \max(k_{ti}, k_{tj})^\alpha \min(k_{ti}, k_{tj})^{2-\alpha}, & \text{softer of } i, j \text{ is flavoured,} \\ \min(k_{ti}^2, k_{tj}^2), & \text{softer of } i, j \text{ is flavourless,} \end{cases} \quad (17)$$

as well as distances to the two beams,

$$d_{iB}^{(F,\alpha)} = \begin{cases} \max(k_{ti}, k_{tB}(\eta_i))^\alpha \min(k_{ti}, k_{tB}(\eta_i))^{2-\alpha}, & i \text{ is flavoured,} \\ \min(k_{ti}^2, k_{tB}^2(\eta_i)), & i \text{ is flavourless,} \end{cases} \quad (18)$$

and an analogous definition of $d_{i\bar{B}}^{(F,\alpha)}$ involving $k_{t\bar{B}}(\eta_i)$ instead of $k_{tB}(\eta_i)$ (both defined as in eqs. (15) and (16)).[9] As in section 2 we have introduced a class of measures, parametrised by $0 < \alpha \leq 2$.

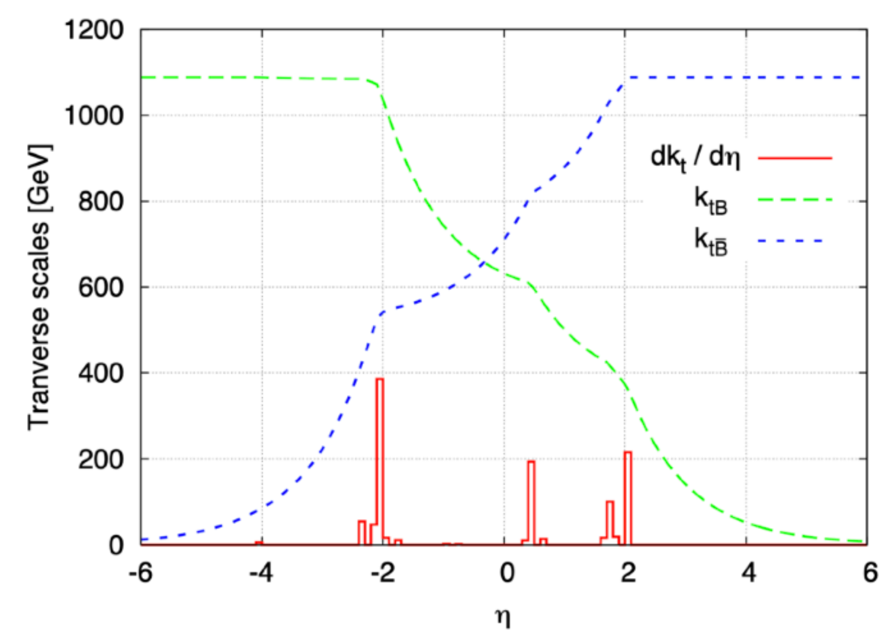
2. Identify the smallest of the distance measures. If it is a $d_{ij}^{(F,\alpha)}$, recombine $i$ and $j$; if it is a $d_{iB}^{(F,\alpha)}$ ($d_{i\bar{B}}^{(F,\alpha)}$) declare $i$ to be part of beam $B$ ($\bar{B}$) and eliminate $i$; in the case where the $d_{iB}^{(F,\alpha)}$ and $d_{i\bar{B}}^{(F,\alpha)}$ are equal (which will occur if $i$ is a gluon), recombine with the beam that has the smaller $k_{tB}(\eta_i)$, $k_{t\bar{B}}(\eta_i)$.

3. Repeat the procedure until all the distances are larger than some $d_{cut}$, or, alternatively, until one reaches a predetermined number of jets.[10,11]

Modified beam distance:

$$k_{tB}(\eta) = \sum_i k_{ti} \left(\Theta(\eta_i - \eta) + \Theta(\eta - \eta_i) e^{\eta_i - \eta}\right)$$

$$k_{t\bar{B}}(\eta) = \sum_i k_{ti} \left(\Theta(\eta - \eta_i) + \Theta(\eta_i - \eta) e^{\eta - \eta_i}\right)$$




IRC flavour safe to all orders, **but** different kinematics (because new distance)

# The flavour dressing algorithm

[Gauld, Huss, GS (2208.11138)]

Flavour assignment *factorised* from jet reconstruction:
**we assign flavour to flavour-agnostic jets**
**in an IRC safe way**

Inputs:
flavour agnostic jets $\{j_k\}$, flavoured clusters $\{\hat{f}_i\}$, association criterion, accumulation criterion

Run a sequential recombination algorithm with flavour-$k_t$-like distances:
- $d(\hat{f}_i, \hat{f}_j)$ between flavoured clusters;
- $d(\hat{f}_i, \hat{j}_k)$ if flavoured cluster $\hat{f}_i$ *associated* to jet $j_k$
- $d_B(\hat{f}_i)$ if $\hat{f}_i$ not associated to any jet

Finally, assign flavour to jet $j_k$ according to collected $\mathrm{tag}_k$ and *accumulation* criterion

# The flavour dressing algorithm: inputs

- *Flavour agnostic jets* $\{j_k\}$*:* set of jets obtained with an IRC safe jet algorithm (e.g. gen-$k_t$ family), possibly after a fiducial selection.
- *Flavoured clusters* $\{\hat{f}_i\}$
- *Association criterion*
- *Accumulation criterion*

# The flavour dressing algorithm: inputs

- *Flavour agnostic jets* $\{j_k\}$

- *Flavoured clusters* $\{\hat{f}_i\}$*:* built out of quarks (e.g. c, b) or stable heavy-flavour hadrons (e.g. D, B), by **dressing them with radiation close in angle, but without touching the soft particles.**

  Exploiting the Soft Drop criterion [Larkoski, Marzani, Soyez, Thaler 1402.2657]

  “Naked” flavoured objects are collinear unsafe

$$\frac{\min(p_{t,a}, p_{t,b})}{(p_{t,a} + p_{t,b})} > z_{\rm cut} \left( \frac{\Delta R_{ab}}{\delta R} \right)^{\beta}$$

- *Association criterion*

- *Accumulation criterion*

# The flavour dressing algorithm: inputs

- *Flavour agnostic jets* $\{j_k\}$
- *Flavoured clusters* $\{\hat{f}_i\}$
- *Association criterion:* whether $\hat{f}_i$ is "associated" to $j_k$
  At parton-level simply if $\hat{f}_i$ is a constituent of $j_k$
  Other options: $\Delta R(\hat{f}_i, j_k) < R_{\mathrm{tag}}$, ghost association, …

  <u>Flavour assignment based only on association is soft unsafe</u>
- *Accumulation criterion*

# The flavour dressing algorithm: inputs

- *Flavour agnostic jets* $\{j_k\}$
- *Flavoured clusters* $\{\hat{f}_i\}$
- *Association criterion*
- *Accumulation criterion:* how to "sum" flavours
  - sum flavoured if unequal number of $\mathrm{f}$ and $\bar{\mathrm{f}}$ (need charge information)
  - sum flavoured if odd number of $\mathrm{f}$ or $\bar{\mathrm{f}}$ (if no charge information)

# Definition of flavoured cluster $\hat{f}_i$

1. Initialise a set with all the flavourless objects $p_i$ (particles used as input to jets) and all the flavoured objects $f_i$ (bare flavours), avoiding double counting if necessary.

2. Find the pair with the smallest angular distance $\Delta R_{ab}$:
   — flavourless $p_a$, $p_b$: combine $p_a$ and $p_b$ into a flavourless $p_{ab}$;
   — flavoured $f_a$, $f_b$: remove both from the set;
   — flavoured $f_a$, unflavoured $p_b$: remove $p_b$ from the set and check a Soft Drop criterion

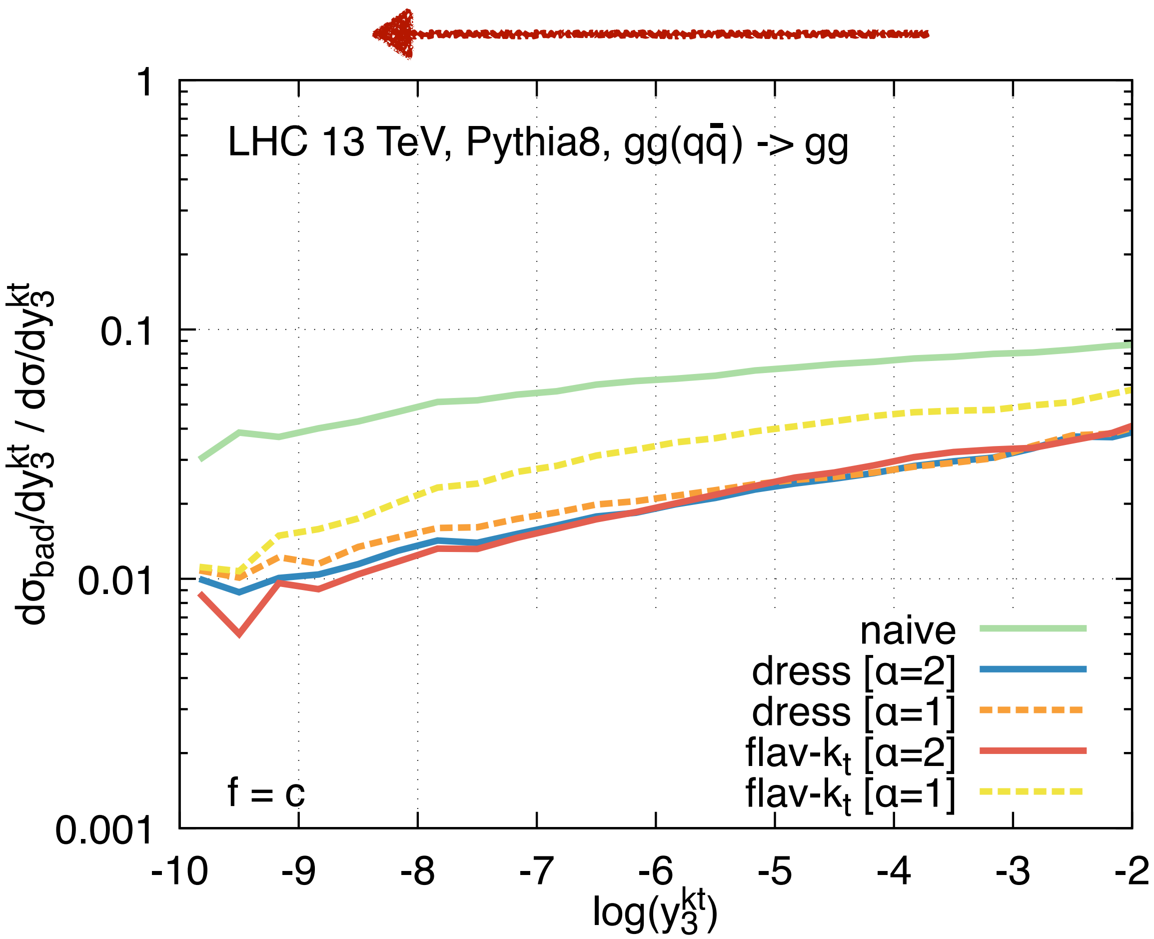
$$\frac{\min(p_{t,a}, p_{t,b})}{(p_{t,a} + p_{t,b})} > z_{\text{cut}} \left( \frac{\Delta R_{ab}}{\delta R} \right)^{\beta}$$

   to **recombine collinear while preserving soft.** [default: $\delta R = 0.1$, $z_{\text{cut}} = 0.1$, $\beta = 2$]
   If satisfied, combine $f_a$ and $p_b$ into a flavoured $f_{ab}$.

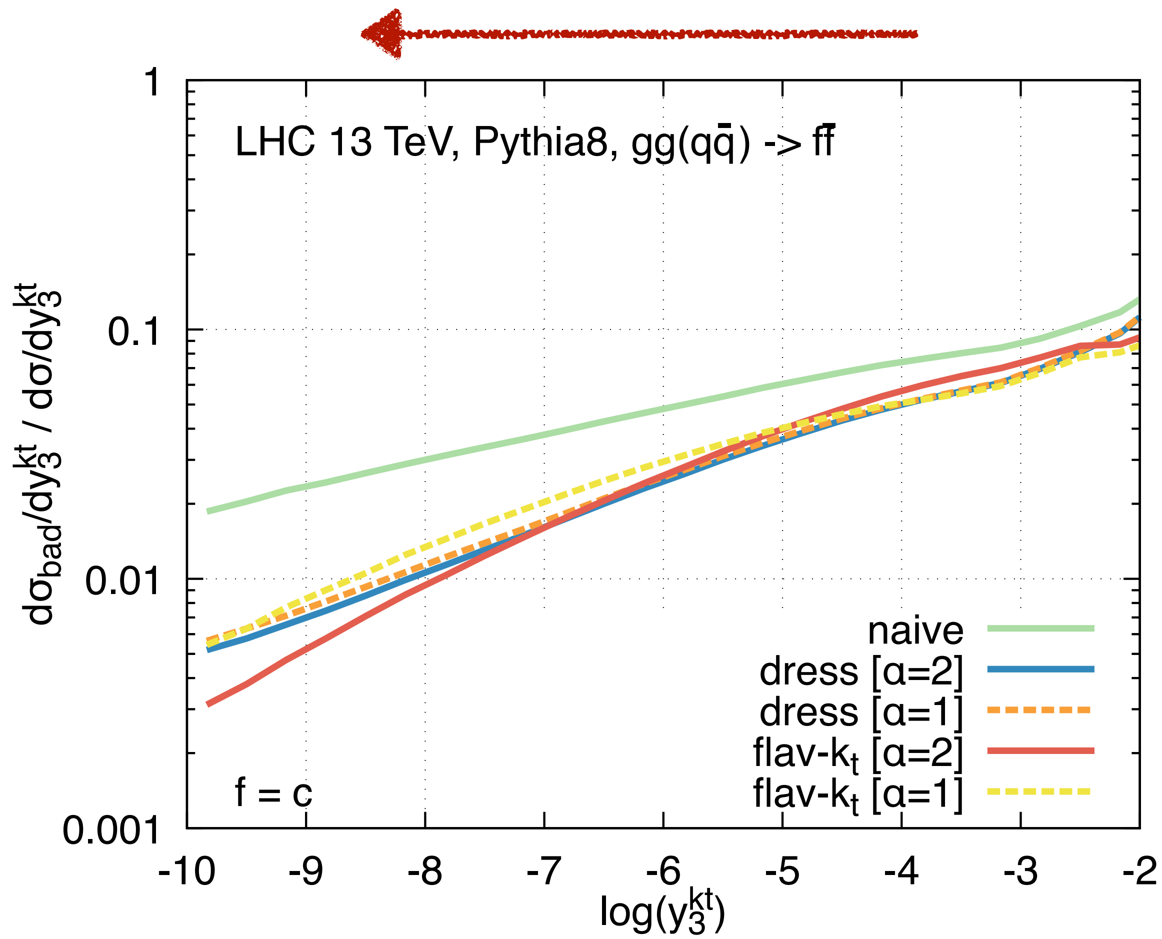
3. Iterate while there are at least two objects in the set until $\Delta R_{ab} > \delta R$.
   The momentum of $\hat{f}_i$ is given by the accumulated momentum into $f_i$.

# IRC sensitivity in $2 \to 2$ QCD events in $pp$

only soft and/or collinear radiation




only soft and/or collinear radiation




Flavour dressing approaches zero
faster than a naive flavour tagging as $y_3^{k_t} \to 0$