

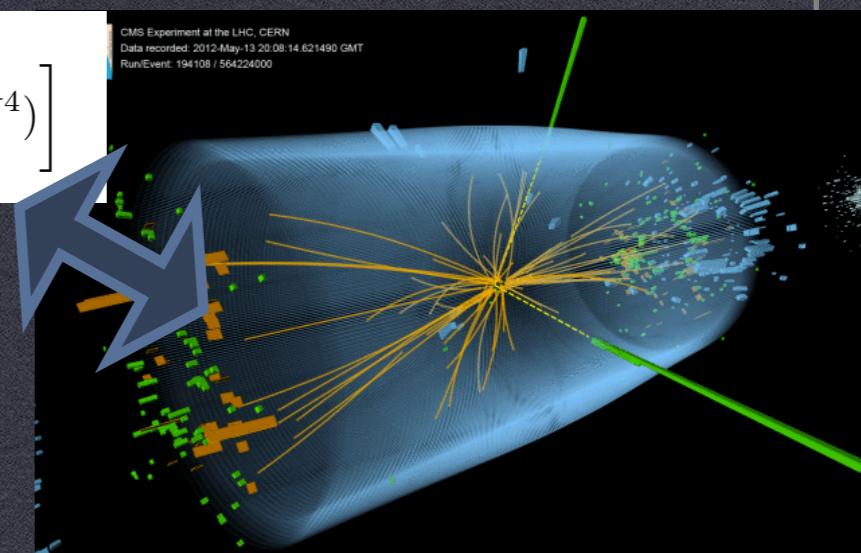
HIGGS AMPLITUDE OBSERVABLES AT COLLIDERS

X^3	φ^6 and $\varphi^4 D^2$	$\psi^2 \varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger \varphi)^3$
$Q_{\tilde{G}}$	$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi \square}$	$(\varphi^\dagger \varphi) \square (\varphi^\dagger \varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^* (\varphi^\dagger D_\mu \varphi)$
$Q_{\tilde{W}}$	$\varepsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$		
$X^2 \varphi^2$	$\psi^2 X \varphi$		
$Q_{\varphi G}$	$\varphi^\dagger \varphi G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$
$Q_{\varphi \tilde{G}}$	$\varphi^\dagger \varphi \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$
$Q_{\varphi W}$	$\varphi^\dagger \varphi W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{\varphi} G_{\mu\nu}^A$
$Q_{\varphi \tilde{W}}$	$\varphi^\dagger \varphi \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{\varphi} W_{\mu\nu}^I$
$Q_{\varphi B}$	$\varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{\varphi} B_{\mu\nu}$
$Q_{\varphi \tilde{B}}$	$\varphi^\dagger \varphi \tilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi G_{\mu\nu}^A$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \varphi W_{\mu\nu}^I$
$Q_{\varphi \tilde{W}B}$	$\varphi^\dagger \tau^I \varphi \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \varphi B_{\mu\nu}$



$hZ\bar{f}f$

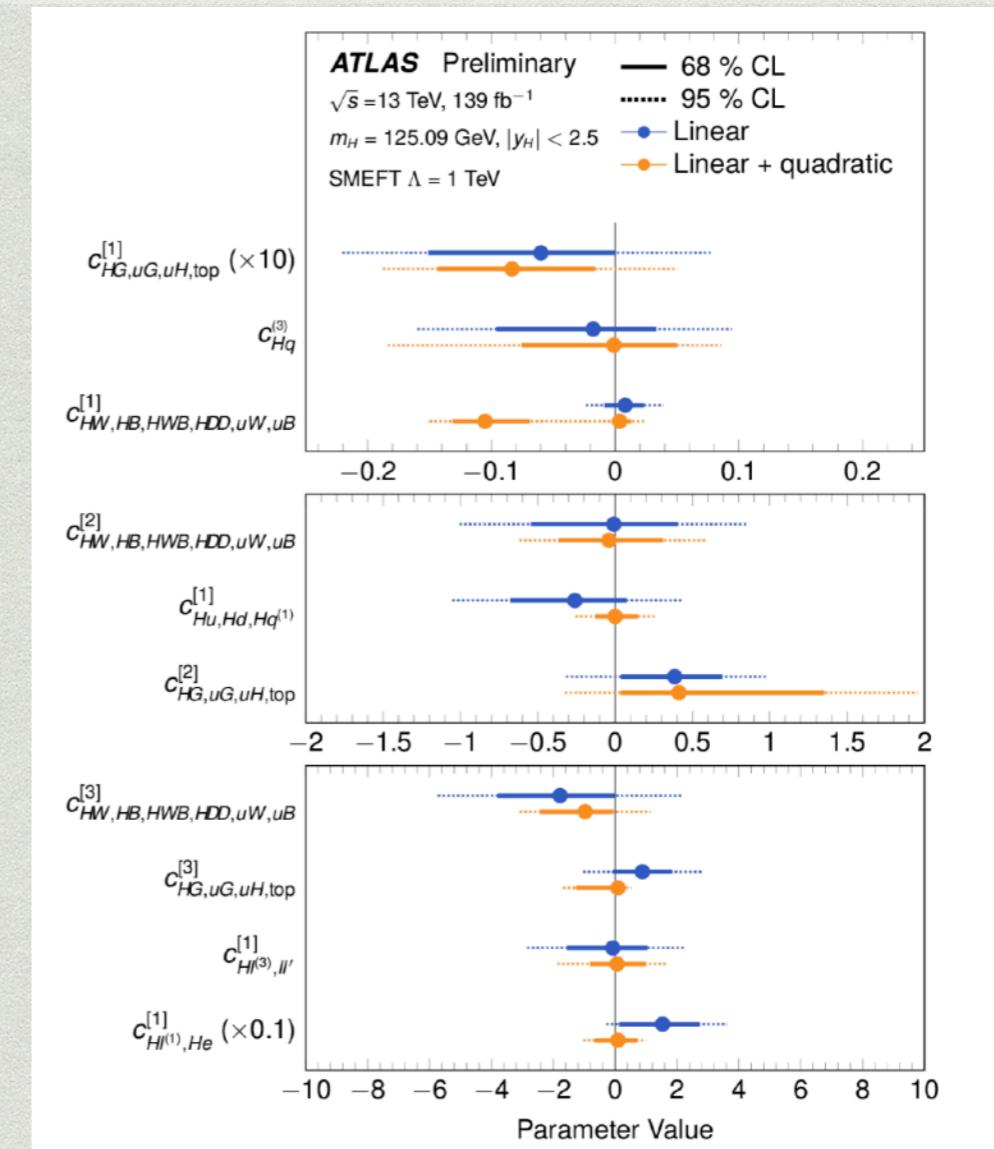
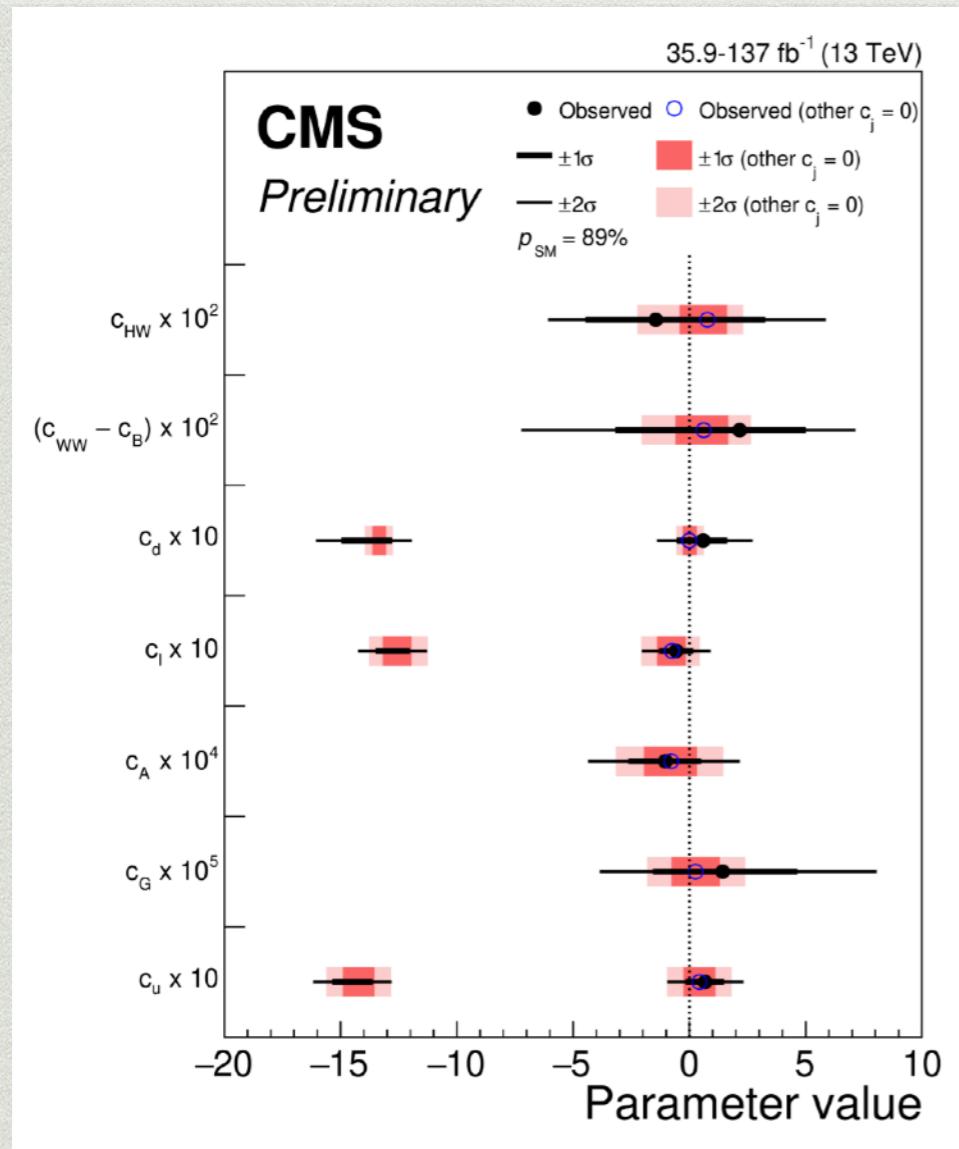
$$C_1 \frac{hZ\bar{f}f}{v} (\bar{u}_{L2} \not{d}_3^* u_{L1}) \left[1 + \alpha_1 \frac{s}{M^2} + \beta_1 \frac{t}{M^2} + O(E^4/M^4) \right]$$



SPENCER CHANG (U. OREGON)
LHCP 24/05/23

BASED ON 2212.06215 (W/ CHEN, LIU, & LUTY) AND 2304.06063 (W/ BRADSHAW)
SEE ALSO DURIEUX ET.AL. (1909.10551, 2008.09652),
MA ET.AL. (2211.16515, 2301.11349)

New Physics via EFT



EFTs parametrize new physics, but make assumptions
 (e.g. linear vs nonlinear EWSB, power counting) and are nonintuitive

On-shell amplitudes as intermediary between theory (EFT, models) and experiment

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Theory

On-shell local amplitudes in one to one correspondence with independent EFT operators

(e.g. SMEFT operator basis from amplitudes
Ma et.al. 1902.06752)

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Experiment

Experiments directly search for amplitudes not Wilson coefficients.

Since EFT is indirect, this motivated signal mapping (e.g. BSM primaries 1405.0181, pseudo-observables 1412.6038, Higgs basis)

EFT operator redundancies and on-shell amplitudes

Redundant Operators

Total Derivatives

$$\partial_\mu \mathcal{O}^\mu \approx 0$$

Equations of Motion

$$\frac{\delta S}{\delta \phi} \mathcal{O} = -(\square \phi + m^2 \phi + \dots) \mathcal{O} \approx 0$$

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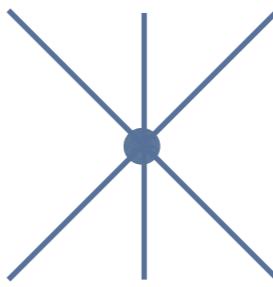
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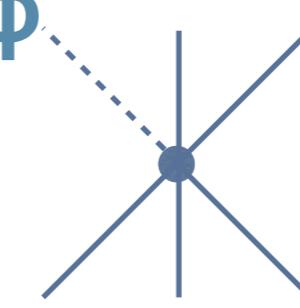
$$\frac{\delta S}{\delta \phi} \mathcal{O} = -(\square \phi + m^2 \phi + \dots) \mathcal{O} \approx 0$$

On-shell Amplitudes

Momentum Conservation


$$\left(\sum_{ext} p_\mu \right) X^\mu = 0$$

Mass Shell


$$(p^2 - m^2) X = 0$$

Independent Amplitudes

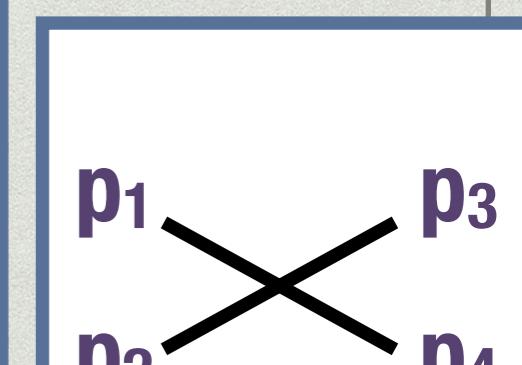
On-shell amplitudes M_i can be related to an operator O_i of lowest mass dimension, work in increasing dimension

Independent Amplitudes

On-shell amplitudes M_i can be related to an operator O_i of lowest mass dimension, work in increasing dimension

Example: hhhh 4-Point Interaction

Dimension	M	# Independents	O
4	1	1	h^4
6	$s+t+u=4m_h^2$	None	None
8	$s^2+t^2+u^2$	1	$h^2 \partial^\mu \partial^\nu h \partial_\mu \partial_\nu h$
10	stu	1	$\partial^\mu \partial^\nu \partial^\rho h \partial_\mu h \partial_\nu h \partial_\rho h$



$$p_1$$

$$p_2$$

$$p_3$$

$$p_4$$

$$s = (p_1 + p_2)^2$$

$$t = (p_1 - p_3)^2$$

$$u = (p_1 - p_4)^2$$

2 to 2 scattering analysis

(w/ Chen, Liu, Luty)

**Amplitude redundancies
($M = 0$), Taylor expansion
of M in
 $\cos \Theta, |p_{\text{initial}}|, |p_{\text{final}}|, E_{\text{com}}$
all coefficients must
vanish**

**Allows numerical
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vanish**

$$\begin{aligned} M(f_1 \bar{f}_2 \rightarrow Z_3 h_4) = & \\ & \epsilon_{3\mu}^* \bar{v}_2 [c_1 p_1^\mu + c_2 p_2^\mu + c_3 p_1^\mu \gamma_5 + c_4 p_2^\mu \gamma_5 + c_5 \gamma^\mu + c_6 p_1^\mu p_3 + c_7 p_2^\mu p_3 \\ & + c_8 \gamma^\mu \gamma_5 + c_9 p_1^\mu p_3 \gamma_5 + c_{10} p_2^\mu p_3 \gamma_5 + c_{11} \gamma^{\mu\nu} p_3 \nu \\ & + \epsilon^{\mu\nu\rho\sigma} p_{1\nu} p_{2\rho} p_{3\sigma} (c_{12} + c_{13} \gamma_5 + c_{14} p_3) \\ & + \epsilon^{\mu\nu\rho\sigma} \gamma_\nu (c_{15} p_{1\rho} p_{2\sigma} + c_{16} p_{1\rho} p_{3\sigma} + c_{17} p_{2\rho} p_{3\sigma}) \\ & + \epsilon^{\mu\nu\rho\sigma} p_{1\nu} p_{2\rho} p_{3\sigma} (c_{18} p_3 \gamma_5) \\ & + \epsilon^{\mu\nu\rho\sigma} \gamma_\nu \gamma_5 (c_{19} p_{1\rho} p_{2\sigma} + c_{20} p_{1\rho} p_{3\sigma} + c_{21} p_{2\rho} p_{3\sigma}) \\ & + c_{22} \epsilon_{\nu\rho\sigma\gamma} \gamma^{\mu\nu} p_1^\rho p_2^\sigma p_3^\gamma \\ & + \epsilon^{\mu\nu\rho\sigma} \gamma_\nu \gamma p_3^\gamma (c_{23} p_{1\rho} p_{2\sigma} + c_{24} p_{1\rho} p_{3\sigma} + c_{25} p_{2\rho} p_{3\sigma}) \\ & + \epsilon^{\mu\nu\rho\sigma} \gamma_{\nu\rho} (c_{26} p_{1\sigma} + c_{27} p_{2\sigma} + c_{28} p_{3\sigma}) \\ & + \epsilon^{\alpha\beta\gamma\delta} \gamma_\alpha p_{1\beta} p_{2\gamma} p_{3\delta} (c_{29} p_1^\mu + c_{30} p_2^\mu + c_{31} p_1^\mu \gamma_5 + c_{32} p_2^\mu \gamma_5)] u_1 \end{aligned}$$

**Allows numerical
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terms (which we can
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$$\begin{aligned}
 M(f_1 \bar{f}_2 \rightarrow Z_3 h_4) = & \\
 \epsilon_{3\mu}^* \bar{v}_2 [& c_1 p_1^\mu + c_2 p_2^\mu + c_3 p_1^\mu \gamma_5 + c_4 p_2^\mu \gamma_5 + c_5 \gamma^\mu + c_6 p_1^\mu p_3 + c_7 p_2^\mu p_3 \\
 & + c_8 \gamma^\mu \gamma_5 + c_9 p_1^\mu p_3 \gamma_5 + c_{10} p_2^\mu p_3 \gamma_5 + c_{11} \gamma^{\mu\nu} p_3 \nu \\
 & + \epsilon^{\mu\nu\rho\sigma} p_{1\nu} p_{2\rho} p_{3\sigma} (c_{12} + c_{13} \gamma_5 + c_{14} p_3) \\
 & + \epsilon^{\mu\nu\rho\sigma} \gamma_\nu (c_{15} p_{1\rho} p_{2\sigma} + c_{16} p_{1\rho} p_{3\sigma} + c_{17} p_{2\rho} p_{3\sigma}) \\
 & + \epsilon^{\mu\nu\rho\sigma} p_{1\nu} p_{2\rho} p_{3\sigma} (c_{18} p_3 \gamma_5) \\
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 & + c_{22} \epsilon_{\nu\rho\sigma\gamma} \gamma^{\mu\nu} p_1^\rho p_2^\sigma p_3^\gamma \\
 & + \epsilon^{\mu\nu\rho\sigma} \gamma_\nu \gamma p_3^\gamma (c_{23} p_{1\rho} p_{2\sigma} + c_{24} p_{1\rho} p_{3\sigma} + c_{25} p_{2\rho} p_{3\sigma}) \\
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 \end{aligned}$$

i	$\mathcal{O}_i^{hZ\bar{f}f}$	CP	$d_{\mathcal{O}_i}$
1	$hZ^\mu \bar{\Psi}_L \gamma_\mu \Psi_L$	+	5
2	$hZ^\mu \bar{\Psi}_R \gamma_\mu \Psi_R$	+	
3	$hZ^{\mu\nu} \bar{\Psi}_L \sigma_{\mu\nu} \Psi_R + \text{h.c.}$	+	6
4	$h\tilde{Z}_{\mu\nu} i \bar{\Psi}_L \sigma^{\mu\nu} \Psi_R + \text{h.c.}$	-	
5	$i hZ^\mu (\bar{\Psi}_L \overset{\leftrightarrow}{\partial}_\mu \Psi_R) + \text{h.c.}$	+	
6	$hZ^\mu \partial_\mu (\bar{\Psi}_L \Psi_R) + \text{h.c.}$	-	6
7	$i hZ^\mu \partial_\mu (\bar{\Psi}_L \Psi_R) + \text{h.c.}$	+	
8	$hZ^\mu (\bar{\Psi}_L \overset{\leftrightarrow}{\partial}_\mu \Psi_R) + \text{h.c.}$	-	
9	$i h\tilde{Z}_{\mu\nu} (\bar{\Psi}_L \gamma^\mu \overset{\leftrightarrow}{\partial}^\nu \Psi_L)$	+	
10	$h\tilde{Z}_{\mu\nu} \partial^\mu (\bar{\Psi}_L \gamma^\nu \Psi_L)$	-	7
11	$i h\tilde{Z}_{\mu\nu} (\bar{\Psi}_R \gamma^\mu \overset{\leftrightarrow}{\partial}^\nu \Psi_R)$	+	
12	$h\tilde{Z}_{\mu\nu} \partial^\mu (\bar{\Psi}_R \gamma^\nu \Psi_R)$	-	

Note: need to include Mandelstams of lower dim operators

More detailed case $hZ\bar{f}f$

$$H_{hZ\bar{f}f} = \frac{2q^5 + 6q^6 + 4q^7}{(1 - q^2)^2}$$

We find for primary operators
**2 at dim 5, 6 at dim 6,
 4 at dim 7**

To get all amplitudes need to
 multiply by arbitrary
 polynomial in s, t

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3	$hZ^{\mu\nu} \bar{\Psi}_L \sigma_{\mu\nu} \Psi_R + \text{h.c.}$	+	6
4	$h\tilde{Z}_{\mu\nu} i \bar{\Psi}_L \sigma^{\mu\nu} \Psi_R + \text{h.c.}$	-	
5	$i hZ^\mu (\bar{\Psi}_L \overset{\leftrightarrow}{\partial}_\mu \Psi_R) + \text{h.c.}$	+	
6	$hZ^\mu \partial_\mu (\bar{\Psi}_L \Psi_R) + \text{h.c.}$	-	6
7	$i hZ^\mu \partial_\mu (\bar{\Psi}_L \Psi_R) + \text{h.c.}$	+	
8	$hZ^\mu (\bar{\Psi}_L \overset{\leftrightarrow}{\partial}_\mu \Psi_R) + \text{h.c.}$	-	
9	$i h\tilde{Z}_{\mu\nu} (\bar{\Psi}_L \gamma^\mu \overset{\leftrightarrow}{\partial}^\nu \Psi_L)$	+	
10	$h\tilde{Z}_{\mu\nu} \partial^\mu (\bar{\Psi}_L \gamma^\nu \Psi_L)$	-	7
11	$i h\tilde{Z}_{\mu\nu} (\bar{\Psi}_R \gamma^\mu \overset{\leftrightarrow}{\partial}^\nu \Psi_R)$	+	
12	$h\tilde{Z}_{\mu\nu} \partial^\mu (\bar{\Psi}_R \gamma^\nu \Psi_R)$	-	

e.g. $\left(\frac{c_1}{v^2} + \frac{c_{1,s}}{v^4} s + \frac{c_{1,t}}{v^4} t + \frac{c_{1,ss}}{v^6} s^2 + \dots \right) hZ^\mu \bar{\Psi}_L \gamma_\mu \Psi_L$

More detailed case $hZff$

$$H_{hZ\bar{f}f} = \frac{2q^5 + 6q^6 + 4q^7}{(1 - q^2)^2}$$

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 multiply by arbitrary
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This has been computed for
3pt: $hhh, hff, hV V$

4pt: $hhhh, hhVV, hhff, hVff, hVvv$
 where $V = W, Z, \gamma, g$

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1	$hZ^\mu \bar{\Psi}_L \gamma_\mu \Psi_L$	+	5
2	$hZ^\mu \bar{\Psi}_R \gamma_\mu \Psi_R$	+	
3	$hZ^{\mu\nu} \bar{\Psi}_L \sigma_{\mu\nu} \Psi_R + \text{h.c.}$	+	6
4	$h\tilde{Z}_{\mu\nu} i \bar{\Psi}_L \sigma^{\mu\nu} \Psi_R + \text{h.c.}$	-	
5	$i hZ^\mu (\bar{\Psi}_L \overset{\leftrightarrow}{\partial}_\mu \Psi_R) + \text{h.c.}$	+	
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9	$i h\tilde{Z}_{\mu\nu} (\bar{\Psi}_L \gamma^\mu \overset{\leftrightarrow}{\partial}^\nu \Psi_L)$	+	
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e.g. $\left(\frac{c_1}{v^2} + \frac{c_{1,s}}{v^4} s + \frac{c_{1,t}}{v^4} t + \frac{c_{1,ss}}{v^6} s^2 + \dots \right) hZ^\mu \bar{\Psi}_L \gamma_\mu \Psi_L$

Pheno Estimate Example ($h \rightarrow \text{Zee}$)

1) Given an operator, e.g. at dim 6

$$i\frac{c}{v^2} h Z^\mu \bar{e}_L \overleftrightarrow{\partial}_\mu e_R + h.c.$$

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**2) Find SMEFT realization
to give conservative proxy for
unitarity bound**

$$(|H|^2 - \frac{v^2}{2})(H^\dagger \overset{\leftrightarrow}{D}^\mu H)(\bar{L}_L \overset{\leftrightarrow}{D}_\mu e_R) H + h.c.$$

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3) Unitarity bounds

(e.g. 2009.11293)

$\text{WW} \rightarrow \text{ee}$: $c \lesssim 0.1 / (\text{TeV}/E_{\max})^3$

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This interferes with SM
amplitude, sensitivity
estimate $N_{\text{new}} \gtrsim \sqrt{N_{\text{SM}}}$

At HL-LHC, requires
 $E_{\max} \lesssim 5 \text{ TeV}$, so this should
be studied in detail

Conclusions

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- * Point to new Higgs decay amplitudes worth exploring at the HL-LHC, e.g. $h \rightarrow ff(Z,W,\gamma)$ and $Z\gamma\gamma$ (and potentially production amplitudes)

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- * Fully characterized general on-shell amplitudes for 3 and 4 point for Higgs (agree w/ spinor helicity results, which are up to dim 8 for SM content)
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- * Point to new Higgs decay amplitudes worth exploring at the HL-LHC, e.g. $h \rightarrow ff(Z,W,\gamma)$ and $Z\gamma\gamma$ (and potentially production amplitudes)
- * Understanding of primary and descendant amplitudes may enable approach to higher order uncertainties (work under discussion w/ Luty, Ma and Wulzer)

Thanks for your attention!



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Extra Slides

Amplitude redundancies

$$\begin{aligned} 0 &= E_{\text{cm}}^N \sum_a C_a \mathcal{M}_a \\ &= P + Qs + Rp_i + Sp_f + Tsp_i + Usp_f + Vp_ip_f + Wsp_ip_f \end{aligned}$$

P, Q, R, S, T, U, V, W are finite polynomials in E_{com} , which due to singularity structures, each polynomial must vanish exactly

$$X_\alpha^i = \frac{\partial d_\alpha}{\partial c_i}$$

Choose random particle masses and numerically take singular value decomposition of this matrix to find number of independent amplitudes (d_α are polynomial coefficients, c_i are operator coefficients)

i	$\mathcal{O}_i^{hZ\bar{f}f}$	CP	$d_{\mathcal{O}_i}$	SMEFT Operator	c Unitarity Bound
1	$hZ^\mu \bar{\Psi}_L \gamma_\mu \Psi_L$	+	5	$i(H^\dagger \overset{\leftrightarrow}{D}_\mu H) \bar{Q}_L \gamma^\mu Q_L$	$\frac{0.6}{E_{\text{TeV}}^2}, \frac{5}{E_{\text{TeV}}^4}$
2	$hZ^\mu \bar{\Psi}_R \gamma_\mu \Psi_R$	+	5	$i(H^\dagger \overset{\leftrightarrow}{D}_\mu H) \bar{u}_R \gamma^\mu u_R$	
3	$hZ^{\mu\nu} \bar{\Psi}_L \sigma_{\mu\nu} \Psi_R + \text{h.c.}$	+	6	$\bar{Q}_L \sigma^{\mu\nu} u_R \sigma^a \tilde{H} W_{\mu\nu}^a + \text{h.c.}$	$\frac{2}{E_{\text{TeV}}^2}, \frac{10}{E_{\text{TeV}}^4}$
4	$h\tilde{Z}_{\mu\nu} i\bar{\Psi}_L \sigma^{\mu\nu} \Psi_R + \text{h.c.}$	-	6	$\bar{Q}_L \sigma^{\mu\nu} u_R \sigma^a \tilde{H} \tilde{W}_{\mu\nu}^a + \text{h.c.}$	
5	$i hZ^\mu (\bar{\Psi}_L \overset{\leftrightarrow}{\partial}_\mu \Psi_R) + \text{h.c.}$	+	6	$(H^\dagger \overset{\leftrightarrow}{D}_\mu H) (\bar{Q}_L \overset{\leftrightarrow}{D}^\mu u_R) \tilde{H} + \text{h.c.}$	
6	$hZ^\mu \partial_\mu (\bar{\Psi}_L \Psi_R) + \text{h.c.}$	-	6	$i(H^\dagger \overset{\leftrightarrow}{D}_\mu H) D^\mu (\bar{Q}_L u_R) \tilde{H} + \text{h.c.}$	$\frac{0.1}{E_{\text{TeV}}^3}, \frac{4}{E_{\text{TeV}}^6}$
7	$i hZ^\mu \partial_\mu (\bar{\Psi}_L \Psi_R) + \text{h.c.}$	+	6	$(H^\dagger \overset{\leftrightarrow}{D}_\mu H) D^\mu (\bar{Q}_L u_R) \tilde{H} + \text{h.c.}$	
8	$hZ^\mu (\bar{\Psi}_L \overset{\leftrightarrow}{\partial}_\mu \Psi_R) + \text{h.c.}$	-	6	$i(H^\dagger \overset{\leftrightarrow}{D}_\mu H) (\bar{Q}_L \overset{\leftrightarrow}{D}^\mu u_R) \tilde{H} + \text{h.c.}$	
9	$i h\tilde{Z}_{\mu\nu} (\bar{\Psi}_L \gamma^\mu \overset{\leftrightarrow}{\partial}^\nu \Psi_L)$	+	7	$i H ^2 \tilde{W}^{a\mu\nu} (\bar{Q}_L \gamma_\mu \sigma^a \overset{\leftrightarrow}{D}_\nu Q_L)$	
10	$h\tilde{Z}_{\mu\nu} \partial^\mu (\bar{\Psi}_L \gamma^\nu \Psi_L)$	-	7	$ H ^2 \tilde{W}^{a\mu\nu} D_\mu (\bar{Q}_L \gamma_\nu \sigma^a Q_L)$	$\frac{0.4}{E_{\text{TeV}}^3}, \frac{1}{E_{\text{TeV}}^4}$
11	$i h\tilde{Z}_{\mu\nu} (\bar{\Psi}_R \gamma^\mu \overset{\leftrightarrow}{\partial}^\nu \Psi_R)$	+	7	$i H ^2 \tilde{B}^{\mu\nu} (\bar{u}_R \gamma_\mu \overset{\leftrightarrow}{D}_\nu u_R)$	
12	$h\tilde{Z}_{\mu\nu} \partial^\mu (\bar{\Psi}_R \gamma^\nu \Psi_R)$	-	7	$ H ^2 \tilde{B}^{\mu\nu} D_\mu (\bar{u}_R \gamma_\nu u_R)$	

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A cross check comes from counting independent operators using the Hilbert series (Lehmann, Martin; Henning, et.al.; ...)

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Note: Consistent with amplitude analysis, higher dimension operators are Mandelstam descendants of a single primary operator h^4 , where denominator expansion gives factors of $s^2+t^2+u^2$ and stu

Hilbert Series (Higgs)

$$H_{h\bar{f}f} = 2q^4, \quad H_{h\gamma Z} = H_{h\gamma\gamma} = H_{hgg} = 2q^5, \quad H_{hZZ} = H_{hWW} = q^3 + 2q^5,$$

$$H_{hhZ} = H_{hh\gamma} = 0, \quad H_{hhh} = q^3,$$

$$H_{\gamma\bar{f}f} = 2q^5, \quad H_{Z\bar{f}f} = H_{W\bar{f}f'} = 2q^4 + 2q^5,$$

$$H_{WWZ} = 5q^4 + 2q^6, \quad H_{WW\gamma} = 2q^4 + 2q^6, \quad H_{ggg} = 2q^6,$$

$$H_{ZZZ} = H_{ZZ\gamma} = H_{Z\gamma\gamma} = H_{Zgg} = 0.$$

$$H_{hZ\bar{f}f} = H_{hW\bar{f}'f} = \frac{2q^5 + 6q^6 + 4q^7}{(1 - q^2)^2}, \quad H_{h\gamma\bar{f}f} = H_{hg\bar{f}f} = \frac{2q^6 + 4q^7 + 2q^8}{(1 - q^2)^2},$$

$$H_{hZ\gamma\gamma} = H_{hZgg} = \frac{3q^7 + 7q^9 + 2q^{11}}{(1 - q^2)(1 - q^4)}, \quad H_{hggg} = \frac{2q^7 + 2q^9 + 4q^{11} + 6q^{13} + 2q^{15}}{(1 - q^4)(1 - q^6)},$$

$$H_{h\gamma gg} = \frac{4q^9 + 4q^{11}}{(1 - q^2)(1 - q^4)}, \quad H_{h\gamma\gamma\gamma} = \frac{2q^{11} + 4q^{13} + 2q^{15}}{(1 - q^4)(1 - q^6)},$$

$$H_{hWW\gamma} = \frac{2q^5 + 14q^7 + 2q^9}{(1 - q^2)^2}, \quad H_{hZZ\gamma} = \frac{8q^7 + 8q^9 + 2q^{11}}{(1 - q^2)(1 - q^4)},$$

$$H_{hWWZ} = \frac{9q^5 + 18q^7}{(1 - q^2)^2}, \quad H_{hZZZ} = \frac{q^5 + 6q^7 + 8q^9 + 7q^{11} + 5q^{13}}{(1 - q^4)(1 - q^6)},$$

$$H_{hh\bar{f}f} = \frac{2q^5 + 2q^8}{(1 - q^2)(1 - q^4)},$$

$$H_{hhWW} = \frac{q^4 + 3q^6 + 5q^8}{(1 - q^2)(1 - q^4)}, \quad H_{hhZZ} = \frac{q^4 + 3q^6 + 2q^8}{(1 - q^2)(1 - q^4)}$$

$$H_{hhZ\gamma} = \frac{2q^6 + 4q^8}{(1 - q^2)(1 - q^4)}, \quad H_{hh\gamma\gamma} = H_{hhgg} = \frac{2q^6 + q^8}{(1 - q^2)(1 - q^4)},$$

$$H_{hhhZ} = \frac{q^7 + q^9 + q^{13}}{(1 - q^4)(1 - q^6)}, \quad H_{hhh\gamma} = \frac{2q^{13}}{(1 - q^4)(1 - q^6)},$$