$t\bar{t}H$ production at NNLO

Javier Mazzitelli

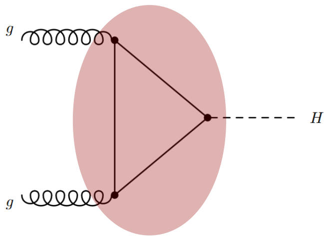
Based on: S. Catani, S. Devoto, M. Grazzini, S. Kallweit, JM, C. Savoini,
Phys.Rev.Lett. 130 (2023) 11, 111902 [arXiv:2210.07846 [hep-ph]]

PAUL SCHERRER INSTITUT

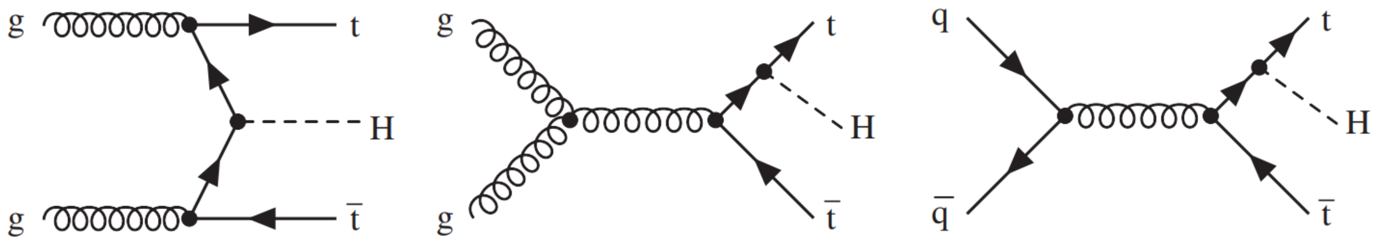
LHCP, May 26th 2023

Introduction

In ggF other contributions
and NP effects can conspire

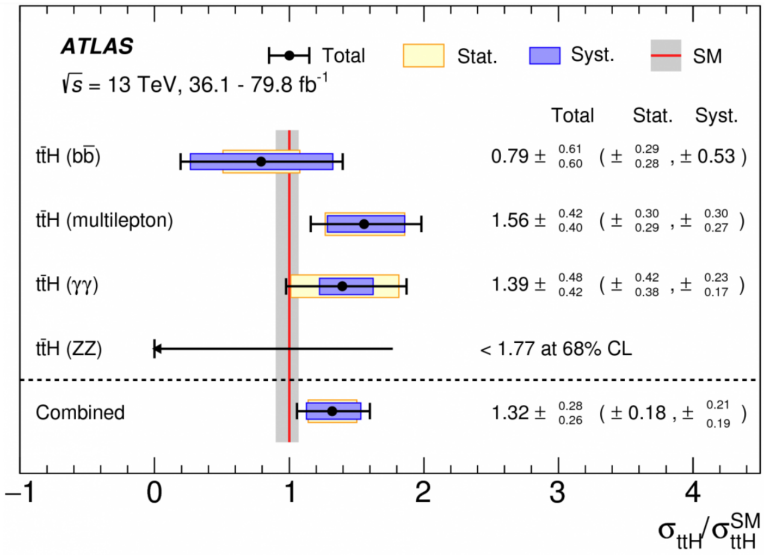


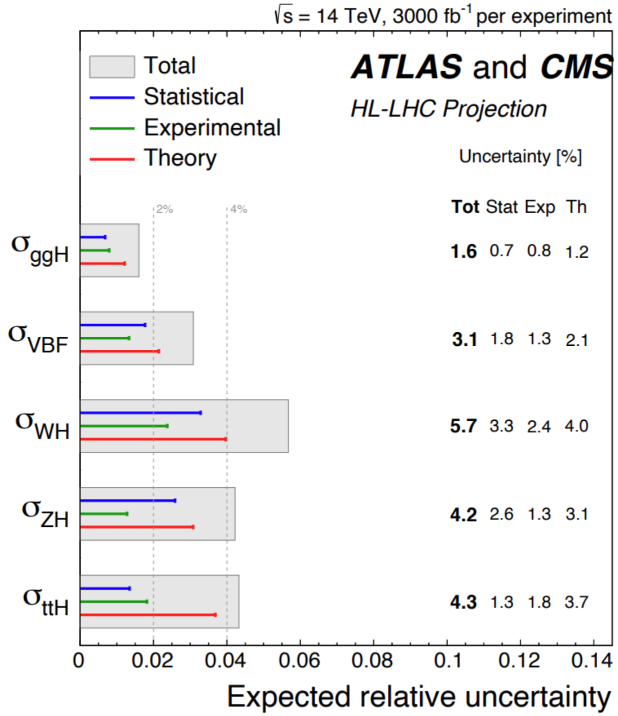
- $t\bar{t}H$ production → 'direct' measurement of the top Yukawa coupling



- Observed 5 years ago by LHC collaborations

[CMS 1804.02610, ATLAS 1806.00425]





- Current experimental uncertainties at O(20%) level
- Experimental precision expected to go down to O(2%) at HL-LHC

  [Cepeda et al.; 1902.00134]
- Precise theoretical predictions are needed to match it!

Theoretical status

NLO QCD

[Beenakker at al.; 0107081, 0211352], [Reina and Dawson; 0107101],
[Reina, Dawson and Wackeroth; 0109066], [Dawson at al.; 0211438],
[Dawson at al.; 0305087]

NLO EW

[Frixione et al.; 1407.0823, 1504.03446],
[Zhang et al.; 1407.1110]

Soft-gluon resummation

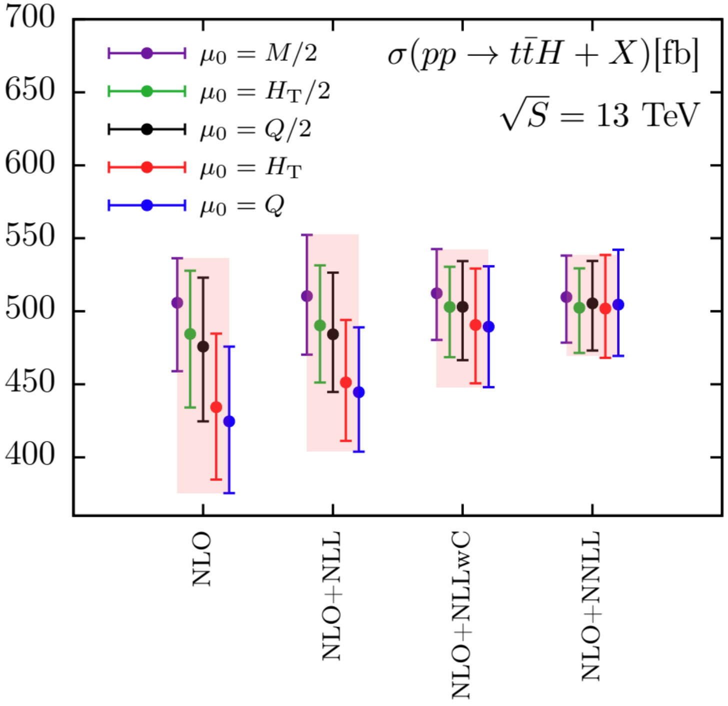
[Kulesza et al.; 1509.02780, 1704.03363], [Broggio et al.; 1510.01914],
[Broggio et al.; 1611.00049], [Broggio et al.; 1907.04343],
[Ju and Yang; 1904.08744], [Kulesza et al.; 2001.03031]

NLO with off-shell effects

[Denner and Feger; 1506.07448], [Denner et al.; 1612.07138]

NLO QCD + PS

[Frederix et al.; 1104.5613], [Garzelli et al.; 1108.0387],
[Hartanto et al.; 1501.04498]



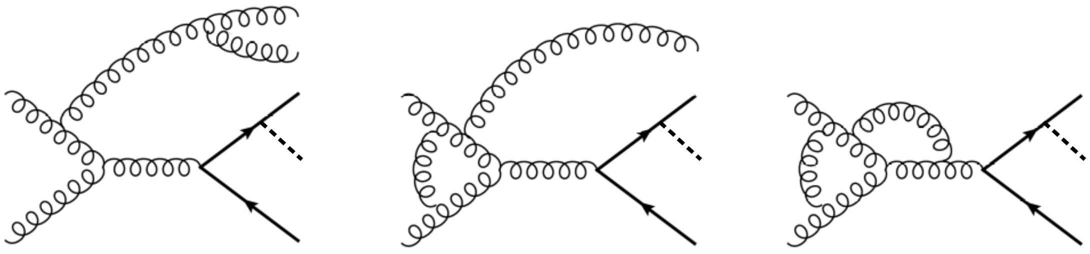
[Kulesza et al.; 2001.03031]

- Current perturbative uncertainties: O(10%)
- NNLO in QCD needed to reduce them!

Challenges in NNLO calculation:

Subtraction of infrared divergencies

Two-loop scattering amplitudes



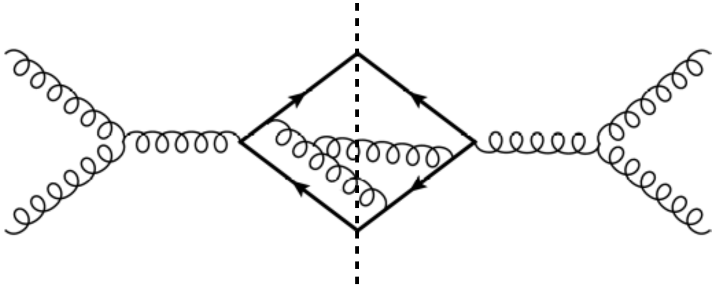
Infrared subtraction

- We use the q_T-subtraction method, originally developed for colour singlet

[Catani, Grazzini; hep-ph/0703012]

- Extended to heavy-quark production: additional soft divergencies from FS emissions

[Catani, Devoto, Grazzini, JM; 2301.11786]



Used for $t\bar{t}$, $b\bar{b}$, both at NNLO and NNLO+PS

[Catani, JM et al.; 1901.04005, 1906.06535, 2005.00557, 2010.11906], [JM et al.; 2012.14267, 2112.12135, 2302.01645]

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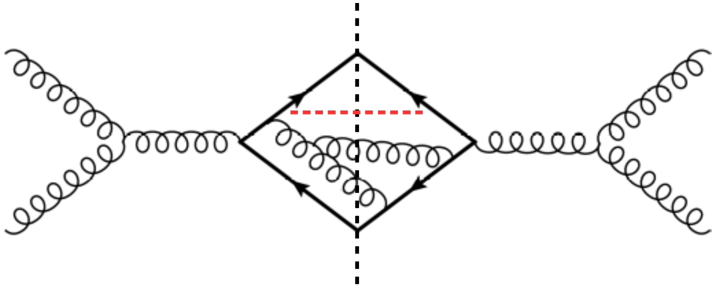

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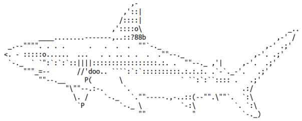
- Further extension needed to deal with heavy-quark + colourless

Remove back-to-back constraint for heavy quarks

Soft function for Heavy quark production in ARbitrary Kinematics

[Devoto, JM; in preparation]



Already applied to $t\bar{t}H$ and $b\bar{b}W$

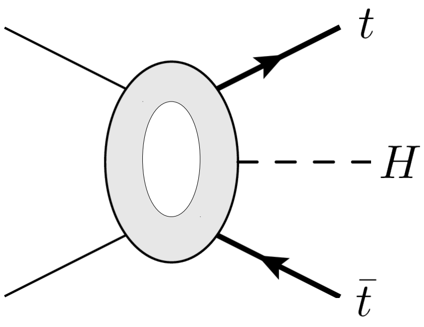
[this talk] [Buonocore, JM, et al.; 2212.04954]

Two-loop corrections: soft Higgs emission

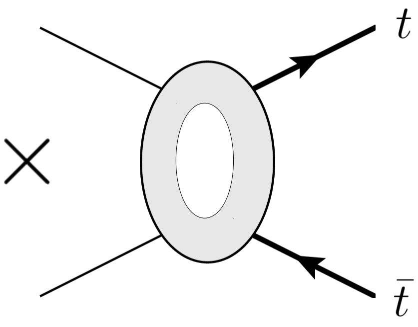
- 2→3 at 2 loops with 3 external masses → beyond current capabilities

Need to rely on some approximation

- We have derived a factorization formula valid in the limit in which the Higgs is **soft**



$$\overset{\text{soft}}{\simeq} F\left(\alpha_s, \frac{m_t}{\mu_R}\right) J(p_H) \times$$



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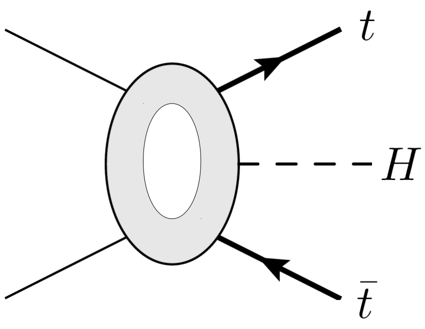

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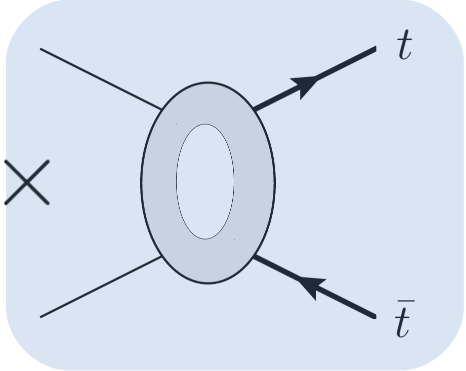

$$1 + \frac{\alpha_s}{2\pi}(-4) + \left(\frac{\alpha_s}{2\pi}\right)^2 \left(-\frac{89}{3} - \frac{46}{3}\log\frac{\mu_R^2}{m_t^2}\right) + \ldots$$

Derived from soft limit of scalar heavy quark form factor, alternatively using Higgs low energy theorems

[Bernreuther et al (2005) Blumlein et al (2017) Fael et al. (2022), Kniehl and Spira (1995)]

$$\sum_i \frac{m_t}{v} \frac{m_t}{p_i \cdot p_H}$$

Known at two loops from top-pair production at NNLO

[Baernreuther, Czakon, Fiedler; 1312.6279]

- Higgs soft current is 'abelian', no higher-order corrections apart from normalization

- This formula can serve as a non-trivial cross check to future Higgs+HQ loop calculations

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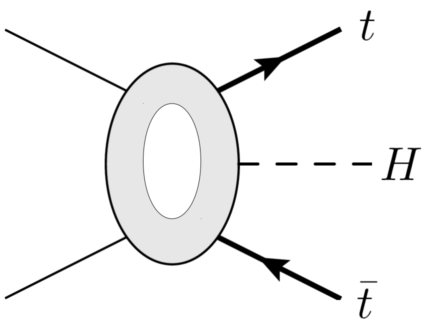

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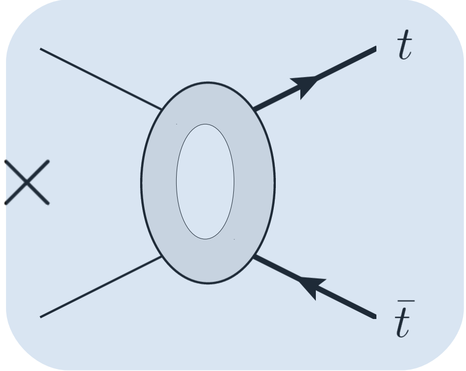

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Known at two loops from
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[Baernreuther, Czakon, Fiedler; 1312.6279]

- We can use the formula to approximate the **only** unkown contribution to $t\bar{t}H$ at NNLO:

$$H^{(2)} = \frac{2\mathrm{Re}\left(\mathcal{M}^{(2)}_{\text{fin}} \mathcal{M}^{(0)*}\right)}{\left|\mathcal{M}^{(0)}\right|^2} \longrightarrow$$

Obs: approximation used both in numerator and denominator (Born improved)

- Mapping needed from $t\bar{t}H$ to $t\bar{t}$ kinematics: Higgs recoil absorbed in initial state particles

[Catani et al.; 1507.06937]

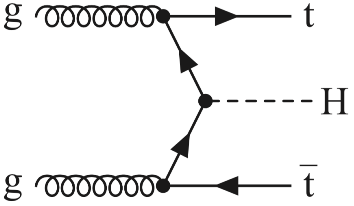
Validation at NLO

σ [fb]	13TeV		100TeV	
	gg	$q\bar{q}$	gg	$q\bar{q}$
LO	261.58	129.47	23055	2323.7
$H^{(1)}$ exact	88.62	7.826	8205	217.0
$H^{(1)}$ approx	61.98	7.413	5612	206.0
Difference	30.1%	5.27%	31.6%	5.06%

- Deviation w.r.t. exact $H^{(1)}$ contribution is about **30% for gg** channel and **5% for $q\bar{q}$** channel

- Quality of approximation independent of c.m. energy

- Better performance in quark channel expected: already at LO Higgs emissions from internal tops in gg channel, which are not captured in the soft limit



- Can we provide **precise NNLO predictions** with this approximation for $H^{(2)}$?

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$H^{(2)}$ approx	-2.980	2.622	-239.4	65.45

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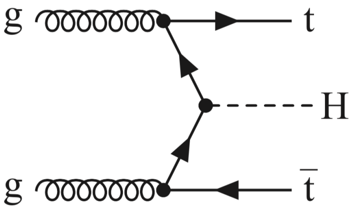


- Can we provide **precise NNLO predictions** with this approximation for $H^{(2)}$?

Yes! Thanks to the small size of the $H^{(2)}$ contribution to the NNLO cross section

Uncertainty estimation

How do we estimate the uncertainties of $H^{(2)}$ approx?

- We use the **deviation from the exact result at NLO** as a reference
- We multiply by a **tolerance factor** of **3**
- We combine **linearly** the uncertainties of the gg and qq channels

Consistency checks for the uncertainty estimation

- We check the effect of changing the recoil prescription
- We change the subtraction scale μ_{IR} at which $H^{(2)}$ is defined

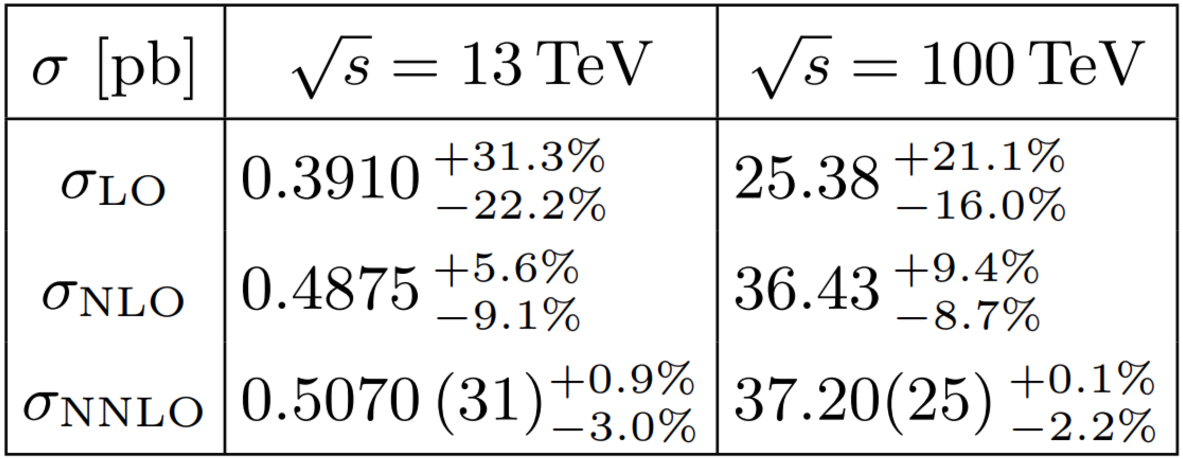
↓

Variations that are consistent or smaller
than our uncertainty estimation

Final uncertainties: $\pm 15\%$ on $\Delta\sigma_{\mathrm{NNLO}}$ $\pm 0.6\%$ on σ_{NNLO}

NNLO results

- Setup: m_t=173.3GeV, m_H=125GeV, NNLO NNPDF31 set, $\mu_0=(2m_t+m_H)/2$



σ [pb]	$\sqrt{s} = 13$ TeV	$\sqrt{s} = 100$ TeV
$\sigma_{\rm LO}$	$0.3910\,^{+31.3\%}_{-22.2\%}$	$25.38\,^{+21.1\%}_{-16.0\%}$
$\sigma_{\rm NLO}$	$0.4875\,^{+5.6\%}_{-9.1\%}$	$36.43\,^{+9.4\%}_{-8.7\%}$
$\sigma_{\rm NNLO}$	$0.5070\,(31)^{+0.9\%}_{-3.0\%}$	$37.20(25)\,^{+0.1\%}_{-2.2\%}$

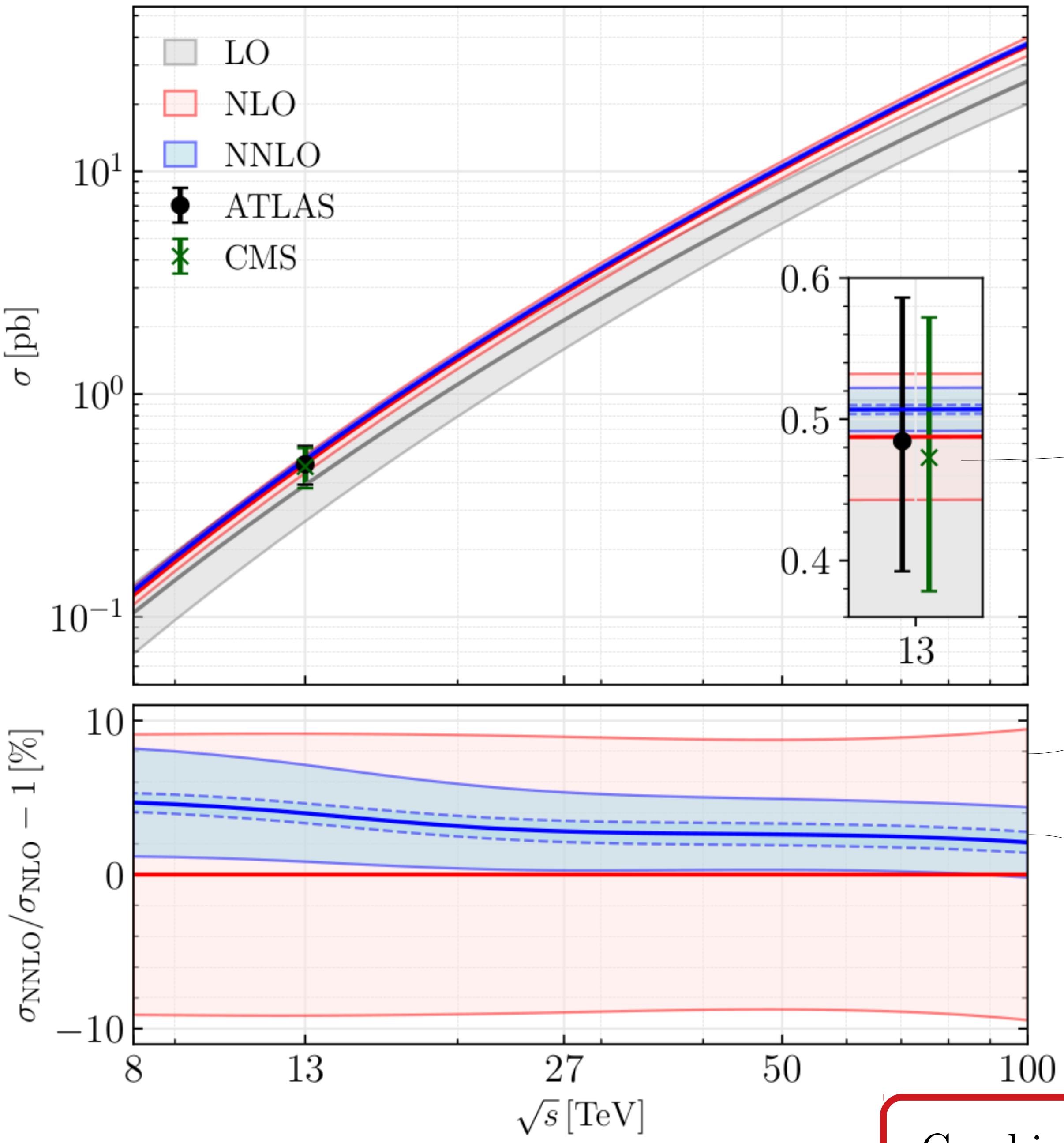
- Effect of NLO corrections is about **+25%** at 13TeV and **+44%** at 100TeV

- Effect of NNLO corrections is about **+4%** at 13TeV and **+2%** at 100TeV

- **Strong reduction** of the perturbative uncertainties at NNLO

- Number in parenthesis includes approximation uncertainty, MC integration uncertainty, and systematic uncertainty from subtraction ($q_T \to 0$ extrapolation)

NNLO results



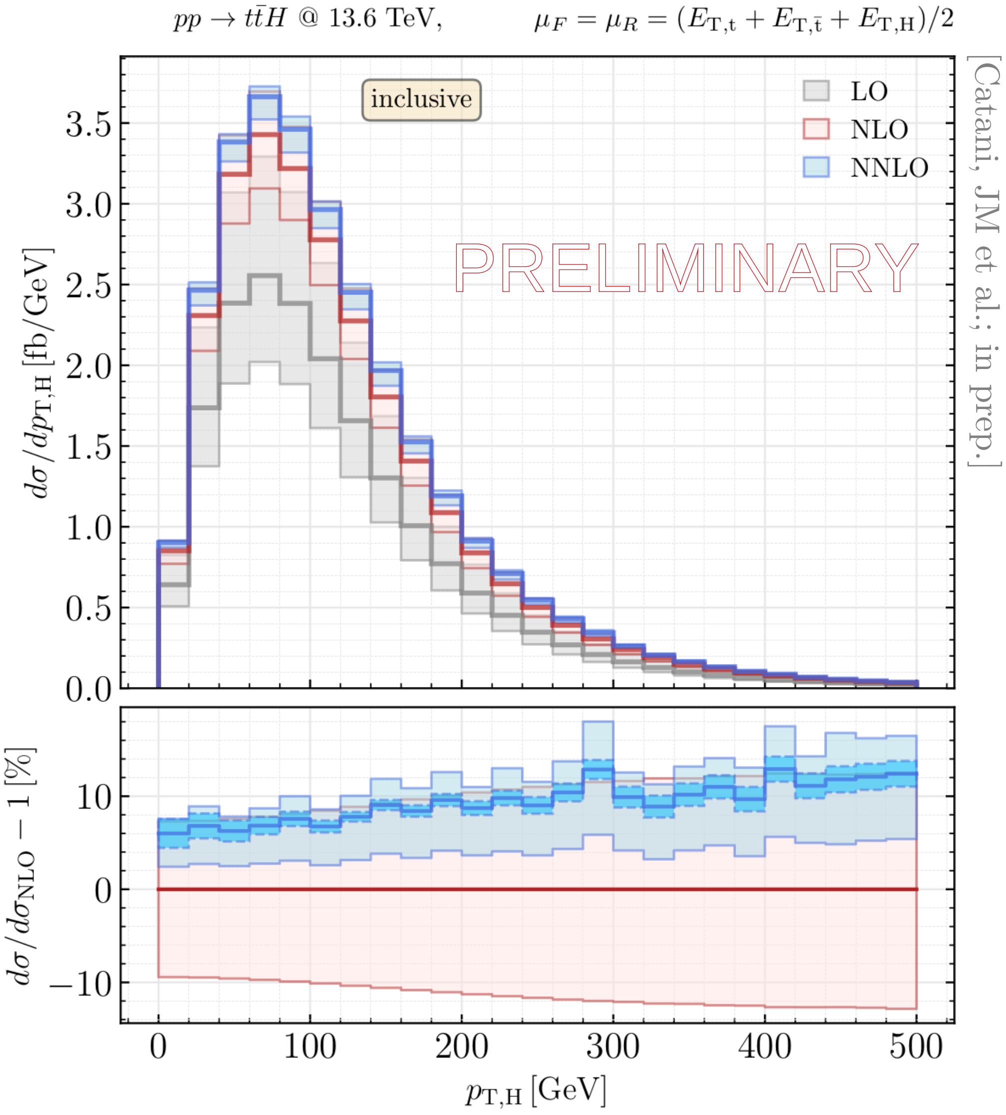
Data from:
[ATLAS 2207.00092]
[CMS 2207.00043]

Bands from symmetrized 7-point scale variation

Dashed band: approximation plus numerical uncertainties

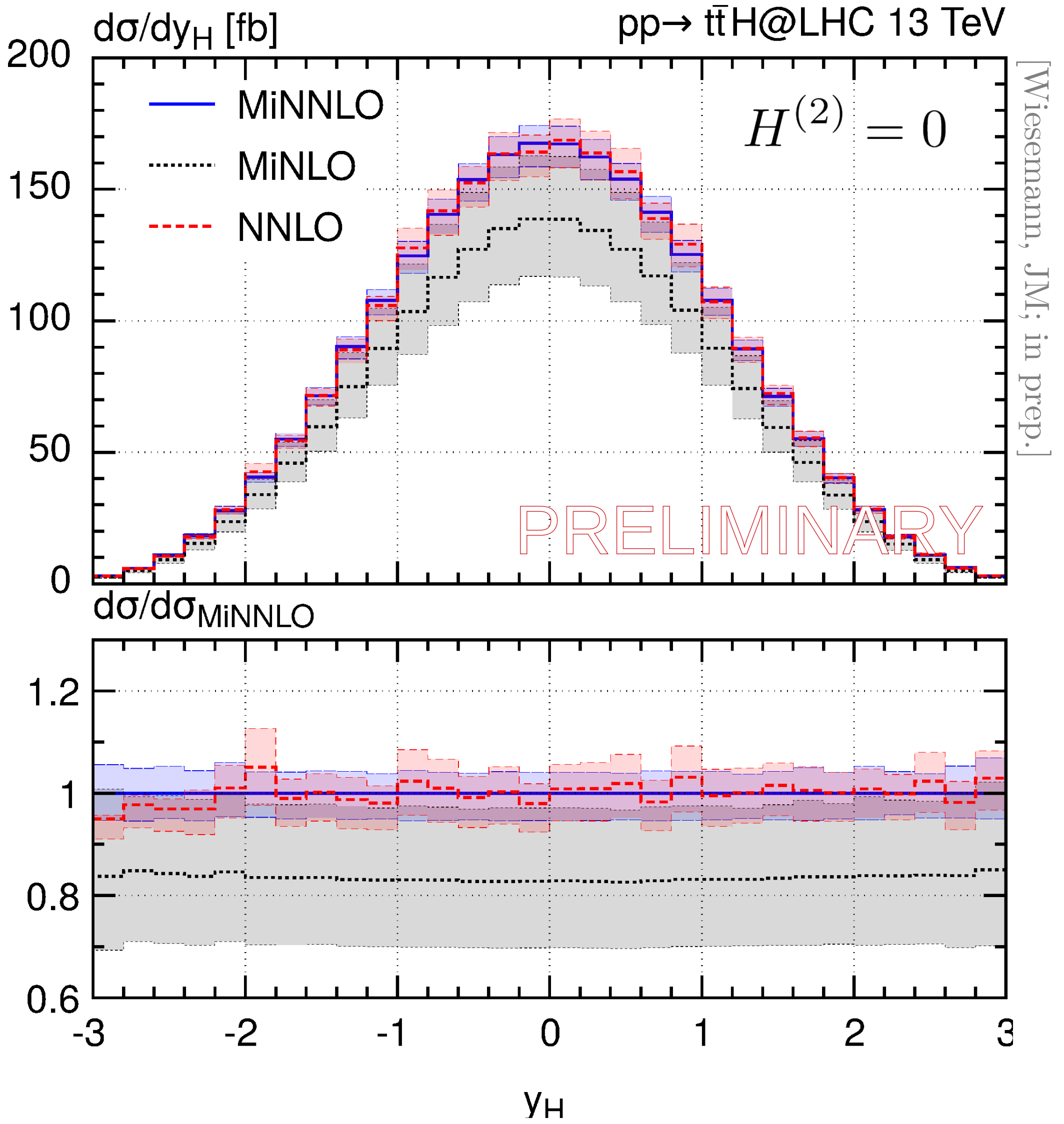
Combination with NLO EW corrections of O(2%) needed for ultimate precision

Future developments



Fully differential NNLO results

[Catani, JM et al.; in prep.]



NNLO+PS event generator

[Wiesemann, JM; in prep.]

Summary

- We have presented the first NNLO calculation for $\mathrm{t\bar{t}H}$ production at hadron colliders

- We used the $\mathrm{q_T}$-subtraction method, now further extended to deal with heavy quark + colourless final states

- Missing two-loop corrections are estimated via a soft Higgs approx., related uncertainties for σ_{NNLO} at the sub-percent level

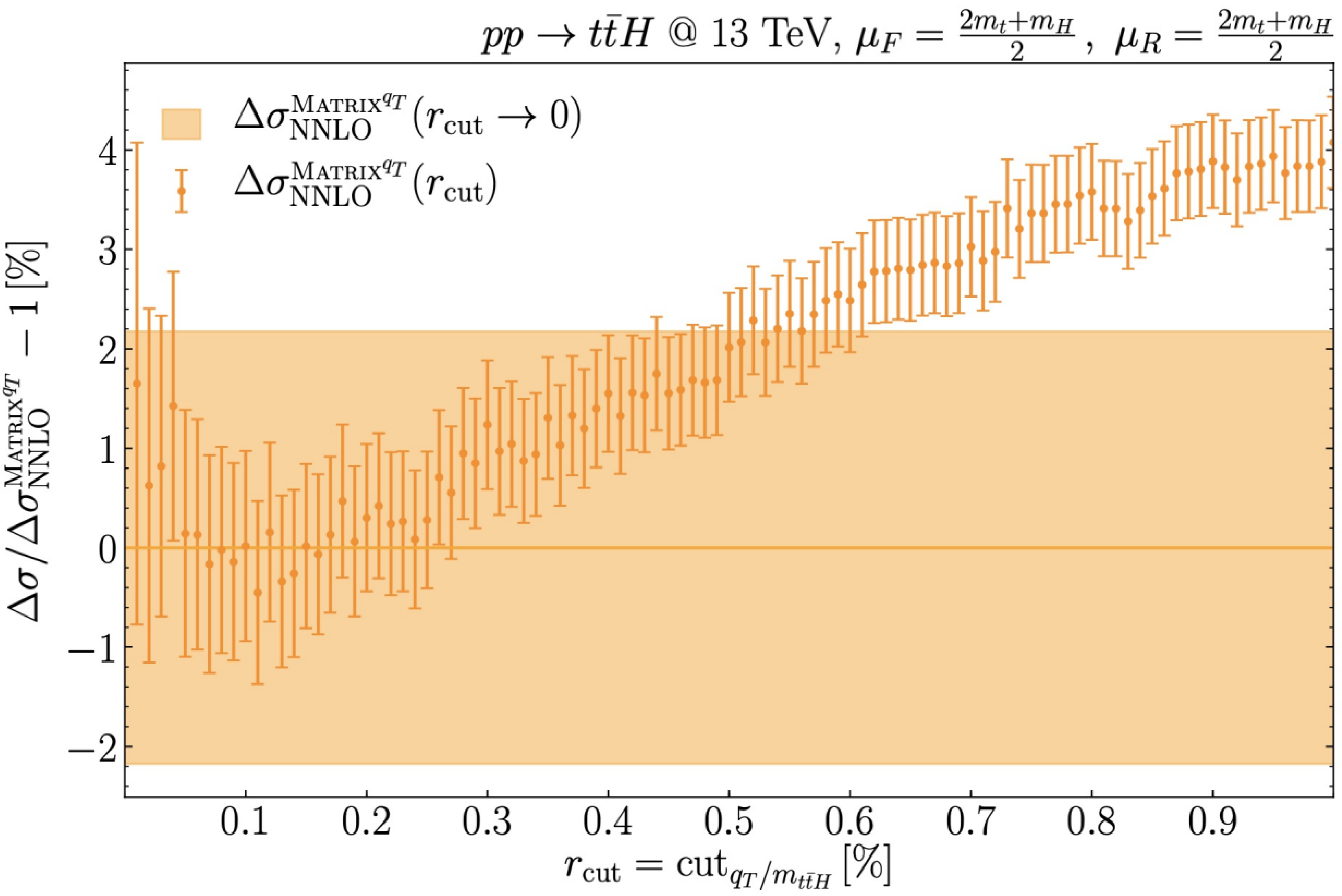
- NNLO corrections are moderate, and leading to a significant reduction of the scale uncertainties w.r.t. NLO

- Further studies are underway:
  fully differential NNLO, NNLO+PS... stay tuned!

Thanks!

Backup slides

$\mathbf{r_{cut} \to 0}$ extrapolation



More numbers on the soft Higgs approx

- Soft Higgs approximation at LO:
  gg channel: factor 2.3 (2.0) larger than exact result at 13 (100) TeV
  $q\bar{q}$ channel: factor 1.11 (1.06) larger than exact result at 13 (100) TeV

- No Born reweighting at LO → worse performance compared to $\mathrm{H}^{(\mathrm{n})}$

- The (differential) cross section within the $\mathrm{q_T}$-subtraction method is

$$d\sigma = \mathcal{H} \otimes d\sigma_{\mathrm{LO}} + [d\sigma_{\mathrm{R}} - d\sigma_{\mathrm{CT}}] \quad \text{with} \quad \mathcal{H} = H(\mu_{\mathrm{IR}})\delta(1-z_1)\delta(1-z_2) + \delta\mathcal{H}(\mu_{\mathrm{IR}})$$

Independent from subtraction scale

$$H(\mu_{\mathrm{IR}}) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{2\pi}\right)^n H^{(n)}(\mu_{\mathrm{IR}}) \quad \text{with} \quad H^{(n)}(\mu_{\mathrm{IR}}) = \frac{2\mathrm{Re}\left(\mathcal{M}^{(\mathrm{n})}_{\mathrm{fin}}(\mu_{\mathrm{IR}})\,\mathcal{M}^{(0)*}\right)}{\left|\mathcal{M}^{(0)}\right|^2}$$

Only approximated piece

Approximation leads to μ_{IR} dependence

- Varying μ_{IR} by a factor of 2:

gg channel: +164%/-25% (13TeV)
+142%/-20% (100TeV)

$q\bar{q}$ channel: +4%/-0% (13TeV)
+3%/-0% (100TeV)