



Non-factorisable contributions to t -channel single-top production

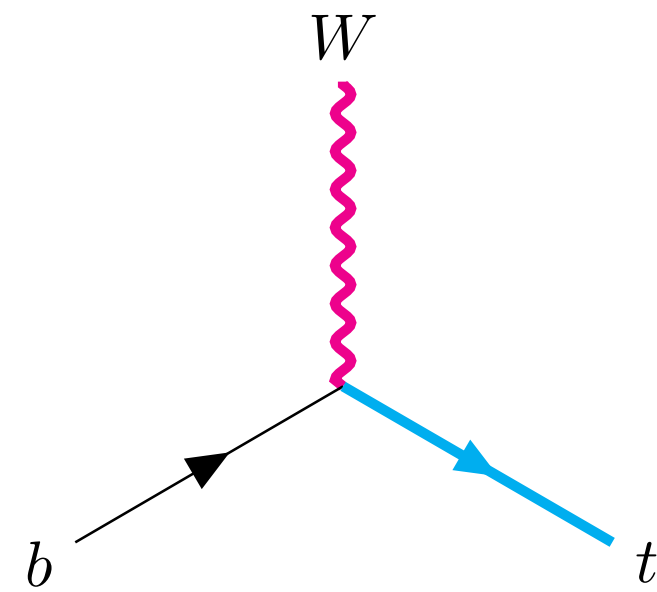
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*Belgrade
24th of May 2023*

In collaboration with: Kirill Melnikov, Jérémie Quarroz, Chiara Signorile-Signorile, and Chen-Yu Wang
Based on 2108.09222 and 2204.05770

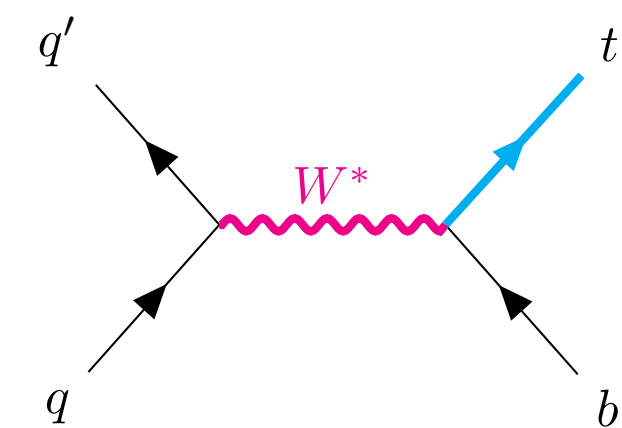
Single-top production: 3 channels

- Electroweak mediated

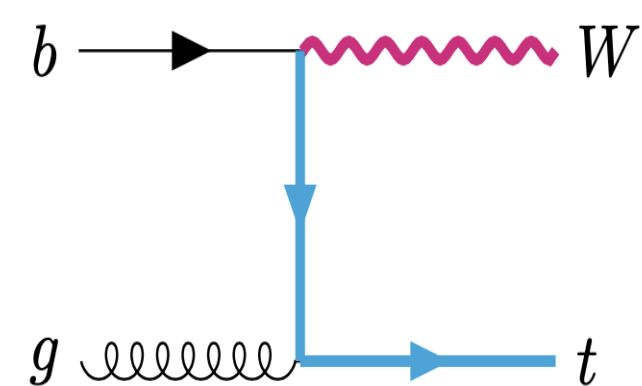


Determination of the **CKM matrix element**
 Anomalous couplings
 Indirect determination of top **width** and **mass**
 Provides constraints on **PDF**

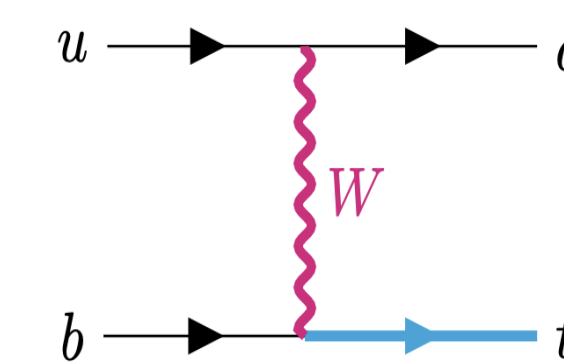
- Production rate at the LHC of comparable of magnitude to top pair: $\sigma_t \sim 0.2 \sigma_{t\bar{t}}$



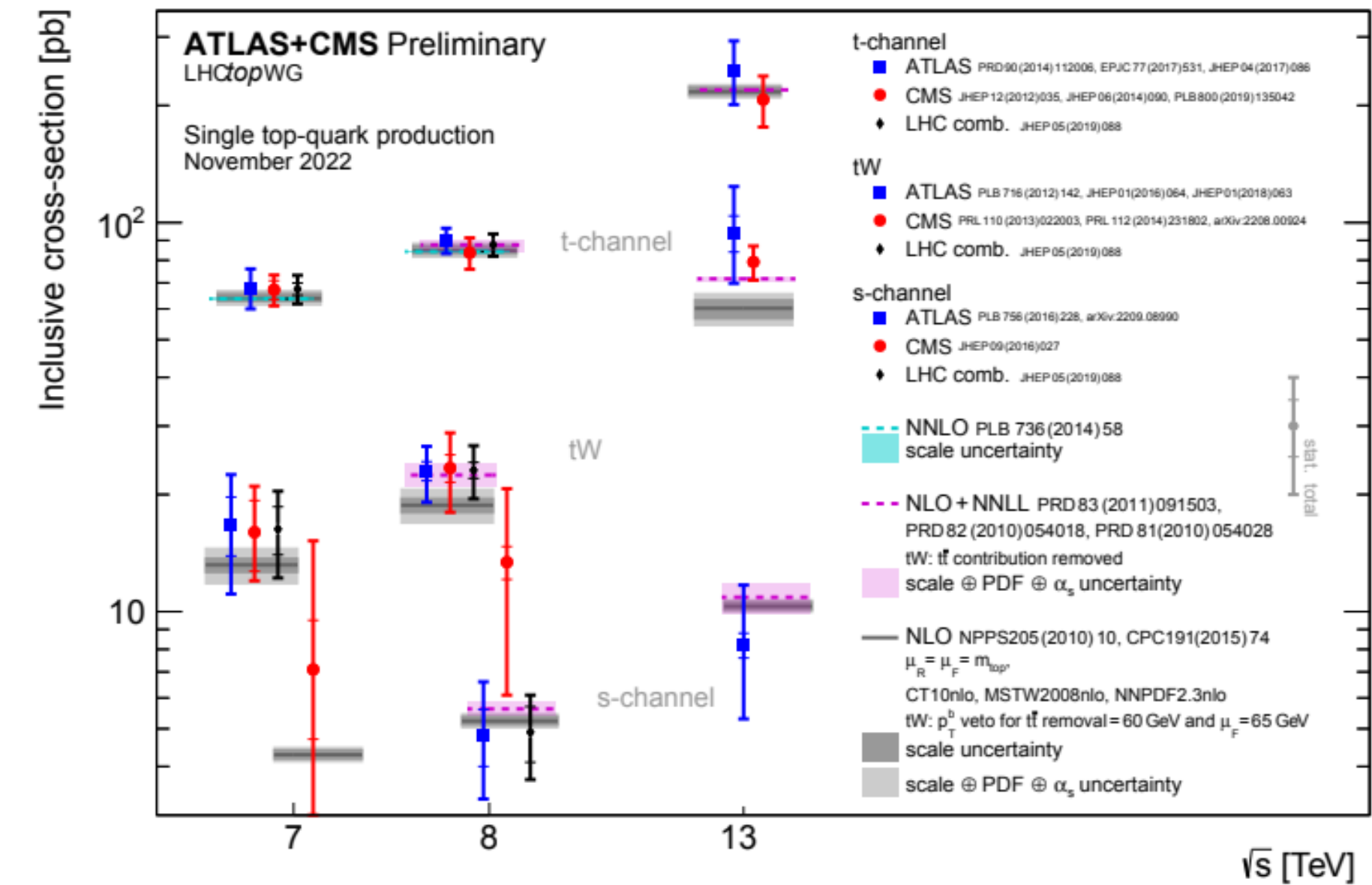
s-channel
 $q q' \rightarrow b t$



associated production
 $g b \rightarrow W t$
 not observed until 2014



t-channel
 $q b \rightarrow q' t$



Single-top production: theory status, s- and tW -channel

s-channel:

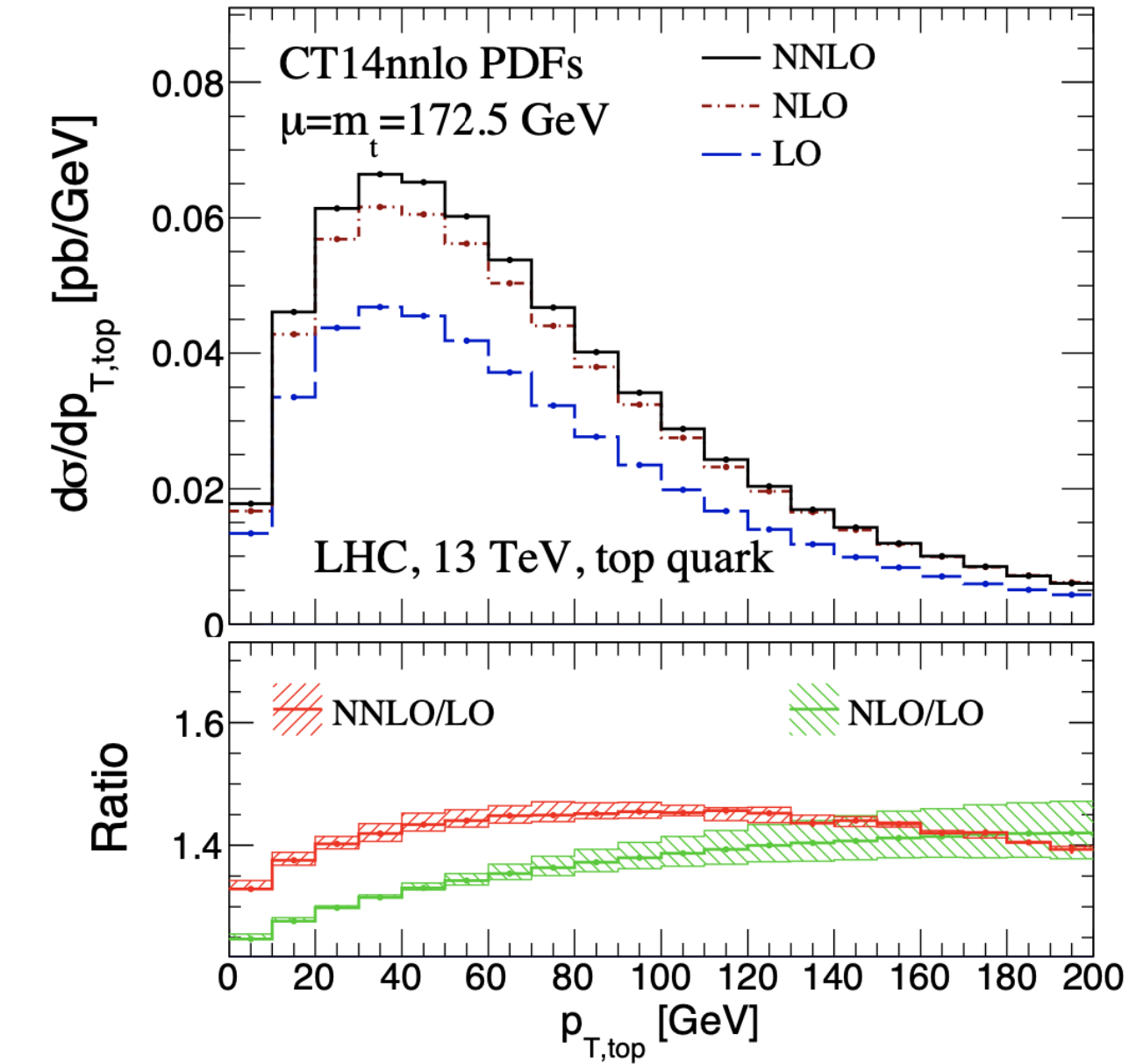
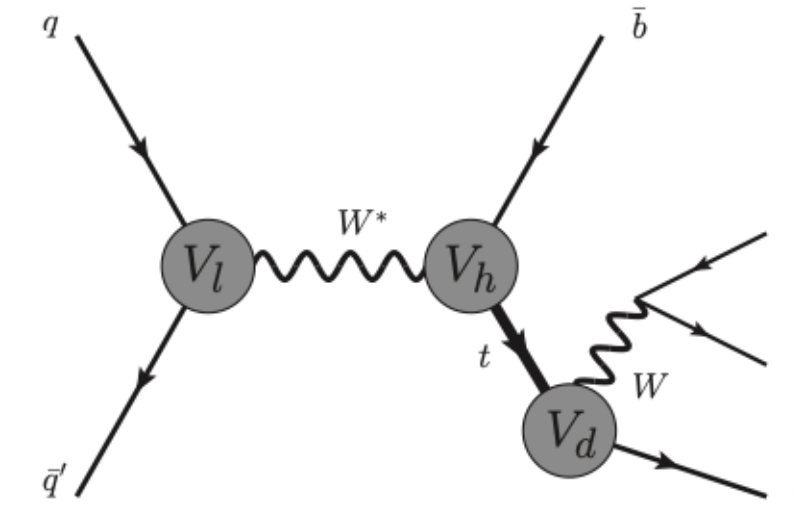
NNLO QCD corrections in production and decay [Liu, Gao '18]

Inclusive corrections are $\sim 5\%$ w.r.t. NLO

In low p_{\perp} region, NNLO corrections can reach 10%

No overlap of NLO and NNLO bands in most regions: NNLO corrections underestimated.

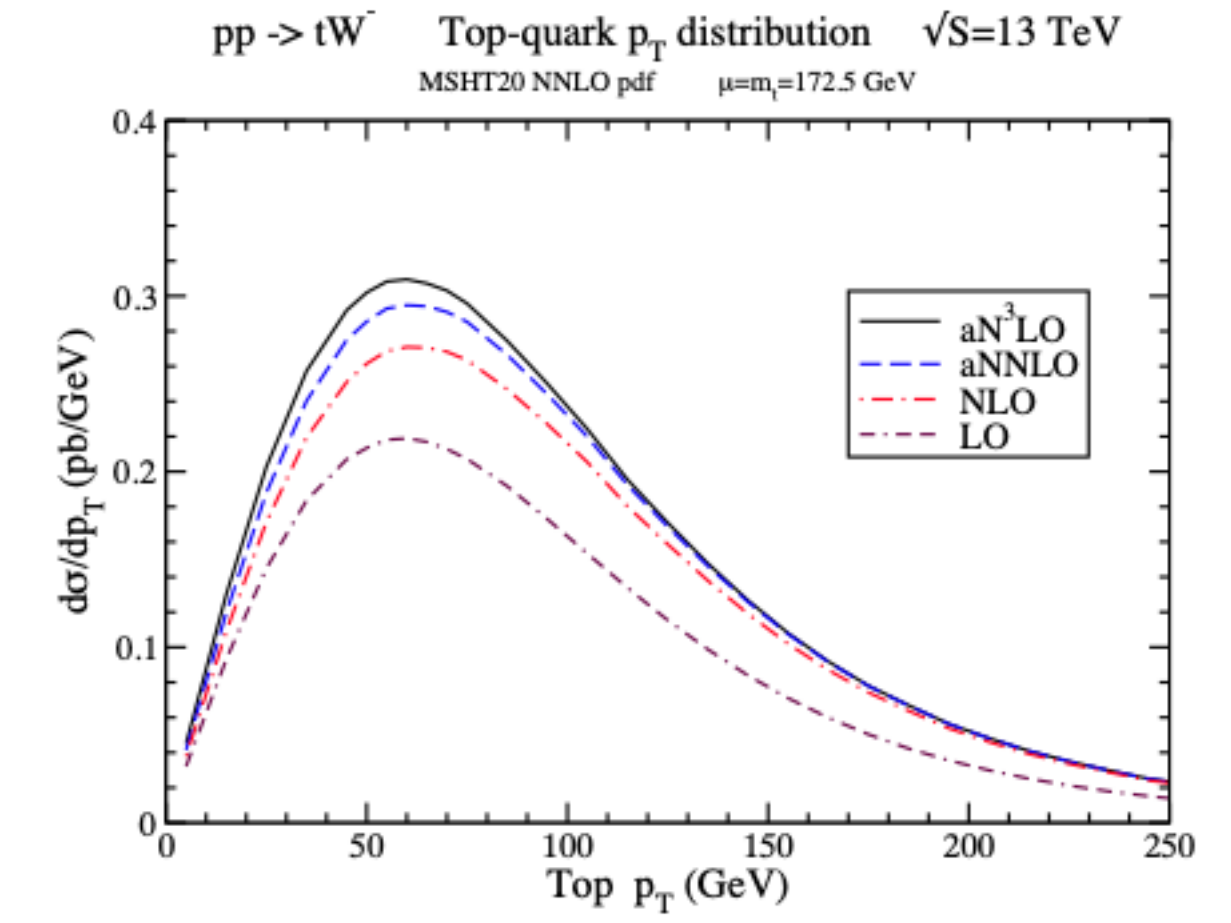
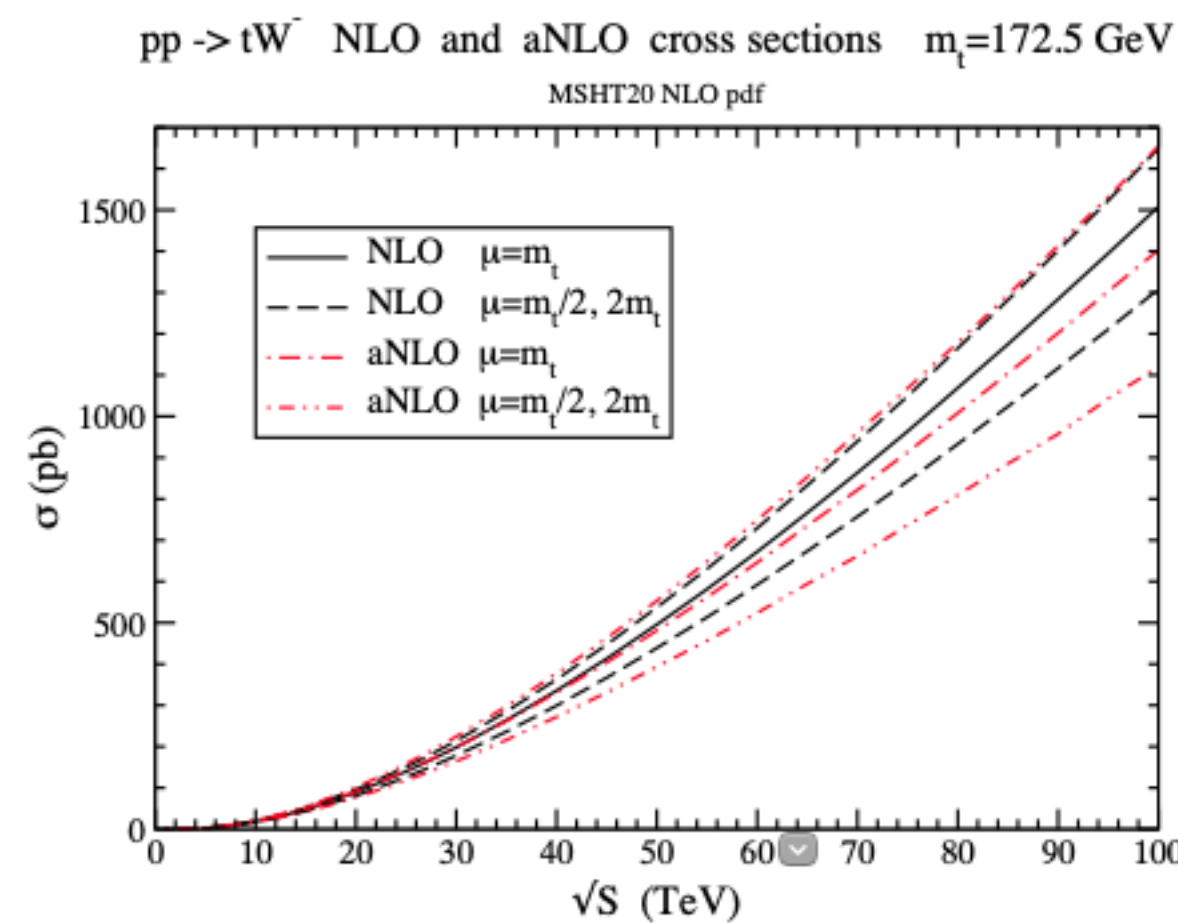
inclusive		LO	NLO	NNLO
8 TeV	$\sigma(t)$ [pb]	$2.498^{+0.17\%}_{-0.74\%}$	$3.382^{+2.36\%}_{-1.81\%}$	$3.566^{+0.95\%}_{-0.78\%}$
	$\sigma(\bar{t})$ [pb]	$1.418^{+0.12\%}_{-0.73\%}$	$1.922^{+2.37\%}_{-1.81\%}$	$2.029^{+1.07\%}_{-0.83\%}$
	$\sigma(t + \bar{t})$ [pb]	$3.916^{+0.15\%}_{-0.73\%}$	$5.304^{+2.36\%}_{-1.81\%}$	$5.595^{+0.99\%}_{-0.80\%}$
	$\sigma(t)/\sigma(\bar{t})$	$1.762^{+0.04\%}_{-0.01\%}$	$1.760^{+0.00\%}_{-0.02\%}$	$1.757^{+0.05\%}_{-0.12\%}$
13 TeV	$\sigma(t)$ [pb]	$4.775^{+2.69\%}_{-3.50\%}$	$6.447^{+1.39\%}_{-0.91\%}$	$6.778^{+0.76\%}_{-0.53\%}$
	$\sigma(\bar{t})$ [pb]	$2.998^{+2.69\%}_{-3.55\%}$	$4.043^{+1.33\%}_{-0.94\%}$	$4.249^{+0.69\%}_{-0.48\%}$
	$\sigma(t + \bar{t})$ [pb]	$7.772^{+2.69\%}_{-3.52\%}$	$10.49^{+1.36\%}_{-0.92\%}$	$11.03^{+0.74\%}_{-0.51\%}$
	$\sigma(t)/\sigma(\bar{t})$	$1.593^{+0.05\%}_{-0.01\%}$	$1.595^{+0.06\%}_{-0.03\%}$	$1.595^{+0.07\%}_{-0.05\%}$



tW -channel:

Approximate approaches used to infer higher-order corrections [Kidonakis, Yamanaka '21]

NNLO QCD corrections not known yet (analytic 2-loop amplitudes known [Chen, Dong, Li, Li, Wang, Wang '22])



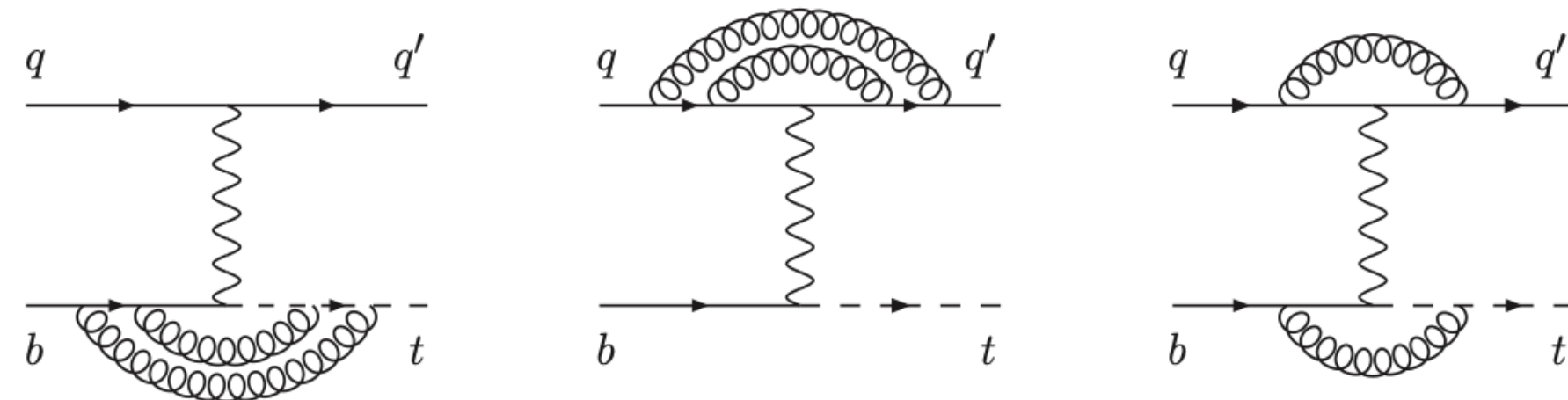
Single-top production: theory status, t -channel (I)

Two types of topologies contribute to the t -channel, single-top production:

- **Factorisable contributions**

- **Structure function approximation** → crosstalk between quark lines neglected due to colour suppression
- **NLO QCD** [Bordes, van Eijk '95] [Campbell, Ellis, Tramontano '04] [Cao, Yuan '05] [Cao, Schwienhorst, Benitez, Brock, Yuan '05] [Harris, Laenen, Phaf, Sullivan, Weinzierl '02] [Schwienhorst, Yuan, Mueller, Cao '11]
- **NNLO QCD**
- First calculated for a **stable top quark** [Brucherseifer, Caola, Melnikov '14]
- **Small effects** on inclusive cross-section and on cross-section with $p_{\perp,t}$ cuts

MSTW2008, lo, nlo, nnlo PDF, $\mu_R = \mu_F = m_t = 173$ GeV @ 8 TeV

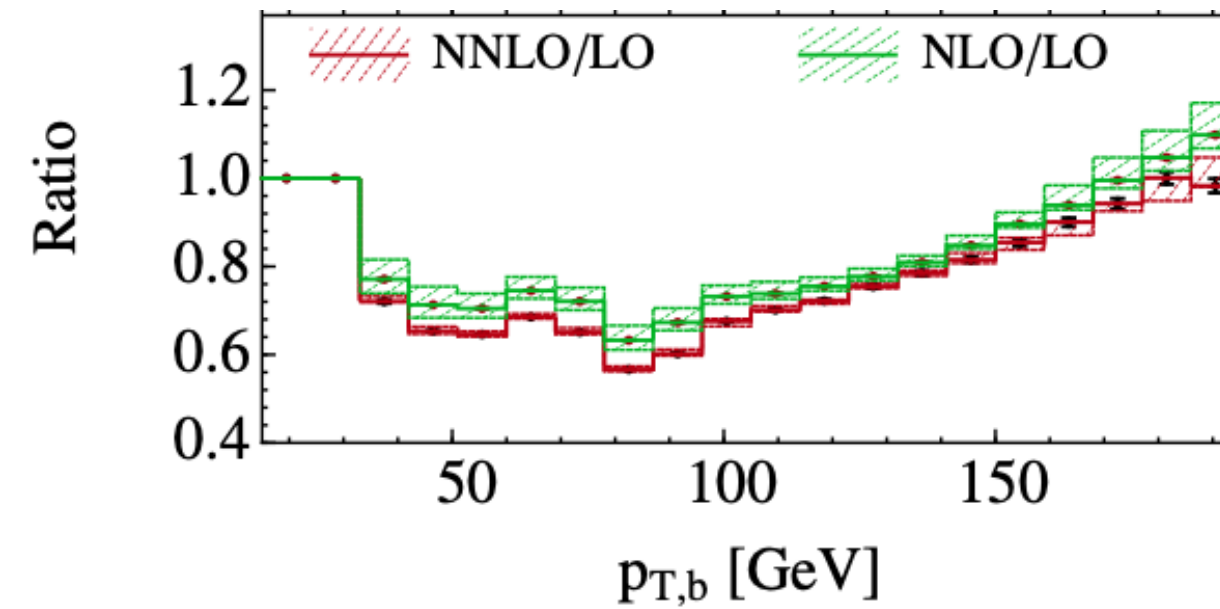
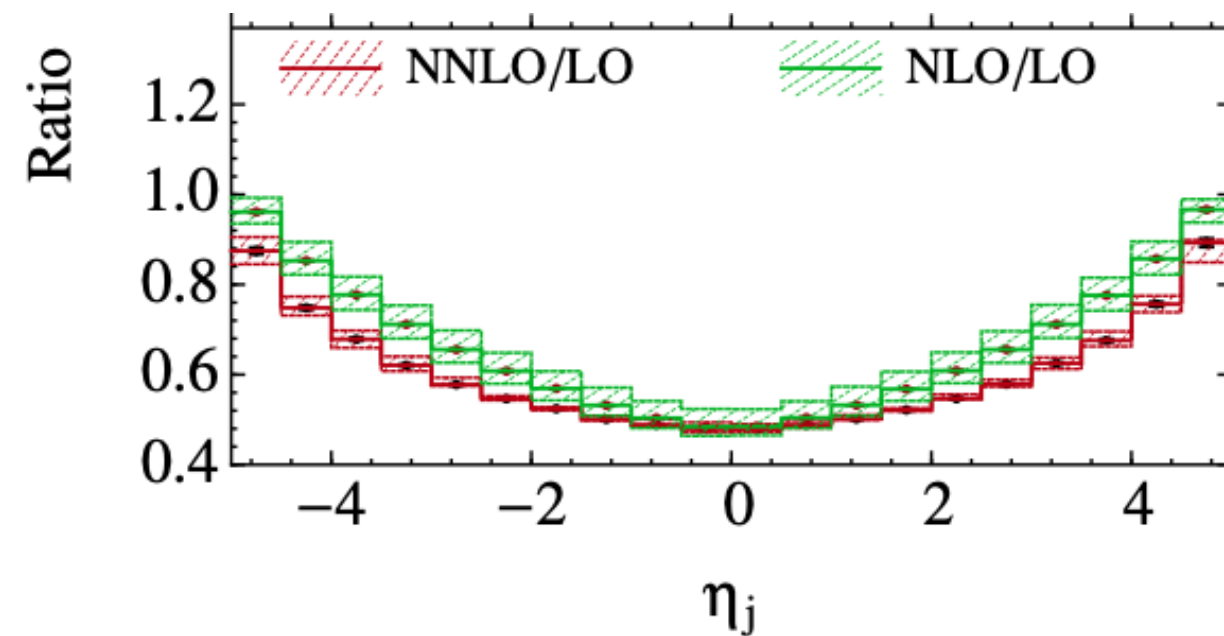
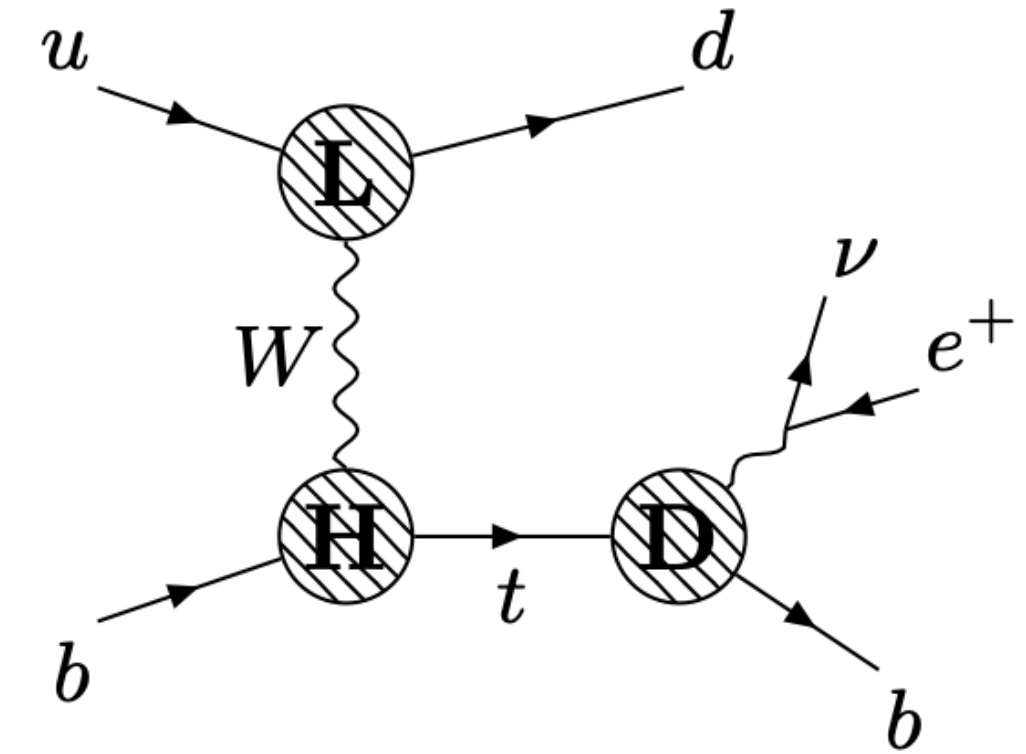


p_{\perp}	$\sigma_{\text{LO}}, \text{pb}$	$\sigma_{\text{NLO}}, \text{pb}$	δ_{NLO}	$\sigma_{\text{NNLO}}, \text{pb}$	δ_{NNLO}
0 GeV	$53.8^{+3.0}_{-4.3}$	$55.1^{+1.6}_{-0.9}$	+2.4%	$54.2^{+0.5}_{-0.2}$	-1.6%
20 GeV	$46.6^{+2.5}_{-3.7}$	$48.9^{+1.2}_{-0.5}$	+4.9%	$48.3^{+0.3}_{-0.02}$	-1.2%
40 GeV	$33.4^{+1.7}_{-2.5}$	$36.5^{+0.6}_{-0.03}$	+9.3%	$36.5^{+0.1}_{+0.1}$	-0.1%
60 GeV	$22.0^{+1.0}_{-1.5}$	$25.0^{+0.2}_{+0.3}$	+13.6%	$25.4^{-0.1}_{+0.2}$	+1.6%

Single-top production: theory status, t -channel (II)

NNLO QCD

- Extension to **top-quark decay in the NW approximation**, including also NNLO in decay
- (computed using SCET/jettiness + projection to Born) [[Berger, Gao, Yuan, Zhu '16, '17](#)]
- Large corrections for some distributions



- Disagreement with earlier calculation of inclusive cross-section
- Independent calculation based on SCET approach [[Campbell, Neumann, Sullivan '21](#)]

CT14, lo, nlo, nnlo PDF, $\mu_R = \mu_F = m_t = 172.5$ GeV @ 14 TeV

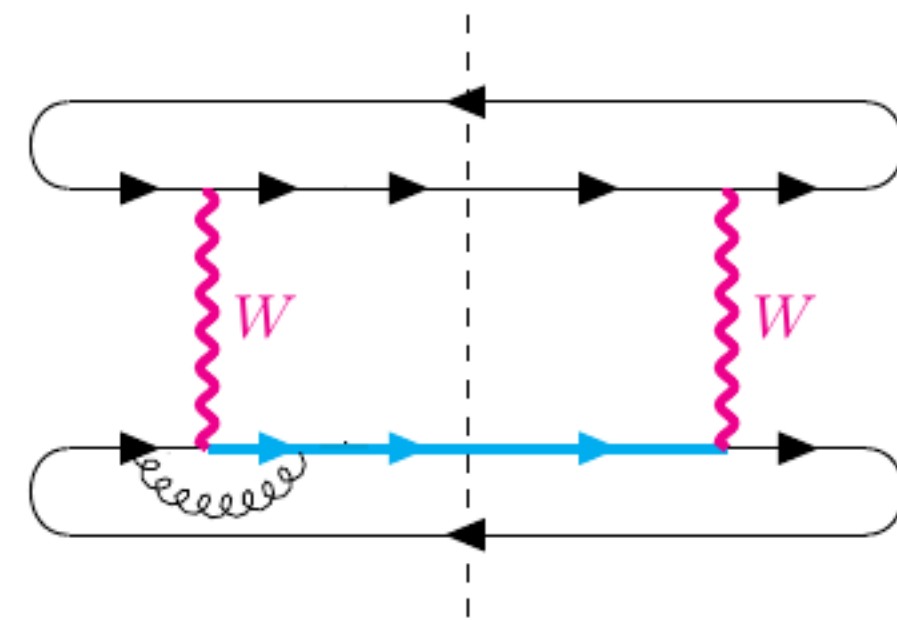
$$\delta\sigma^{\text{NNLO}} \sim -0.7\% \sigma^{\text{NLO}}$$

	7 TeV pp		14 TeV pp	
	top	anti-top	top	anti-top
$\sigma_{\text{LO}}^{\mu=m_t}$	$37.1^{+7.1\%}_{-9.5\%}$	$19.1^{+7.3\%}_{-9.7\%}$	$134.6^{+10.0\%}_{-12.1\%}$	$78.9^{+10.4\%}_{-12.6\%}$
$\sigma_{\text{LO}}^{\text{DDIS}}$	$39.5^{+6.4\%}_{-8.6\%}$	$19.9^{+7.0\%}_{-9.3\%}$	$140.9^{+9.4\%}_{-11.4\%}$	$80.7^{+10.2\%}_{-12.3\%}$
$\sigma_{\text{NLO}}^{\mu=m_t}$	$41.4^{+3.0\%}_{-2.0\%}$	$21.5^{+3.1\%}_{-2.0\%}$	$154.3^{+3.1\%}_{-2.3\%}$	$91.4^{+3.1\%}_{-2.2\%}$
$\sigma_{\text{NLO}}^{\text{DDIS}}$	$41.8^{+3.3\%}_{-2.0\%}$	$21.5^{+3.4\%}_{-1.6\%}$	$154.4^{+3.7\%}_{-1.4\%}$	$91.2^{+3.1\%}_{-1.8\%}$
	PDF $+1.7\%$ -1.4%	PDF $+2.2\%$ -1.5%	PDF $+1.7\%$ -1.1%	PDF $+1.9\%$ -0.9%
$\sigma_{\text{NNLO}}^{\mu=m_t}$	$41.9^{+1.2\%}_{-0.7\%}$	$21.9^{+1.2\%}_{-0.7\%}$	$153.3(2)^{+1.0\%}_{-0.6\%}$	$91.5(2)^{+1.1\%}_{-0.9\%}$
$\sigma_{\text{NNLO}}^{\text{DDIS}}$	$41.9^{+1.3\%}_{-0.8\%}$	$21.8^{+1.3\%}_{-0.7\%}$	$153.4(2)^{+1.1\%}_{-0.7\%}$	$91.2(2)^{+1.1\%}_{-0.9\%}$
	PDF $+1.3\%$ -1.1%	PDF $+1.4\%$ -1.3%	PDF $+1.2\%$ -1.0%	PDF $+1.0\%$ -1.0%

Non-factorisable corrections: why? (I)

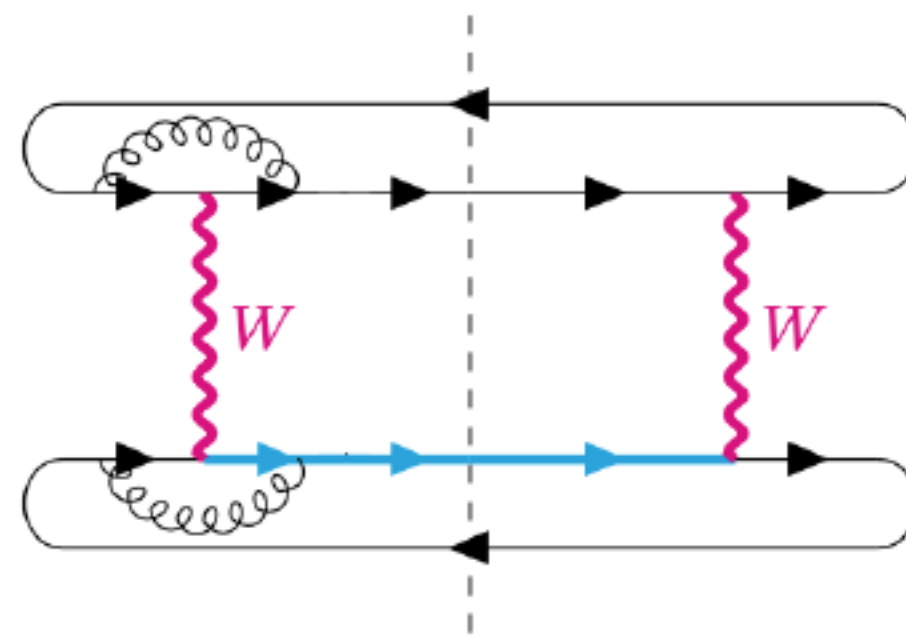
NLO

Factorisable contributions



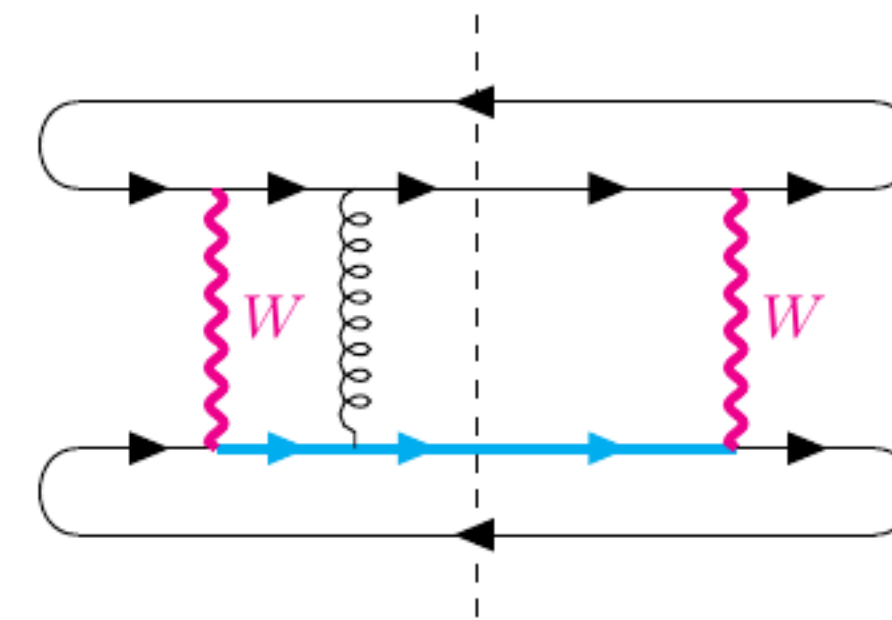
$$\text{tr}(t^a t^a) = \frac{N_c^2 - 1}{2}$$

NNLO

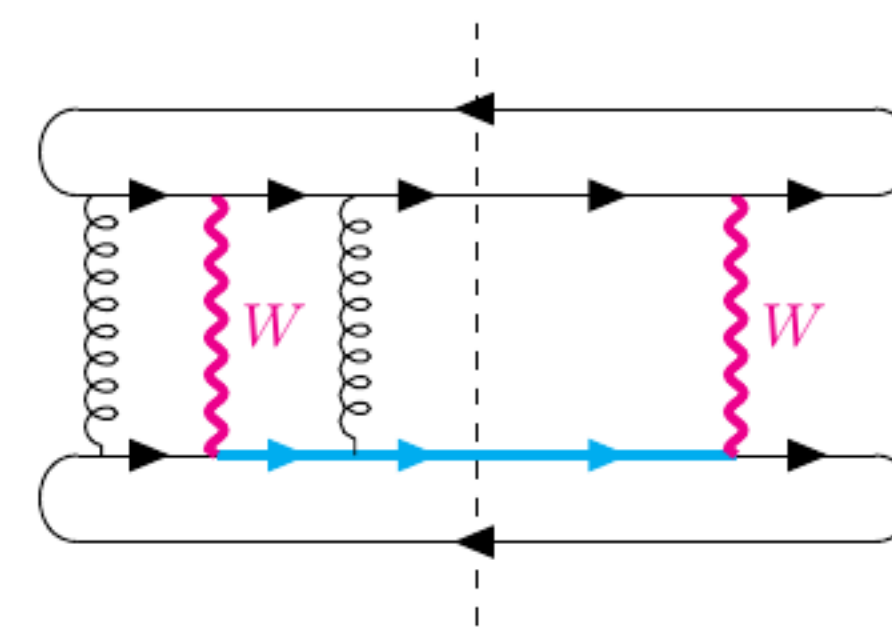


$$\text{tr}(t^a t^a) \text{tr}(t^b t^b) = \frac{(N_c^2 - 1)^2}{4}$$

Non-factorisable contributions



$$\text{tr}(t^a) = 0$$



$$\text{tr}(t^a t^b) \text{tr}(t^a t^b) = \frac{N_c^2 - 1}{4}$$

Non-factorisable corrections: why? (II)

Non-factorisable contributions vanish at NLO due to their colour structure, and are suppressed by a factor $N_c^2 - 1 = 8$ at NNLO.

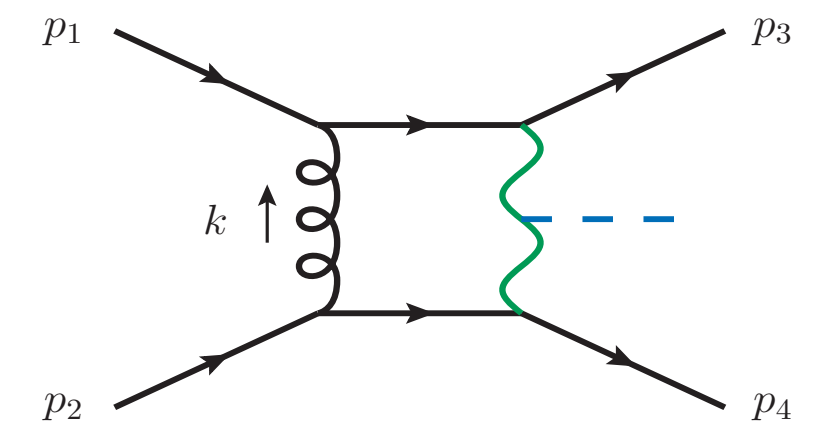
However:

Factorisable predictions are already small, about a few percent

The actual size of NNLO non-factorisable corrections **cannot be inferred from NLO contributions**

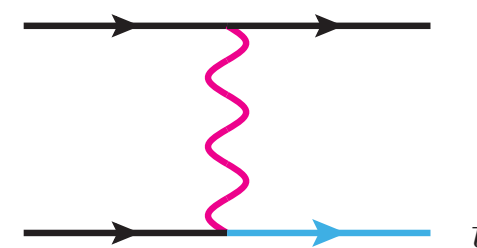
Non-factorisable corrections could be **enhanced by a factor $\pi^2 \simeq 10$** related to a **Glauber phase**

- shown for Higgs production in weak boson fusion in the eikonal approximation [*Liu, Melnikov, Penin '19*]
- loop effect that, in principle, does not require a scattering to occur



$$\sigma = \underbrace{\sigma_0}_{\text{virtual contribution}} + \underbrace{\left(\frac{p_{\perp}}{\sqrt{s}}\right)^2 \sigma_1 + \dots}_{\text{real contribution}}$$

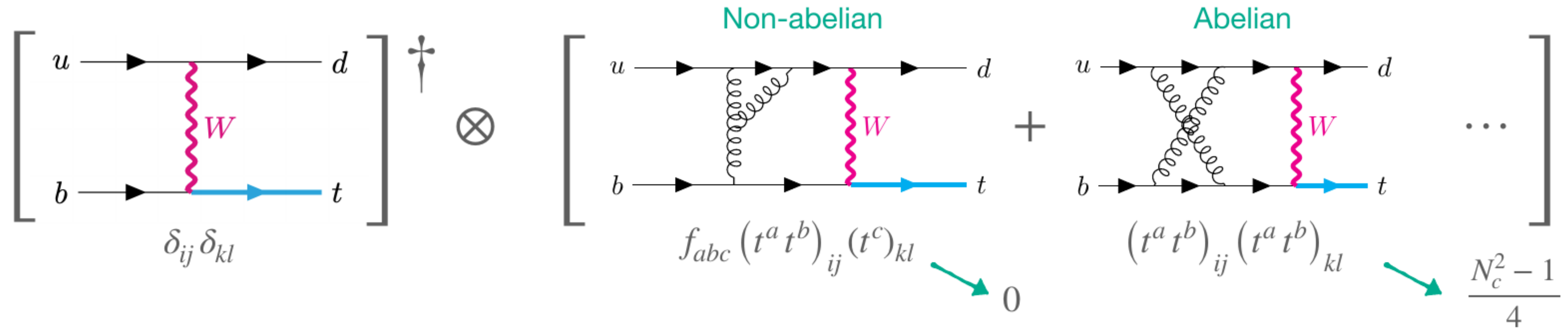
π^2 (circled in red) points to the coefficient of the real contribution.



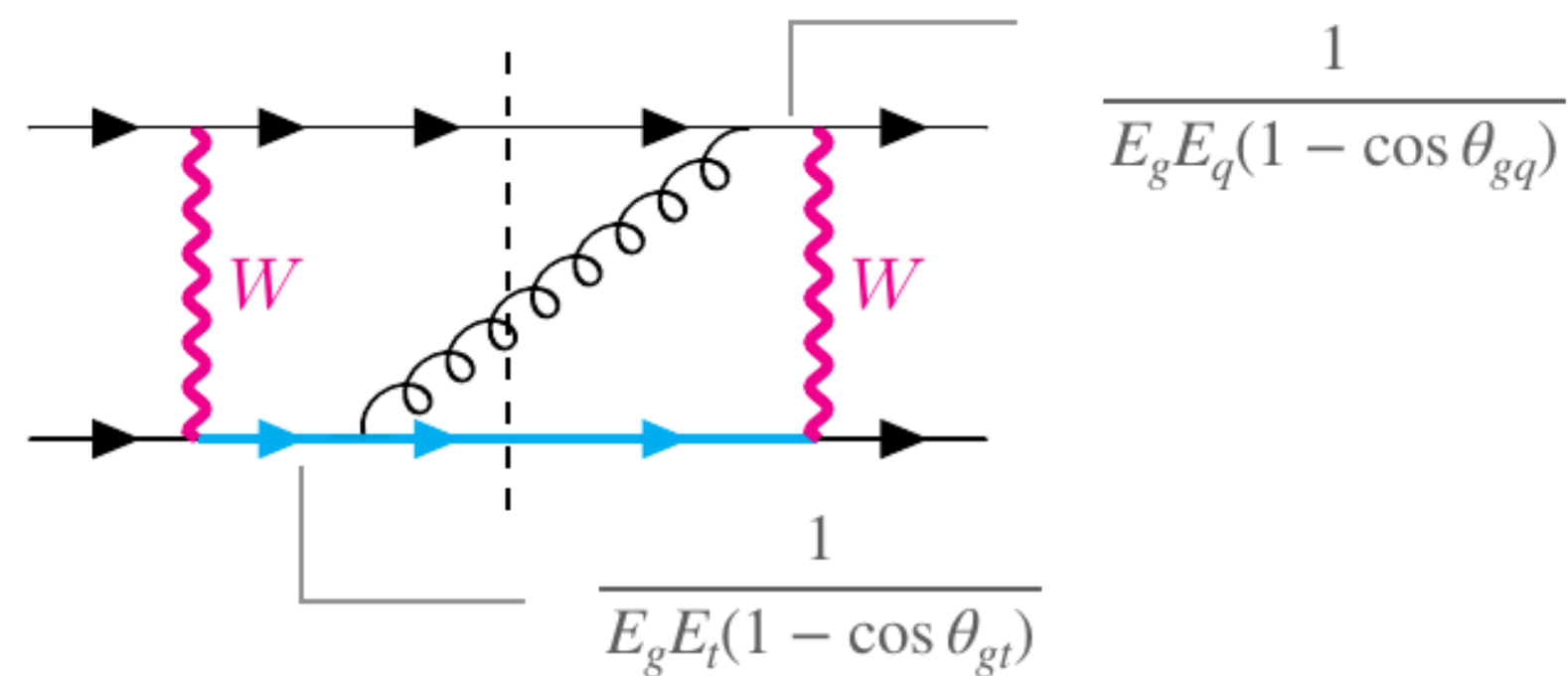
$$p_{\perp}^t \sim 40 \text{ GeV} \quad \sqrt{s} \sim 300 \text{ GeV}$$

Non-factorisable corrections: ingredients of the calculation (II)

Non-factorisable contributions have to **connect upper and lower quark lines** and are effectively **Abelian**

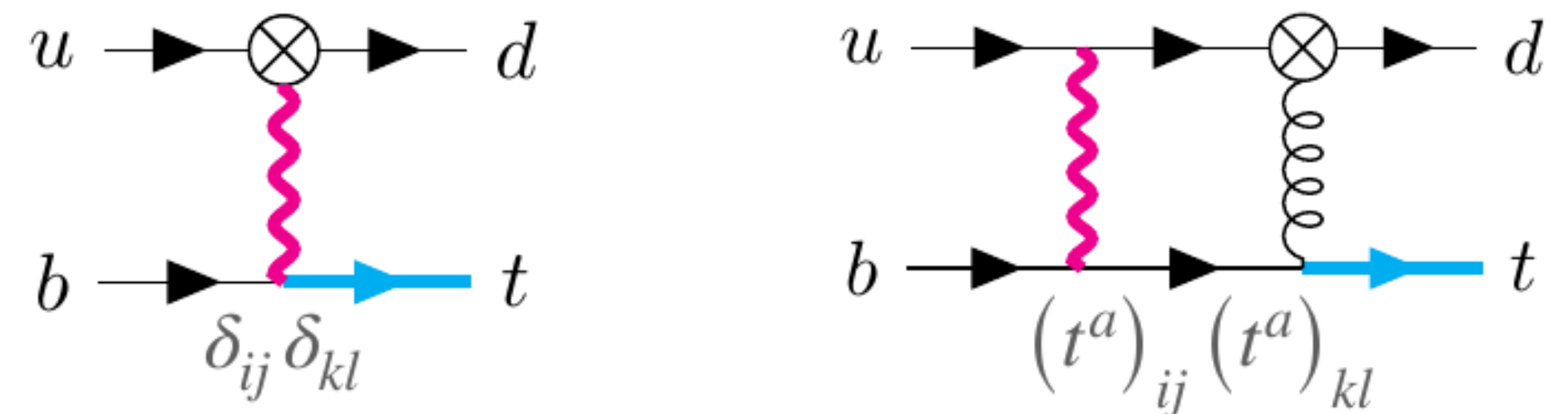


The infrared structure is simplified: **no collinear singularities**



All IR singularities are of **soft origin**.

Non-factorisable contributions are **UV finite**



Renormalisation simply consists of $\alpha_s^{\text{bare}} = \alpha_s \mu^{2\epsilon} S_\epsilon$

Single top production: Results at 13 TeV (I)

Differential cross section:

pp collision: $\sqrt{s} = 13 \text{ TeV}$, PDFs: CT14_lo@LO, CT14_nnlo@NNLO $m_W = 80.379 \text{ GeV}$, $m_t = 173.0 \text{ GeV}$, $\alpha_S(m_t) = 0.108$, $\mu_F = m_t$

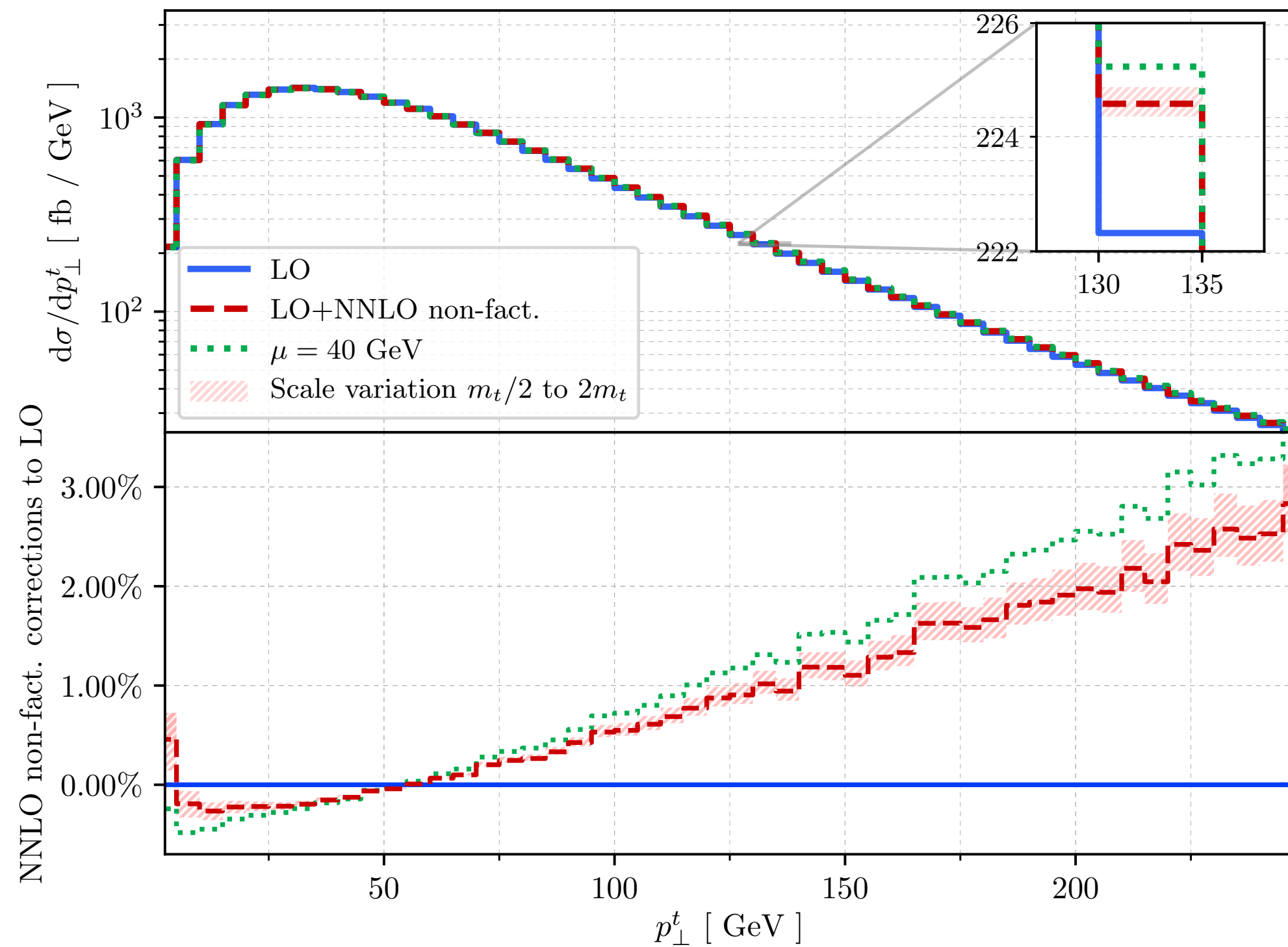
$$\frac{\sigma_{pp \rightarrow X+t}}{1 \text{ pb}} = 117.96 + 0.26 \left(\frac{\alpha_S(\mu_R)}{0.108} \right)^2$$

- Non-factorisable corrections are $0.22_{+0.05}^{-0.04} \%$ LO for $\mu_R = m_t$.
- **Theoretical uncertainties** are estimated through **scale variation**: $\mu_R \in [m_t/2, 2m_t]$.
- **Unclear optimal scale choice**: non-factorisable corrections appear for the first time at NNLO → no indication from lower orders.
- For $\mu_R = 40 \text{ GeV}$ (typical **momentum transfer scale** of top quark) non-factorisable corrections are 0.35% LO.
- In comparison, **NNLO factorisable** corrections to **NLO cross section** are around 0.7% .

Single top production: Results at 13 TeV (II)

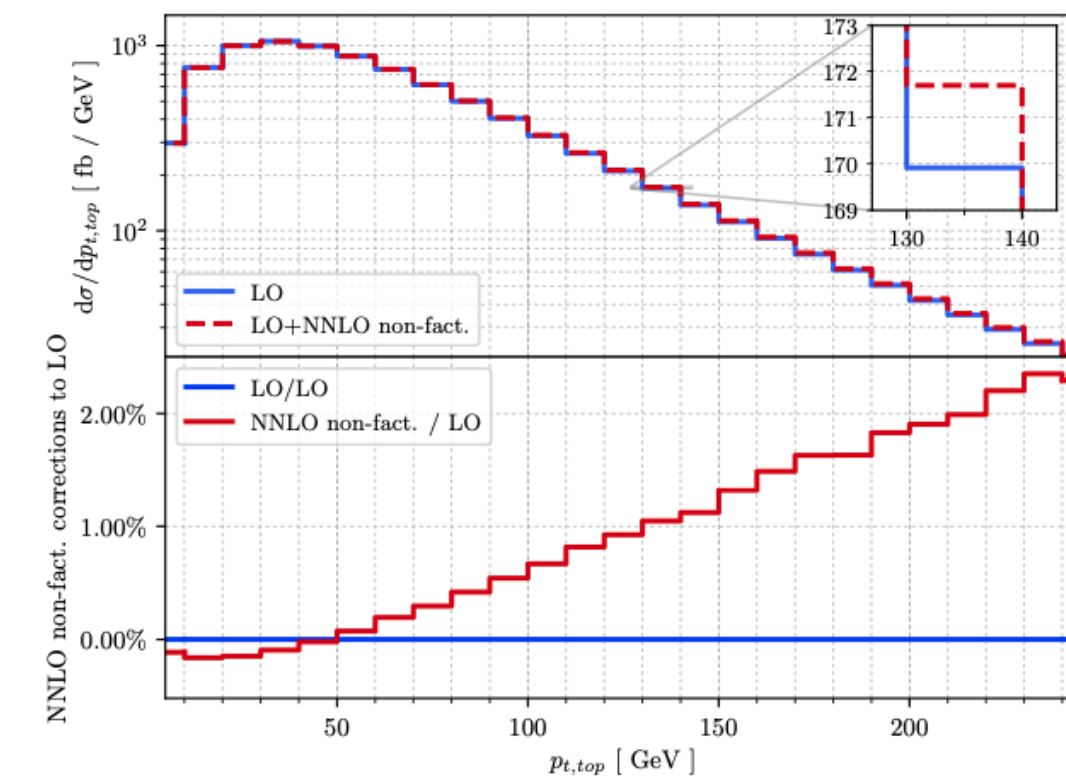
Differential cross section:

pp collision: $\sqrt{s} = 13 \text{ TeV}$, PDFs: CT14_lo@LO, CT14_nnlo@NNLO $m_W = 80.379 \text{ GeV}$, $m_t = 173.0 \text{ GeV}$, $\alpha_S(m_t) = 0.108$, $\mu_F = m_t$



[Brønnum-Hansen, Melnikov, Quarroz, Signorile-Signorile, Wang '22]

- Non-factorisable corrections are p_{\perp}^t dependent.
- Non-factorisable corrections are small and negative at low values of p_{\perp}^t
- They **vanish** at $p_{\perp}^t \sim 50 \text{ GeV}$ (in agreement with results for virtual corrections)
- **Factorisable corrections** vanish around $p_{\perp}^t \sim 30 - 40 \text{ GeV}$
- **Factorisable** and **non-factorisable** corrections are comparable in the region around the maximum of the p_{\perp}^t distribution.



[Brønnum-Hansen, Melnikov, Quarroz, Wang '21]

Results at 100 TeV

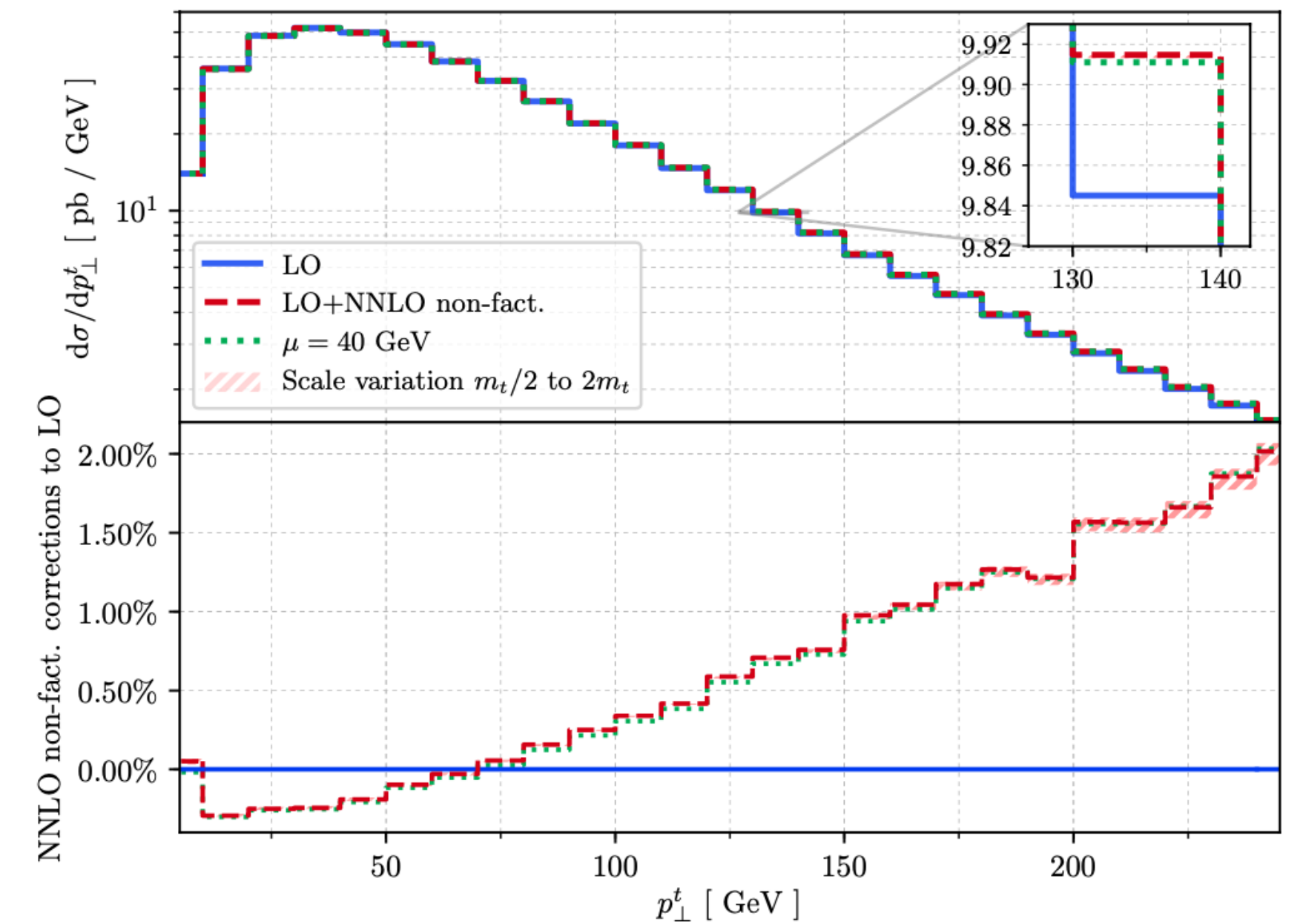
Differential cross section:

pp collision: $\sqrt{s} = 100 \text{ TeV}$ PDFs: CT14_lo@LO, CT14_nnlo@NNLO $m_W = 80.379 \text{ GeV}$, $m_t = 173.0 \text{ GeV}$, $\alpha_S(m_t) = 0.108$, $\mu_F = m_t$

$$\frac{\sigma_{pp \rightarrow X+t}}{1 \text{ pb}} = 2367.0 + 3.8 \left(\frac{\alpha_S(\mu_R)}{0.108} \right)^2$$

- Non-factorisable corrections are **0.16 % LO** for $\mu_R = m_t$.
- p_{\perp}^t peaks around 40 GeV, changes sign around 70 GeV.
- For $\mu_R = 40 \text{ GeV}$ non-factorisable corrections are **0.25 % LO**.

$p_{\perp}^{t,\text{cut}}$	$\sigma_{\text{LO}} \text{ (pb)}$	$\mu_R = m_t$		$\mu_R = 40 \text{ GeV}$	
		$\sigma_{\text{NNLO}}^{\text{nf}} \text{ (pb)}$	$\delta_{\text{NNLO}} \text{ [%]}$	$\sigma_{\text{NNLO}}^{\text{nf}} \text{ (pb)}$	$\delta_{\text{NNLO}} \text{ [%]}$
0 GeV	2367.02	$3.79^{+0.84}_{-0.63}$	$0.16^{+0.04}_{-0.03}$	5.95	0.25
20 GeV	2317.03	$3.89^{+0.86}_{-0.64}$	$0.17^{+0.04}_{-0.03}$	6.11	0.26
40 GeV	2216.61	$4.14^{+0.92}_{-0.69}$	$0.19^{+0.04}_{-0.03}$	6.50	0.29
60 GeV	2121.88	$4.28^{+0.95}_{-0.71}$	$0.20^{+0.04}_{-0.03}$	6.71	0.32



Recent work on non-factorisable VBF Higgs production

presented in arXiv:2305.08016

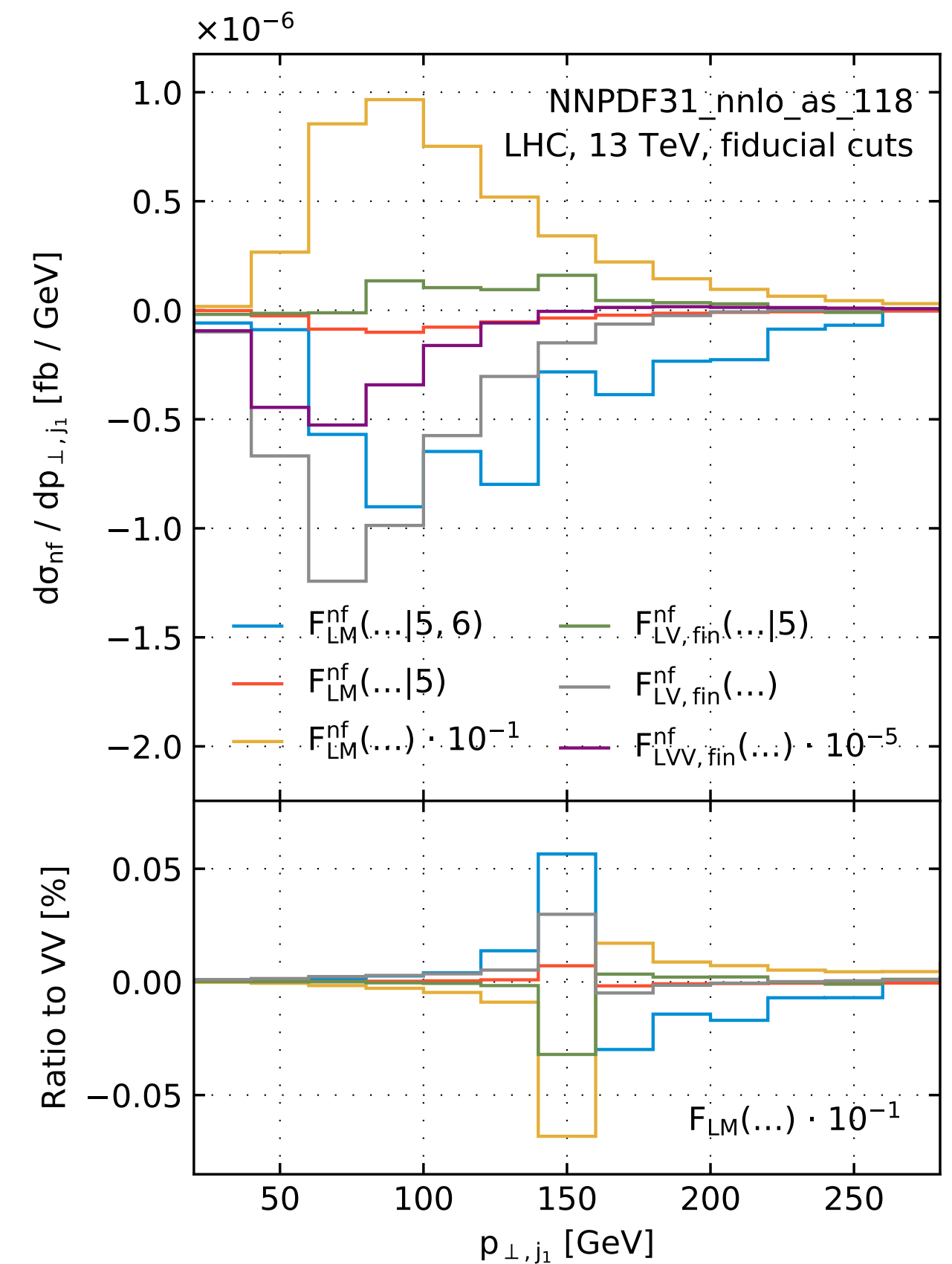
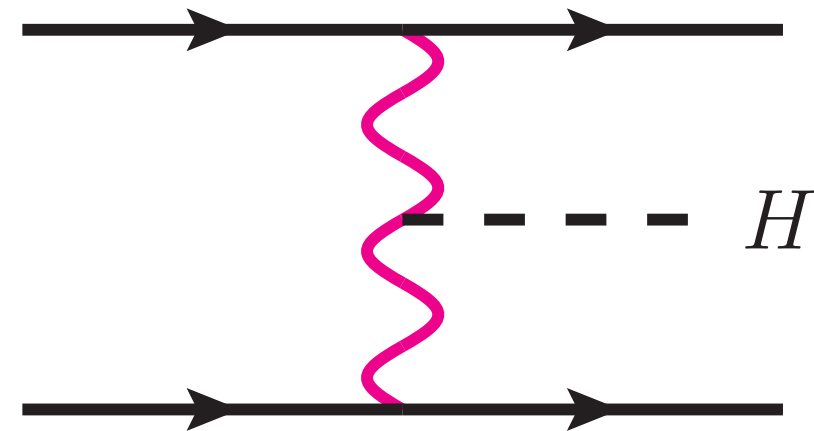
Differential cross section:

pp collision: $\sqrt{s} = 13 \text{ TeV}$, PDFs: NNPDF31-nnlo-as-118

$m_H = 125.0 \text{ GeV}$, $m_W = 80.398 \text{ GeV}$, $m_Z = 91.1876 \text{ GeV}$, $\alpha_S(m_Z) = 0.118$

$$\sigma_{\text{nf}} = -3.1 \text{ fb} \quad \mu_R = \mu_F = \sqrt{\frac{m_H}{2} \sqrt{\frac{m_H^2}{4} + p_{\perp,H}^2}}$$

- Non-factorisable corrections are **0.5 % of factorisable through NNLO**
- Double-virtual accounts for **99.99 %** due to strong suppression of other contributions within fiducial volume
- For $\mu_R = \mu_F$ scale variation is **$\mathcal{O}(40) \%$**



[Asteriadis, Brønnum-Hansen, Melnikov '23]

Thank you for your attention!